Number Systems

1

Unit Overview

In this unit you will extend your knowledge of numbers and expressions to the entire set of integers and develop an understanding of rational numbers. You will apply your understanding of rational numbers as you solve problems.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- critique
- ascend
- descend

Math Terms

- absolute value
- subset
- rational number
- terminating decimal
- repeating decimal

ESSENTIAL QUESTIONS

? Why is it important to understand properties and operations involving integers and negative rational numbers?

? How can models be used to interpret solutions of real-world problems?

EMBEDDED ASSESSMENTS

These assessments, following activities 2 and 4, will give you an opportunity to demonstrate how you can use your understanding of the number system to solve mathematical and real-world problems.

Embedded Assessment 1:

Positive Rational Numbers and Adding and Subtracting Integers p. 23

Embedded Assessment 2:

Rational Number Operations and Multiplying and Dividing Integers p. 47

UNIT 1

Getting Ready

1. Determine the value of each of the following expressions.

a. 32×21

b. $30{,}000 \div 10$

c. $478 + 593$

d. $101 - 68$

2. Determine the value of each of the following expressions.

a. 2.2×1.3

b. $39.5 + 8.74$

c. $33.4 - 2.11$

d. $470.4 \div 5.6$

3. Determine the value of each of the following expressions

a. $\frac{2}{5} + \frac{3}{10}$

b. $\frac{5}{6} - \frac{1}{3}$

c. $\frac{4}{5} \times \frac{7}{8}$

d. $\frac{6}{7} \div \frac{3}{4}$

4. Which property is illustrated by each example? Choose from the Associative, Commutative, and Distributive properties.

a. $6 + 8 = 8 + 6$

b. $(2 + 3) + 4 = 2 + (3 + 4)$

c. $2 \times 3 + 2 \times 5 = 2(3 + 5)$

5. Draw a number line like the one shown and graph the following points on the number line. Label each point with its letter.

a. 8 **b.** 3.5 **c.** $5\frac{1}{3}$

6. Order the following sets of numbers from least to greatest.

a. $\frac{1}{2}, \frac{2}{5}, \frac{3}{8}, \frac{7}{10}$

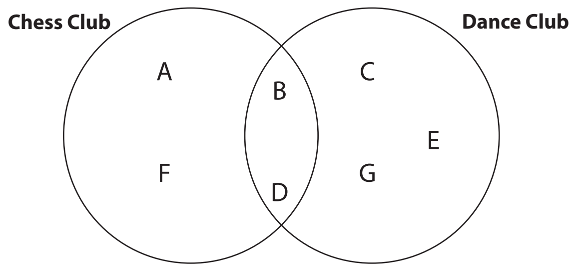
b. 32.51, 2.53, 514.37

7. Tell the value of each of the following expressions.

a. $|12|$ **b.** $|-13|$

c. $|-5| + |5|$ **d.** $|3 + 7| - |-7|$

8. This Venn diagram provides a visual representation of six students' memberships in after-school clubs. What does the diagram tell you about the club memberships of Student B and Student G? Explain.



Operations on Positive Rational Numbers

ACTIVITY 1

Paper Clips, Airplanes, and Spiders

Lesson 1-1 Adding and Subtracting Decimals

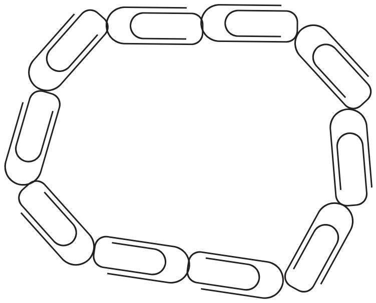
My Notes

Learning Targets:

- Solve problems with decimals, using addition and subtraction.
- Justify solutions with decimals, using addition and subtraction.
- Estimate decimal sums and differences.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Use Manipulatives, Think-Pair-Share, Discussion Groups

How long do you think it would take to make a paper clip chain that is 10 paper clips long? Last year, the student with the best time was able to do this in 26.25 seconds. Do you think you can do it in less time?



Work with your group. You will need

- Paper clips that are all the same size.
- A digital stopwatch that records time to the nearest hundredth of a second.

1. One at a time, each person makes a chain of 10 paper clips while the other students keep time with the stopwatch and record the amount of time. Fill in the times for your group in a chart like the one at the right.

Group Member	Time (in seconds)

2. Without computing an exact sum, estimate the total amount of time it took for your group to make their chains. How did you come up with this estimate?

3. Now compute the total time. Is your computed result reasonable? How can you justify your result?

4. Compare the fastest time in your group with last year's best time. Without computing an exact difference, estimate the difference in the times. How did you come up with this estimate?

DISCUSSION GROUP TIPS

If you do not understand something in group discussions, ask for help or raise your hand for help. Describe your questions as clearly as possible, using synonyms or other words when you do not know the precise words to use.

My Notes

ACADEMIC VOCABULARY

To **critique** is to analyze and discuss the details of something.

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  28.5
+29.75
   27
 32.87
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MATH TIP

These are some of the tools you can use to solve problems in this math course:

- calculator
- manipulatives
- pencil and paper

Which tool would you select to solve the problem in Item 7?

5. Now compute the difference. Is your computed result reasonable? How can you justify your result?

6. Critique the reasoning of others. Julio's group did the paper clip chain activity and got the following times (in seconds): 28.5; 29.75; and 27. He wrote the numbers in a column and added, as shown at the left. What error did he make? Write your answer here and also explain the error to your group using clear descriptions and correct math language.

7. What is the correct sum of the times for Julio's group?

8. Write a rule for Julio to use when adding or subtracting decimals so that he does not make this type of error again.

Check Your Understanding

Find each sum or difference. Justify your results.

9. $5.03 + 13.7 + 108$ **10.** $3.084 - 1.7$ **11.** $159 - 88.99$

12. Ping is buying a sandwich for $5.95 and a bottle of juice for $1.75. He is going to pay with a $10 bill.

a. How can he estimate how much change he should receive?

b. What is his exact amount of change?

LESSON 1-1 PRACTICE

Find each sum. Justify your results.

13. $9.08 + 14.6$ **14.** $12 + 1.12$ **15.** $7.009 + 2.02$

16. $0.66 + 6$ **17.** $11.05 + 14.6 + 46$ **18.** $59 + 5.9 + 0.59$

Find each difference. Justify your results.

19. $8.644 - 3.7$ **20.** $21.56 - 9.56$

21. $36.8 - 36.55$ **22.** $7 - 0.007$

23. Construct viable arguments. Theo bought these items: Shoes: $19.99; socks: $4.19; T-shirt: $8.50; pants: $27.75. How can he estimate the total cost?

24. Find the actual total cost of Theo's items.

25. Ana took Ali out for lunch. Their lunches cost $13.28 and $14.25, including tax and tip. Ana paid with two $20 bills. How much change did Ana receive?

My Notes

Learning Targets:

- Estimate decimal products and quotients.
- Solve problems involving multiplication and division of decimals.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Think-Pair-Share

Whitney did the paper clip chain activity but dropped some of the paper clips on the floor. The timekeeper in her group said that her time for completing the chain was 2.8 times as long as last year's best time of 26.25 seconds.

1. **Reason quantitatively.** Estimate the amount of time it took Whitney to complete her chain. How did you determine your estimate?

2. Explain what you already know about multiplying decimals.

You do not have to vertically align the decimal points when you multiply, but you do have to keep track of the number of decimal points in each of the numbers you multiply.

Example A

Find the exact amount of time it took Whitney to complete her chain.

Step 1: Set up the multiplication.

$$\begin{array}{r} 26.25 \\ \times\ 2.8 \\ \hline \end{array}$$

Step 2: Multiply. Locate the decimal point in the product.

$$\begin{array}{rl} 26.25 & \leftarrow 2 \text{ decimal places} \\ \times\ 2.8 & \leftarrow 1 \text{ decimal place} \\ \hline 21000 & \\ 5250 & \\ \hline 73.500 & \leftarrow 2 + 1 = 3 \text{ decimal places} \end{array}$$

Solution: It took Whitney 73.5 seconds to complete her chain. This should be close to your estimate in item 1 and therefore reasonable.

Try These A

Find each product. Justify your results.

a. 8.5×2.3 **b.** 0.03×14 **c.** 1.08×2.014

Check Your Understanding

3. Jerry multiplied 3.04×7.091 and got the product 2.155664. Is his answer reasonable? Why or why not?
4. Joanie multiplied 0.78×0.34 and got the product 26.52. What error did she make?

My Notes

MATH TIP

$0.045\overline{)1.8}$ is the same as the fraction $\frac{1.8}{0.045}$. When you multiply the numerator and denominator by the same number, the value does not change:

$$\frac{1.8 \times 1000}{0.045 \times 1000} = \frac{1800}{45}$$

You must also keep track of decimal points when dividing.

Example B

A new road is 1.8 km long. Each lot along the road will be 0.045 km long. How many lots will there be along the road?

Step 1: Set up the division. $0.045\overline{)1.8}$

Step 2: Multiply the divisor by 1000 to make 0.045 a whole number. You must also multiply the dividend, 1.8, by 1000. Then divide. Make sure to place the decimal point in the quotient above the decimal point in the dividend.

$$\begin{array}{r} 40. \\ 45\overline{)1800.} \\ \underline{180} \\ 00 \\ \underline{0} \\ 0 \end{array}$$

Solution: There will be 40 lots along the road.

Try These B

Find each quotient.

a. $300.6 \div 18$ **b.** $3.24 \div 3.6$ **c.** $28.8 \div 0.24$

Check Your Understanding

5. Curtis divided 27.16 by 2.8 and got 0.97. Is his answer reasonable? Why or why not?

6. Write a set of directions for dividing 3.6 by 0.25. Then find the quotient.

LESSON 1-2 PRACTICE

Find each quotient.

7. $601.2 \div 18$ **8.** $3.24 \div 7.2$

9. $80 \div 32$ **10.** $7.2 \div 0.12$

11. Josiah paid $19.75 for 2.5 pounds of coffee beans. What was the cost of the beans per pound?

12 Keisha bought 1.2 pounds of Swiss cheese that was selling for $5.95 per pound. How much did Keisha pay for the Swiss cheese?

13. Make sense of problems. Ralph has a spool with 9.8 meters of wire. How many 0.14 meter pieces of wire can he cut from the spool?

My Notes

Learning Targets:

- Solve problems with fractions using addition, subtraction, multiplication, and division.
- Estimate with fractions.

SUGGESTED LEARNING STRATEGIES: Use Manipulatives, Create Representations

How far can you throw a paper airplane? According to a recent entry in Guinness Book of World Records, the record holder threw a paper airplane a distance of $207\frac{1}{3}$ feet.

Work with your group to make a paper airplane. Listen to group members' ideas and share your own. Ask and respond to questions to help the group accomplish this task. Your teacher will give you a set of directions on how to make an airplane if you need one.

Test your airplane. Mark a starting line on the classroom floor, and then measure the distance the plane flies to the nearest $\frac{1}{12}$ of a foot. Record the three best distances in the table in the My Notes space.

Best Distances (ft)

1. Write, but do not evaluate, expressions that could be used to answer each question.
   a. What was the distance between the record and your best distance?

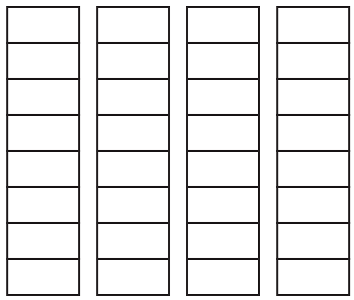
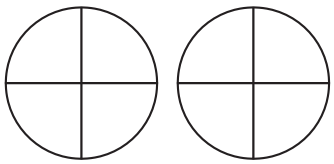
   b. If another group had a best distance that was $1\frac{1}{4}$ times your group's best distance, what would that distance be?

   c. How many times your group's best distance is the world record?

   d. What is the average of your three best distances?

My Notes

2. In the table below, represent the processes for operations on fractions with models, numbers, and words. Shade or mark the models to show each operation. Then use words to explain the process. Finally, find the answer to the operation.

Operation	With Model	Explanation in words	Answer
a. $\frac{17}{8} + \frac{11}{8}$			
b. $1\frac{1}{4} - \frac{3}{4}$			

3. How is the process of adding $\frac{3}{8} + \frac{1}{4}$ different from the addition shown in 2a?

Check Your Understanding

On a middle school track team, the record time for a completing an obstacle course is $8\frac{1}{3}$ minutes.

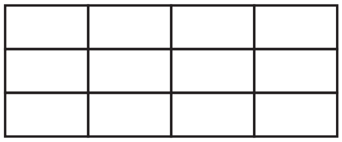
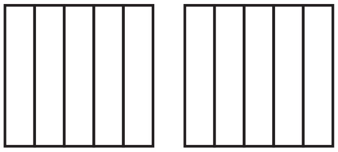
4. Alana's time for completing the obstacle course was $13\frac{1}{3}$ minutes. What is the difference between Alana's time and the record time?

5. Leesa's best time for completing the obstacle course was $10\frac{1}{2}$ min, and Sandy's best time was $9\frac{1}{5}$ min. What was the total of their best times?

6. How much less is the team record time than the sum of Leesa's time and Sandy's time?

My Notes

7. In the table below, represent the processes for operations on fractions with models, numbers, and words. Shade the models to show each operation. Then use words to explain the process. Finally, find the answer to the operation.

Operation	With Model	Explanation in words	Answer
a. $\frac{1}{4} \cdot \frac{2}{3}$			
b. $1\frac{3}{5} \div \frac{2}{5}$			

8. Yanni threw his paper airplane $15\frac{1}{2}$ feet. Adrian threw his paper airplane $\frac{3}{4}$ of Yanni's distance. What is the distance Adrian threw his paper airplane? Explain how you found your answer.

9. Mr. Adams has poster paper that is $10\frac{3}{4}$ feet long. He wants to make sheets that are $\frac{1}{4}$ of a foot long to make paper airplanes. How many sheets can he make?

My Notes

Check Your Understanding

Evaluate each expression you wrote for item 1 of this lesson to determine how close you are to setting a new Guinness World Record for throwing a paper airplane. Show your work.

10. What is the difference between the record and your best distance?

11. If another student threw a paper airplane $1\frac{1}{4}$ times farther than you did, what would that distance be?

12. How many times farther than your distance was the record holder able to throw the paper airplane?

13. Find your average distance by calculating the mean of the three trials.

LESSON 1-3 PRACTICE

Find each sum or difference.

14. $35\frac{3}{4}+18\frac{3}{4}$

15. $\frac{5}{6}+\frac{1}{3}$

16. $11\frac{1}{12}+\frac{7}{8}$

17. $\frac{13}{17}-\frac{5}{17}$

18. $12\frac{2}{9}-\frac{5}{6}$

19. $41\frac{9}{11}-27\frac{1}{3}$

20. The recipe for a cake calls for the following dry ingredients: $\frac{1}{3}$ cup of sugar; $\frac{3}{4}$ cup of cornmeal; and $\frac{1}{2}$ cup of flour. What is the total number of cups of dry ingredients called for?

21. In 1935, American athlete Jesse Owens set a world record for the long jump by jumping 26 ft $8\frac{1}{16}$ in. In 1960, Ralph Boston set a new world record by jumping 26 ft $11\frac{1}{4}$ in. How much longer was Ralph Boston's jump?

Find each product or quotient.

22. $\frac{1}{8}\cdot 5$

23. $\frac{7}{10}\cdot 3\frac{3}{7}$

24. $9\frac{2}{7}\cdot 2\frac{2}{13}$

25. $\frac{3}{8}\div 2$

26. $\frac{2}{3}\div\frac{4}{7}$

27. $1\frac{5}{6}\div 5\frac{2}{5}$

28. Lilly jogged $3\frac{1}{4}$ miles each day for 24 days last month. How many miles did she jog in all?

29. Lester jogs $5\frac{3}{4}$ miles on each day that he jogs. Last month, he jogged a total of 115 miles. How many days did he jog last month?

30. **Reason quantitatively.** Parmesan cheese was on sale for \$13.60 per pound. Wesley bought a piece of the Parmesan cheese that weighed $1\frac{1}{8}$ pounds. How much did he pay?

Learning Targets:

- Convert a fraction to a decimal.
- Understand the difference between terminating and repeating decimals.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Think-Pair-Share

Sarai is researching spiders. She read that outside the United States, it is not unusual to find a camel spider that is $6\frac{3}{8}$ inches long. Her classmate Akeem is researching insects. He read an article about an insect known as a titan beetle that was $6\frac{1}{3}$ inches long.

It can sometimes be helpful to compare numbers expressed in fraction form by converting the fractions to decimals. Some decimal forms of fractions *terminate*, and some decimal forms *repeat*.

Example A

Express $6\frac{3}{8}$, the length in inches of the camel spider Sarai researched, as a decimal.

Step 1: Write the mixed number $6\frac{3}{8}$ as an improper fraction.

$$6\frac{3}{8} = 6 + \frac{3}{8} = \frac{48}{8} + \frac{3}{8} = \frac{51}{8}$$

Step 2: Divide the numerator by the denominator.

$$\begin{array}{r} 6.375 \\ 8\overline{)51.000} \\ -48 \\ 30 \\ -24 \\ 60 \\ -56 \\ 40 \\ -40 \\ 0 \end{array}$$

The remainder is 0, so the decimal form of $6\frac{3}{8}$ is a ***terminating decimal***.

Solution: The decimal form of $6\frac{3}{8}$ is 6.375.

My Notes

CONNECT TO SCIENCE

Spiders belong to the class Arachnida and are commonly mistaken for insects. One major difference between arachnids and insects is that arachnids have eight legs and insects have six legs.

MATH TERMS

A **terminating decimal** has a finite or limited number of digits following the decimal point.

My Notes

Example B

Express $6\frac{1}{3}$, the length in inches of the titan beetle, as a decimal.

Step 1: Write the mixed number as an improper fraction.

$$6\frac{1}{3} = 6 + \frac{1}{3} = \frac{18}{3} + \frac{1}{3} = \frac{19}{3}$$

Step 2: Divide the numerator by the denominator.

$$\begin{array}{r} 6.333 \\ 3\overline{)19.000} \\ \underline{-18} \\ 10 \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 10 \\ \underline{-9} \\ 1 \end{array}$$

The remainder repeats so the digits in the quotient repeat. The quotient is a ***repeating decimal***.

The *bar notation* indicates which digits repeat.

Solution: The decimal form of $6\frac{1}{3}$ is $6.\overline{3}$.

MATH TERMS

A **repeating decimal** has one or more digits following the decimal point that repeat endlessly.

Try These A–B

Express each mixed number as a decimal. Indicate whether the decimal is terminating or repeating.

a. $7\frac{7}{8}$ **b.** $2\frac{1}{6}$ **c.** $5\frac{3}{4}$ **d.** $12\frac{5}{9}$

DISCUSSION GROUP TIPS

As you interact with your group in solving problems, you may hear math terms and other words that may be new to you. As for clarification of their meaning, and make notes to help you learn and use vocabulary heard during classroom instruction and interactions.

Check Your Understanding

1. Compare the answers of Examples A and B. Which answer is greater? How do you know?
2. **Critique the reasoning of others.** Nathan converted $\frac{5}{11}$ to a repeating decimal and wrote $0.4\overline{5}$ as the answer. What error did he make?

CONNECT TO AP

In Calculus, answers are rounded to three decimal places.

LESSON 1-4 PRACTICE

Express each fraction or mixed number as a decimal. Identify the repeating decimals.

3. **a.** $\frac{2}{3}$ **b.** $\frac{5}{8}$ **c.** $\frac{4}{5}$
4. **a.** $3\frac{3}{16}$ **b.** $8\frac{2}{9}$ **c.** $11\frac{7}{11}$
5. Which is greater, 0.32 or $0.\overline{3}$? How do you know?
6. Philip takes $2\frac{3}{5}$ hours to clean his room. Ashton takes $2\frac{5}{8}$ hours to clean his room. Who took less time to clean up his room?
7. **Look for and make use of structures.** What kinds of denominators generate repeating decimals?

ACTIVITY 1 PRACTICE

Lesson 1-1

In items 1–4, estimate each sum or difference. Explain how you determined your estimate.

1. $3.77 + 1.39$
2. $4.35 + 3.8 + 4.129 + 3.672$
3. $17.129 - 9.7$
4. $38.8 - 12.2$

Evaluate each expression in items 5–8.

5. $2.9 + 0.29$
6. $0.34 + 495.5 + 99.008$
7. $87.6 - 53.909$
8. $48 - 0.48$
9. At one time, the world record for running 100 yd backward was 13.5 seconds. If the record is now 12.7 seconds, how many seconds faster is the new record?
10. In 1985, American swimmer Tom Jager completed a 50-meter freestyle swim in 22.40 seconds. In 1990, he was able to complete the swim in 21.81 seconds. How many seconds slower was his 1985 swim?
11. Linda is running in a marathon, which is 26.2 miles long. Checkpoint 1 is 3 miles past the start; checkpoint 2 is 2.5 miles after checkpoint 1; and checkpoint 3 is 3.75 miles after checkpoint 2. When Linda makes it to checkpoint 3, how many miles does she have to run to complete the marathon?

Lesson 1-2

Evaluate each expression in items 12–15.

12. $1.4 \cdot 27$
13. $0.17 \cdot 0.6$
14. $14.127 \div 5.1$
15. $6.58 \div 9.4$
16. Without doing the computation, explain why or why not 12.702 is a reasonable value for the expression $5.8 \cdot 2.19$.
17. Without doing the computation, explain why or why not 14.766 is a reasonable value for the expression $3.21 \cdot 0.46$.
18. Without doing the computation, explain why or why not 19.7 is a reasonable value for the expression $122.14 \div 6.2$.
19. Three people bought books for a total of \$12.42. If they shared the cost equally, how much did each person pay?
    A. \$6.21 **B.** \$4.14
    C. \$4.00 **D.** \$4.52
20. Cheryl makes \$8.40 an hour. If she works 10.75 hours in a week, how much will she earn for the week?
    A. \$9.30 **B.** \$90.30
    C. \$900.30 **D.** \$9000.30
21. Daniel is buying a video game that costs \$52.99. The sales tax is found by multiplying the cost of the video game by 0.07. How much is the sales tax for the video game? What is the total cost, including tax?

22. Cory earns $9.50 per hour for the first 40 hours he works in a week. For any hours over 40 hours per week, his hourly rate is multiplied by 1.5. How much does he earn if he works 43.5 hours in one week?

Lesson 1-3

Evaluate each expression in items 23–26.

23. $4\frac{1}{2}+1\frac{2}{7}+3\frac{1}{3}$

24. $132\frac{1}{6}-99\frac{5}{6}$

25. $\frac{1}{10}\cdot\frac{3}{11}$

26. $21\div3\frac{1}{2}$

27. A machine can make a box in $1\frac{3}{10}$ seconds. How many boxes can the machine make in 1 hour?

28. Carrie has a 10-ft plank of wood. She wants to cut 3 pieces that are each $2\frac{2}{3}$ feet long from the plank. How long will the plank be after she cuts off the three pieces?

29. A large carton of juice holds 12 cups. How many $\frac{3}{4}$-cup servings does the carton hold?

30. Gary is $61\frac{1}{8}$ inches tall. His friends Gino and Gilbert are $56\frac{1}{2}$ inches tall and $63\frac{1}{8}$ inches tall. What is the average height of the three friends?

31. Can you think of situations in which it might be preferable to compute with decimals rather than fractions or to compute with fractions rather than decimals? Give examples of each situation and tell why you think that number form is preferable.

Lesson 1-4

For items 32–37, write the fraction as a decimal. Then identify the decimal as terminating or repeating.

32. $\frac{3}{5}$

33. $\frac{1}{6}$

34. $\frac{5}{9}$

35. $\frac{9}{20}$

36. $\frac{13}{25}$

37. $\frac{10}{11}$

38. Which fraction is equivalent to a repeating decimal?

A. $\frac{3}{12}$

B. $\frac{6}{12}$

C. $\frac{8}{12}$

D. $\frac{9}{12}$

39. Order the numbers from least to greatest:

$1\frac{4}{5}, 1.78, 1\frac{5}{6}, \frac{7}{4}, 1.\overline{7}, 1\frac{8}{11}$

40. Two turtles are competing in a race. Turtle A reaches the finish line in $1\frac{3}{7}$ hours. Turtle B finished in $1\frac{2}{5}$ hours. Which turtle had the faster time?

41. Emily says that she can convert $\frac{18}{25}$ to a decimal by using equivalent fractions instead of dividing 18 by 25. Use Emily's method to convert $\frac{18}{25}$ to a decimal.

MATHEMATICAL PRACTICES
Critique the Reasoning of Others

42. Nilsa converted $\frac{1}{12}$ to a repeating decimal and wrote $0.\overline{083}$ as the answer. What error did she make?

Addition and Subtraction of Integers

Elevation Ups and Downs

Lesson 2-1 Adding Integers

ACTIVITY 2

My Notes

Learning Targets:

- Add two or more integers.
- Identify and combine opposites.
- Solve real-world problems by adding integers.

SUGGESTED LEARNING STRATEGIES: Close Reading, Marking the Text, Create Representations, Quickwrite

A passenger jet that ascends $+5$ miles and then descends -3 miles will end at an elevation 2 miles above where it began.

$$+5 + (-3) = +2$$

A similar relationship holds in chemistry. An ionic bond is formed by an attraction between two oppositely charged ions. *Cations* are positively charged ions, and *anions* are negatively charged ions. Sodium (Na) has one cation with a $+1$ charge, and chlorine (Cl) has one anion with a -1 charge. When put together, sodium chloride (NaCl), table salt, is formed, and it has a charge of 0.

$$+1 + (-1) = 0$$

CONNECT TO SCIENCE

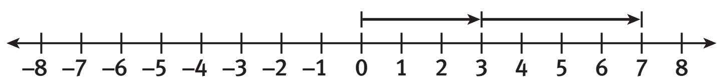
Cations and anions are the building blocks of molecules, which are the building blocks of all matter in the universe.

1. Write an equation to represent the resulting charge when each of the following ionic bonds of cations and anions are formed.

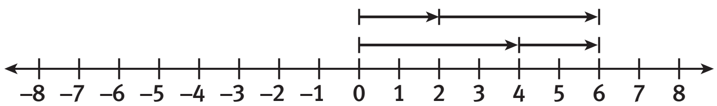
a. 5 cations and 3 anions **b.** 2 cations and 7 anions

The equations you wrote are examples of integers being added. One way to visualize integer addition is to use number lines. You can then connect the number line representations to equations and develop rules for adding integers.

2. Explain how the number line shows the sum of 3 and 4. What is the sum? Write the equation.

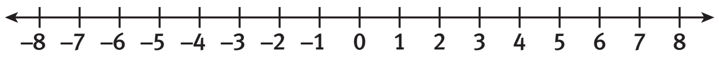


3. What property of addition is shown by the number line? Explain your reasoning.



My Notes

4. Use the number line to find the sum $(-3) + (-5)$.



$(-3) + (-5) =$ ______

Your results can be summarized with this rule:

- To add two integers with the same sign, add the ***absolute values*** of the integers. The sum has the same sign as the addends.

MATH TERMS

The **absolute value** of a number is its distance from zero on a number line. Distance, or absolute value, is always positive, so $|-6| = 6$ and $|6| = 6$.

Example A

Add: $15 + 23$

The signs are the same, so add the absolute values.

$$|15| + |23| = 15 + 23 = 38$$

Since both addends are positive, the sum is positive.

Solution: $15 + 23 = +38$

Example B

Add: $(-12) + (-7)$

The signs are the same, so add the absolute values.

$$|-12| + |-7| = 12 + 7 = 19$$

Since both addends are negative, the sum is negative.

Solution: $(-12) + (-7) = -19$

Try These A–B

Add.

a. $(-14) + (-36)$

b. $19 + 16$

c. $26 + 45$

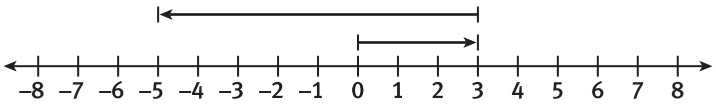
d. $(-28) + (-28)$

5. A scuba diver descended to an elevation of -43 feet, stopped descending, and then descended 17 feet more. What was the diver's final elevation?

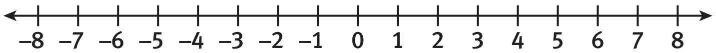
My Notes

You can also use a number line to add two integers with different signs.

6. **Model with mathematics.** Explain how the number line shows the sum of 3 and -8. What is the sum?



7. Draw arrows and use the number line below to find the sum $(-6) + 9$.



$(-6) + 9 =$ ______

Your results can be summarized with this rule:

- To add two integers with different signs, find the difference of the absolute values of the integers. The sum has the sign of the integer with the greater absolute value.

MATH TIP

These are some of the tools you can use to solve problems in this math course:

- calculator
- manipulatives
- pencil and paper

Which tool would you select to solve the problem in Item 7?

Example C

Add: $-13 + 8$

The signs of the addends are different. Find the difference of the absolute values: $|-13| - |8| = 5$

Use the sign of the integer with the greater absolute value. The integer with the greater absolute value is -13, so the sum is negative.

Solution: $-13 + 8 = -5$

Try These C

Add.

a. $21 + (-14)$ **b.** $11 + (-17)$ **c.** $(-32) + 19$

8. Why are 89 and -89 called opposites? Use a number line to explain.

9. Find the sum of 89 and -89.

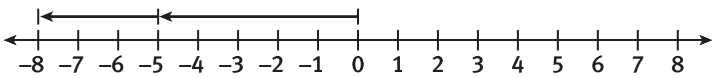
My Notes

10. Why do you think that 89 and -89 are called additive inverses?

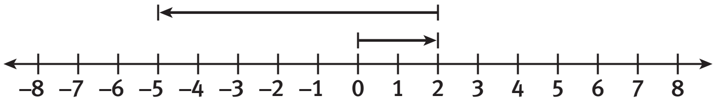
Check Your Understanding

Write the sum shown by the arrows.

11.



12.



13. What is the sum of any integer and its opposite?

14. Identify a real-life situation where opposite quantities combine to make 0.

LESSON 2-1 PRACTICE

Find each sum.

15. Add.

a. $-21 + 25$ **b.** $(-13) + (-21)$

c. $46 + (-58)$ **d.** $(-39) + 16$

e. $28 + (-24) + (-3)$ **f.** $15 + (-42) + (-5)$

16. A mountain climber camped at an elevation of 18,492 feet. The following day the climber descended 2,516 feet to another campsite. Write a numerical expression you can evaluate to find the elevation of the second campsite. Then find the elevation.

17. Explain how to determine if the sum of two integers with different signs is positive or negative.

18. **Reason quantitatively.** If you stood at sea level, the base of the Hawaiian volcano Mauna Kea would be at the bottom of the ocean, at 19,680 feet below you. The top would be 33,476 feet above the base. Write a numerical expression you can evaluate to find the elevation of the top of Mauna Kea above sea level. Then find the elevation.

19. Justify Steps 1 and 2 in the evaluation of the expression $5 + ((-7) + 3) + (-6)$.

Step 1 $5 + ((-7) + 3) + (-6) = 5 + (3 + (-7)) + (-6)$

Step 2 $= (5 + 3) + (-7) + (-6)$

$= 8 + (-13)$

$= -5$

Lesson 2-2
Subtracting Integers

My Notes

Learning Targets:

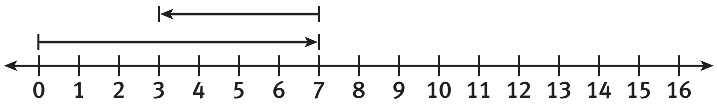
- Subtract integers.
- Find distances using absolute value.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations, Quickwrite

Long before you knew anything about integers, you were able to subtract whole numbers.

$$7 - 4 = 3$$

You may have used a number line to show subtraction.



1. Compare the above graph with the one you would draw to find the sum $7 + (-4)$.

2. Compare the graph you would draw to find $12 - 5$ (the difference between the whole numbers 12 and 5) and the one you would draw to find the sum $12 + (-5)$.

MATH TIP

These are some of the tools you can use to solve problems in this math course:

- calculator
- manipulatives
- pencil and paper
- ruler

Which tools would you select to solve the problem in Item 2?

These examples show that you can convert a subtraction problem to an addition problem: $9 - 2 = 9 + (-2)$. This leads to the rule:

- To subtract an integer, add its opposite.

Example A

Subtract: $-12 - (-5)$

Step 1: To -12, add the opposite of -5. $-12 - (-5) = -12 + 5$

Step 2: Find the difference of the absolute values. $|-12| - |5| = 12 - 5 = 7$

Step 3: Use the sign of the integer with the greater absolute value. $|-12| > |5|$, so $= -12 + 5 = -7$

Solution: $-12 - (-5) = -7$.

Try These A

Subtract.

a. $16 - (-4)$ **b.** $7 - 12$ **c.** $-9 - 9$

My Notes

Check Your Understanding

3. Write the subtraction problem as an addition problem.

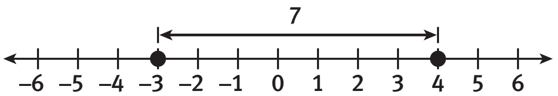
a. $19 - 6$ **b.** $-4 - (-8)$ **c.** $-3 - 5$
d. $0 - 12$ **e.** $13 - 14$ **f.** $-2 - (-2)$

4. Subtract.

a. $14 - 7$ **b.** $-11 - (-7)$ **c.** $-12 - 8$
d. $6 - (-6)$ **e.** $21 - 30$ **f.** $-17 - (-20)$

5. Tristan rewrote the expression $6 - (-8)$ as $6 - (+8)$. Was he correct? Why or why not?

You can find the *distance* between -3 and 4 by counting the number of units from -3 to 4 on a number line. The distance is 7 units.

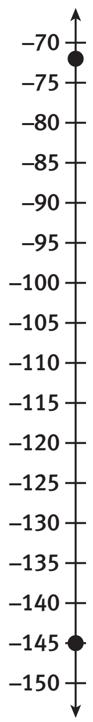


Another way to find the distance is to find the absolute value of the difference of -3 and 4.

$$|-3 - 4| = |-7| = 7$$

The order of the subtraction does not matter. The result will be the same:

$$|4 - (-3)| = |4 + (+3)| = |7| = 7$$



Example B

A team of divers was at an elevation of 145 feet below the surface of the water, or -145 ft. Another team was directly above the first team at an elevation of -72 ft. What was the distance between the teams?

Step 1: Visualize the problem.

Think of a vertical number line with points at -145 and -72.

Step 2: Write and evaluate an absolute value expression to find the distance.

$$|-145 - (-72)| = |-145 + (72)| = |-73| = 73$$

Solution: The distance between the teams is 73 feet.

Try These B

Find the distance between each pair of numbers.

a. -34 and 7 **b.** -42 and -78 **c.** 29 and 4

My Notes

Check Your Understanding

Write an absolute value expression you can use to find the distance between each pair of numbers. Then find the distance.

6. 15 and −15
7. −47 and 53
8. −24 and −42
9. Howard needs to find the distance between 178 and −395 on a number line. Write two absolute value expressions Howard can write to find the distance.

LESSON 2-2 PRACTICE

10. At noon, a hot-air balloon was at an elevation of 2,400 feet. One hour later it was at an elevation of 1,700 feet.
    a. Write a numerical expression you can use to find the change in altitude from noon to 1 P.M.
    b. Evaluate your expression and explain what it means.
11. Yesterday's high temperature was −8°F. Today's high temperature is −3°F.
    a. Write a numerical expression you can use to find the change in temperature from yesterday to today.
    b. Evaluate your expression and explain what it means.
12. A submarine is at −750 feet, or 750 feet below sea level. It descended 300 feet, then ascended 550 feet, and then descended 425 feet.
    a. Write and evaluate a numerical expression to find the submarine's final elevation.
    b. The submarine next descended to the ocean floor, which was at an elevation of −2,250 feet. How far did the submarine descend?
13. **Reason abstractly.** If you subtract a negative number from another number, will the other number increase or decrease? Explain.
14. Justify Steps 1 and 2 in the evaluation of the expression 23 − 48 + 7.

Step 1: $23 - 48 + 7 = 23 + (-48) + 7$
Step 2: $= 23 + 7 + (-48)$
$= 30 + (-48)$
$= -18$

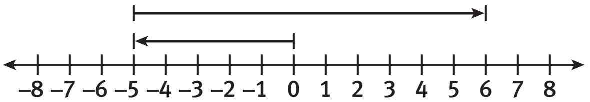
Classify each statement in Items 15–16 as *true* or *false*. If false, explain why.

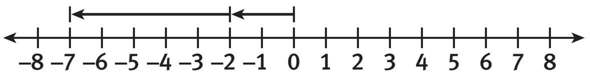
15. The difference of two integers can never be 0.
16. The difference of two negative integers is always a negative integer.
17. Which expression can you use to find the distance between 19 and −31?
    A. $|19 - 31|$ **B.** $|19| - |31|$
    C. $|-31 - 19|$ **D.** $|-31| - |19|$

ACTIVITY 2 PRACTICE

Lesson 2-1

For Items 1–2, write the sum shown by the arrows.

1.


2.


For Item 3–6, draw a number line from −8 to 8. Illustrate the move along the number line to find each sum.

3. $5 + (-7)$
4. $-5 + 3$
5. $-6 + 10$
6. $-2 + (-5)$

For Item 7 and 8, write an addition expression to represent each problem situation. Then solve the problem by finding the sum.

7. At 8:00 A.M., the temperature was −6°F. By noon, the temperature had risen by 9°F. What was the temperature at noon?
8. Jamal reached into a bag and pulled out a handful of counters. He pulled out 16 negative counters and 27 positive counters. What was the combined value of the counters?

Classify each statement in Item 9–10 as *true* or *false*. If false, explain why.

9. The sum of two integers cannot be 0.
10. The sum of two negative integers is always a negative integer.
11. What number must you add to −6 to get a sum of zero? Explain.

In Items 12–16, find each sum.

12. $56 + (-48) + (-30)$
13. $-45 + (-45) + (-45)$
14. $97 + (-112) + 15$
15. $-38 + 7 + 59$
16. $-154 + (-89) + 226$

Lesson 2-2

Write each subtraction problem as an addition problem. Then find the difference.

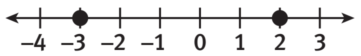
17. $5 - 7$
18. $4 - (-3)$
19. $-6 - 1$
20. $-2 - (-5)$

For Items 21–22, write a subtraction expression to represent each problem situation. Then solve the problem by finding the difference.

21. At 8:00 P.M., the temperature was 16°F. By midnight, the temperature had fallen by 19°F. What was the temperature at midnight?
22. Gina was touring New Orleans, which has an elevation of 5 feet below sea level, or −5 ft. A helicopter flew over her at an elevation of 186 ft. How far above the ground where Gina was standing was the helicopter?

In Item 23–24, evaluate each expression.

23. $132 - 178 + 59$
24. $-6.75 + 8 - 2.2$
25. What number must you subtract from −13 to get a difference of 0? Explain.
26. Identify a situation involving money where opposites combine to make 0.
27. Write and evaluate an absolute value expression to find the distance between the two points graphed below.



MATHEMATICAL PRACTICES
Model with Mathematics

28. Which expression can you use to find the distance between 28 and −53?
 A. $|28 - 53|$
 B. $|28| - |53|$
 C. $|-53 - 28|$
 D. $|-53| - |28|$

Positive Rational Numbers and Adding and Subtracting Integers

OFF TO THE RACES

Embedded Assessment 1
Use after Activity 2

Write your answers on notebook paper. Show your work.

The Middle School Track and Field Championships are held every year on the last day of school. The table gives the best times and distances in three events from previous years.

Event	Record
100-Meter Run	13.76 sec
400-Meter Run	1 min, 5.21 sec
High Jump	$5\frac{7}{12}$ ft

1. In his three high jumps, Kevin jumped $4\frac{3}{4}$ feet, $4\frac{5}{6}$ feet, and $4\frac{2}{3}$ feet.

a. Find the mean of the heights. Explain how you found the answer.

b. Estimate how much higher than his best jump Kevin would have had to jump to tie the record. Explain how you made your estimate.

c. How much higher than his best jump would Kevin have to jump to tie the record? Find the exact answer.

d. Consider only the fractional parts of the three mixed numbers that make up Kevin's three heights. Find the fractions which, written as decimals, would be repeating decimals, and write them as repeating decimals.

2. Elena completed the 100-meter run in 15.58 seconds.

a. How much faster would she have had to run to tie the record?

b. If she could have run 400 meters at the same rate as she ran 100 meters, would she have broken the record? Find the difference between her time for 400 meters and the record time.

c. The 400-meter run consists of four laps around a 100-meter track. What was the record holder's average time per lap?

Times and distances are sometimes given by comparing them with the record for the event. A negative number indicates the amount by which a record has been broken. A positive number indicates the amount by which the record has failed to be broken.

3. In the discus throw, Devan scored 7, Joel scored +15, and Greg scored the opposite of Devan.

a. By how much did Greg's distance exceed Joel's?

b. Leo's score was 4 less than Greg's. What was Leo's score?

c. Order the scores from greatest to least.

4. Explain how you can use absolute value to compare a score with the record for the event when scores are given as integers.

Positive Rational Numbers and Adding and Subtracting Integers

OFF TO THE RACES

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-d, 2a-c, 3a-c, 4)	• Clear and accurate understanding of operations with fractions, decimals, and integers. • Effective understanding and accuracy in ordering and comparing integers.	• Operations with fractions, decimals, and integers that are usually correct. • Correct comparison of integers by ordering a set or using absolute value.	• Operations with fractions, decimals, and integers that are sometimes correct. • Partially correct comparison or ordering of integers; incorrect use of absolute value.	• Incorrect or incomplete computation in operations with fractions, decimals, and integers. • No comparison or ordering of integers.
Problem Solving (Items 1d, 2a-c, 3a-b)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a-d, 2a-c, 3a-c, 4)	• Clear and accurately written expressions involving operations with fractions, decimals, and integers. • Clear and correct ordering and comparison of integers. • Correct use of absolute value to compare scores.	• Some difficulty in writing the best expression for a problem situation, but can get correct answers. • Correct conversion of fractions to decimals. • An understanding of ordering integers. • An understanding of absolute value.	• Errors in writing expressions for a given problem situation. • Errors in ordering rational numbers (for example, orders least to greatest instead of greatest to least). • Incorrect use of absolute value to compare scores.	• Inaccurately written expressions. • Inaccurate conversion of fractions to decimals. • Incorrect ordering of rational numbers. • Little or no understanding of absolute value.
Reasoning and Communication (Items 1a-b, 4)	• Precise use of appropriate math terms and language to explain finding a mean and estimating a difference. • A thorough understanding of using absolute value to compare scores.	• An adequate explanation of finding a mean and estimating a difference. • An adequate explanation of how to use absolute value to compare scores.	• A misleading or confusing explanation of finding a mean or estimating a difference. • Partial understanding of absolute value.	• An incomplete or inaccurate description of finding a mean or estimating a difference. • Little or no understanding of absolute value.

Multiplication and Division of Integers

What's the Sign?

Lesson 3-1 Multiplying Integers

Learning Targets:

- Multiply two or more integers.
- Apply properties of operations to multiply integers.
- Solve real-world problems by multiplying, adding, and subtracting integers.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Summarizing, Paraphrasing, Create Representations

Kaleena's brother is a helicopter pilot who performs rescue operations for the Coast Guard. Kaleena is doing research to learn how a helicopter moves up and down. She learns that the helicopter her brother flies takes about 3 minutes to ***ascend*** to an altitude of 900 feet from ground level.

1. What is the vertical rate of ascent, in feet per second, when a helicopter ascends 900 feet in 3 minutes?

2. Would it be more appropriate to represent this rate of ascent as a positive integer or a negative integer? Explain your reasoning.

3. What is the vertical rate of descent, in feet per second, when a helicopter descends 900 feet in 5 minutes?

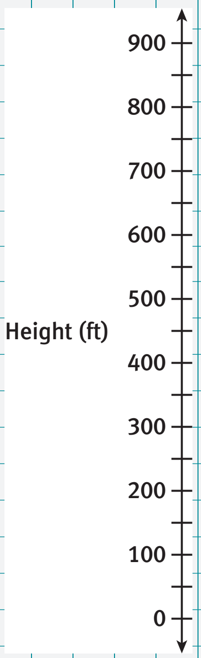
4. Would it be more appropriate to represent this rate of descent as a positive integer or a negative integer? Explain your reasoning.

My Notes

ACADEMIC VOCABULARY

Ascend means to "move upward." ***Descend*** means "to move downward."

My Notes



MATH TIP

These are some of the tools you can use to solve problems in this math course:

- calculator
- manipulatives
- pencil and paper

Which tool would you select to draw diagrams in Items 6 and 7?

Kaleena's brother sent her a video of him taking off and ascending. Kaleena decides to represent the ascent of the helicopter using a vertical number line.

5. Use the rate of ascent you found in Item 1. On the number line at the left, mark the height of the helicopter at 10-second intervals, from 0 to 3 minutes.

6. Use △+ to represent the helicopter ascending for 10 seconds at the rate you found in Item 1. Draw a diagram to represent the total ascent of the helicopter.

7. Use ▽− to represent the helicopter descending for 10 seconds at the rate you found in item 3. Draw a diagram to represent the total descent of the helicopter.

In Items 6 and 7, you represented multiplication of positive and negative numbers using triangle symbols. You can also use counters to represent multiplication problems.

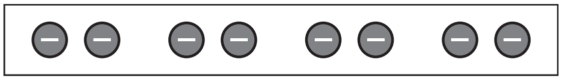
8. If ⊖ represents −10, what does ⊖⊖⊖ represent?

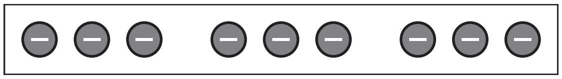
9. Use multiplication to write an equation illustrated by each diagram. Each counter stands for 10.

a. ⊕ ⊕ ⊕ ⊕ ⊕ ⊕

b. ⊖ ⊖ ⊖ ⊖ ⊖ ⊖

My Notes

c. 

d. 

10. Use your results from Item 9 to answer the following.

a. What is the sign of the product of a positive integer and a positive integer?

b. What is the sign of the product of a positive integer and a negative integer?

11. a. To find the sign of the product of two negative integers, start by filling in all the squares in the multiplication table below except for the 9 shaded squares in the lower right corner of the table.

·	3	2	1	0	−1	−2	−3
3							
2							
1							
0							
−1							
−2							
−3							

b. Now use patterns in the rows and columns you completed to fill in the 9 shaded squares.

12. What patterns did you see in the table that helped you fill in the shaded squares?

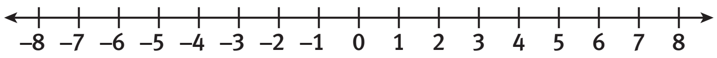
13. What rule can you use to multiply two negative integers?

My Notes

Check Your Understanding

14. Find each product.
a. $-5(7)$ **b.** $9 \cdot 5$ **c.** $-8(-8)$ **d.** $12(-4)$

15. Show how to find the product $4(-2)$ using the number line.



What is the product $4(-2)$?

16. Evaluate.
a. $(-1)^2$ **b.** $(-1)^3$ **c.** $(-1)^4$ **d.** $(-1)^5$
e. Write a rule you can use to evaluate -1 to any power.

LESSON 3-1 PRACTICE

17. A Navy submarine descended from sea level at a rate of 7 feet per second.
a. Write the descent rate as an integer.
b. Find the submarine's elevation after 10 minutes.

18. In golf, *par* is a score of zero. One golfer scored 3 under par each day of a 4-day tournament.
a. Write the golfer's daily score as an integer.
b. Find the golfer's final score for the entire tournament.

19. Frances has no money in her checking account. She writes 3 checks for \$35 each. The bank imposes a \$15 penalty because she has overdrawn her account. How much money is in her account now?

20. A hot-air balloon leaves the ground and ascends at a rate of 6 feet per second for 3 minutes. Then it descends at a rate of 3 feet per second for 2 minutes. Finally, it ascends at a rate of 4 feet per second for 5 minutes. How far above the ground is the balloon now?

21. **Construct viable arguments.** You used a multiplication table to show that the product of two negative integers is positive. The following proof uses a different approach to show that $(-1)(-1) = 1$. Justify each step. You can use the step "Simplify" if necessary.

$$\begin{array}{lrcl}
\textbf{Step 1:} & -1(1+(-1)) & = & -1(1) + (-1)(-1) \\
\textbf{Step 2:} & -1(1+(-1)) & = & -1 \quad + (-1)(-1) \\
\textbf{Step 3:} & -1(0) & = & -1 \quad + (-1)(-1) \\
\textbf{Step 4:} & 0 & = & -1 \quad + (-1)(-1) \\
\textbf{Step 5:} & 1 & = & (-1)(-1)
\end{array}$$

My Notes

Learning Targets:

- Divide integers.
- Solve real-world problems by dividing integers and possibly adding, subtracting, or multiplying integers as well.

SUGGESTED LEARNING STRATEGIES: Think Aloud, Think-Pair-Share, Look for a Pattern

The table gives the elevations of four neighboring California towns. A surveyor wanted to calculate the average elevation of the towns. To do so, the surveyor needed not only to add integers but also to divide them.

City	Elevation (ft)
Coachella	−71
Indio	−9
La Quinta	120
Mecca	−180

Because division is the inverse operation of multiplication, you can use that relationship to find the rules for dividing positive and negative integers.

1. **Make use of structure.** The equation $3 \cdot 4 = 12$ shows that the numbers 3, 4, and 12 are related by multiplication. Write two equations to show that 3, 4, and 12 are related by division.

2. Use the fact that $5(-2) = -10$ to write two equations showing that 5, -2, and -10 are related by division.

3. Use the fact that $(-7)(-3) = 21$ to write two equations showing that -7, -3, and 21 are related by division.

4. Use your results above to complete these statements:

   The quotient of two integers with the same sign is ___________.

   The quotient of two integers with different signs is ___________.

5. What is the average elevation of the four California towns?

MATH TIP

Remember that you can express division in three ways. For example, the following all mean 12 divided by 3.

$$12 \div 3$$

$$\frac{12}{3}$$

$$3\overline{)12}$$

My Notes

Check Your Understanding

6. Find each quotient.

a. $24 \div (-6)$ **b.** $40 \div 8$ **c.** $-49 \div (-7)$ **d.** $-36 \div 4$

7. Simplify each fraction.

a. $-\frac{25}{5}$ **b.** $\frac{-25}{5}$ **c.** $\frac{25}{-5}$

d. What conclusion can you draw about the placement of the negative sign in expressions like those above?

LESSON 3-2 PRACTICE

8. Find the number that goes in each blank.

a. $14 \times$ ______ $= -266$ **b.** $-23 \times$ ______ $= 345$

c. $18 \times$ ______ $= 306$ **d.** $-11 \times$ ______ $= -341$

9. Evaluate each expression.

a. $-4 \times (-3) \div (-6)$ **b.** $30 \div (-2) \div (-5)$

c. $|4 \times (-15)| \div (-12)$ **d.** $[13 + (-19)] \times (-7) \div (-3)$

10. The temperature of a pot of water fell from 72°F to 36°F in 4 minutes. Find the average change in temperature per minute.

11. The price of one share of stock in BadInvestment.com plunged 14 points in 4 weeks. Find the average change in the stock price per day.

12. The low temperatures in Colton for 5 consecutive days were −8°F, −13°F, −4°F, −9°F, and −16°F. What was the average low temperature for the 5 days?

13. **Reason quantitatively.** Find two integers with a sum of 16 and a quotient of −9.

14. Use a related multiplication equation to show why the equation $\frac{5}{n} = 0$ has no solution.

15. The product of two integers, $m \times n$, is negative. Is $m \div n$, the quotient of the same integers, positive, negative, or impossible to find without knowing the values of m and n? Explain.

ACTIVITY 3 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 3-1

1. $3(-5)$ **2.** $-12(4)$

3. $0(-6)$ **4.** $-8(-10)$

5. $13(3)$ **6.** $7(-1)$

Evaluate.

7. $-8 \cdot |-8|$

8. $|-3| \cdot |-11|$

9. $-|7 - 13| \cdot (-|13 - 7|)$

10. $-14 \cdot (-|-5|)$

11. $-5 \cdot |-9| + 3 \cdot |4|$

12. $-|6(-4)|-7|(-3)(-2)|$

Find the number that goes in the blank.

13. $-10 \times$ ______ $= -20$

14. $5 \times$ ______ $= -45$

15. $-12 \times$ ______ $= 84$

16. $9 \times$ ______ $= 99$

17. $90 = -15 \times 3 \times$ ______

18. $-84 = -2 \times (-3) \times$ ______

Write $<$ or $>$ in the box.

19. $-3(-5) \square 4(-4)$

20. $-5(5) \square 6(-4)$

21. $8(5) \square 13(3)$

22. $-7(8) \square -11(-5)$

23. In the 3×3 array below, the product of the integers in each row and each column is the same number. The numbers in four of the squares are given. Find the remaining five numbers.

−18	−4	3
		2

24. An airplane descends at a rate of 500 feet per minute. Write and evaluate an expression to show how far the plane will descend in 6 minutes.

25. Starting at sea level, a diver descends into the ocean at a rate of 12 feet per minute. Write and evaluate an expression to show how far the diver will descend in 7 minutes.

26. Between low tide and high tide, the width of a beach changes by −17 feet per hour. Write and evaluate an expression to show how much the width of the beach changes in 3 hours.

State whether the product is positive or negative.

27. $(-3)5$ **28.** $(-2)(-10)$

29. $(-6)3$ **30.** $11(20)$

31. Two numbers, m and n, are integers, with $m < n$. Is it always true that $m^2 < n^2$. Explain your reasoning

Lesson 3-2

Complete the table.

Product	Related Quotients
$3 \cdot 7 = 21$	$21 \div 3 = 7$
	$21 \div 7 = 3$
32. $10(-4) =$ ______	
33. $-5(-9) =$ ______	
34. $-20(6) =$ ______	

Simplify.

35. $\frac{33}{-11}$

36. $-\frac{54}{9}$

37. $\frac{72}{8}$

38. $\frac{-32}{-2}$

39. Which of the following expressions is not equivalent to the others?

A. $\frac{-2}{-3}$ **B.** $-\frac{2}{3}$

C. $\frac{-2}{3}$ **D.** $\frac{2}{-3}$

40. Which expression gives the least product or quotient?

A. $-4(-2)$ **B.** $-3 \cdot 3$

C. $-15 \div 5$ **D.** $-36 \div (-4)$

Evaluate.

41. $64 \div [-8 \div (-2)]$

42. $[64 \div (-8)] \div (-2)$

43. $\frac{-45}{9} \cdot \frac{-15}{-5}$

44. $\frac{100}{-20} \cdot \frac{-15}{-5}$

Find the number that goes in the blank.

45. $-30 \div$ ______ $= -5$

46. $56 \div$ ______ $= -8$

47. $48 \div$ ______ $= 16$

48. $-76 \div$ ______ $= 19$

49. $3 = 48 \div (-4) \div$ ______

50. $-2 = -100 \div 10 \div$ ______

Write $<$ or $>$ in the box.

51. $32 \div (-8) \square -5 \div (-1)$

52. $-60 \div 4 \square 32 \div (-2)$

53. $0 \div (-49) \square 49 \div (-1)$

54. $33 \div (-33) \square -32 \div 16$

55. Explain how multiplication and division are related.

56. Over the past five weeks, the average daily temperature in Wellington has dropped 40 degrees Fahrenheit. Write and evaluate an expression to show the average temperature change per week.

57. The high temperatures in Weston for 7 consecutive days were −14°C, −10° C, −3°C, 6°C, 8°C, −4°C, and −11°C. What was the average high temperature for the 7 days?

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

58. Is there a greatest integer value for x that makes the inequality $\frac{x}{-5} > 4$ true? If so, what is it? Explain your reasoning.

Operations on Rational Numbers

ACTIVITY 4

Let's Be Rational!

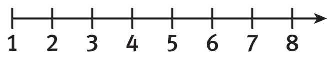
Lesson 4-1 Sets of Rational Numbers

Learning Targets:

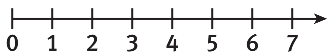
- Given a rational number, determine whether the number is a whole number, an integer, or a rational number that is not an integer.
- Describe relationships between sets of rational numbers.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Think-Pair-Share, Create Representations

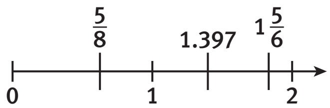
The history of numbers is the story of the gradual filling in of the number line. Ancient peoples had no concept of zero and needed numbers only to count items, such as cattle. Their number line consisted of the *natural numbers* 1, 2, 3, . . .



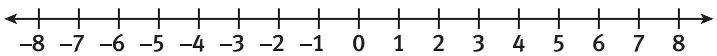
The idea of zero occurred to the ancient Babylonians as well as to the Mayans of Mesoamerica. Adding zero to the natural numbers on the number line creates the set of *whole numbers.*



Points between the whole numbers were known to the ancient Greeks. They comprise *fractions, decimals* and *mixed numbers.*



Negative numbers have been used in China and India for more than a thousand years. They did not come into wide use in Europe until the 17th century. The whole numbers and their negative-number opposites form the set of *integers*.



These three sets of numbers are ***subsets*** of the set of ***rational numbers***. A rational number is a number that can be expressed as a ratio $\frac{a}{b}$, where both a and b are integers and $b \neq 0$. The number -5, for example, can be expressed as the ratio $\frac{-15}{3}$.

My Notes

WRITING MATH

Use *ellipses*—three periods in a row—to represent all the numbers in an infinite sequence. For example, 0, 1, 2, 3, 4, 5, … represents the unending sequence of whole numbers.

MATH TERMS

A **subset** is a set whose elements are all in the original set. Every set is a subset of itself.

A **rational number** is a number that can be expressed as a ratio $\frac{a}{b}$, where both a and b are integers and $b \neq 0$.

My Notes

1. Show that each number is a rational number by expressing it as a ratio of two integers.

a. 27 **b.** 0.75 **c.** $4\frac{2}{3}$

d. -9 **e.** -0.43 **f.** -1.8

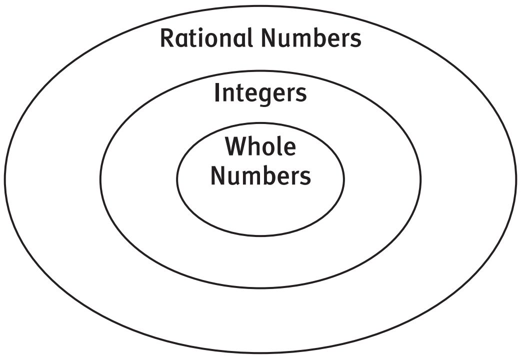
2. Classify each rational number as a whole number, as an integer, or as a rational number that is not an integer.

a. -34 **b.** 1.57

c. 0 **d.** $\frac{13}{14}$

3. The Venn diagram shows the relationships among whole numbers, integers, and rational numbers. Write the following numbers in their correct places in the diagram:

$13, 11\frac{9}{10}, 4.78, -803, -7\frac{5}{6}, 0, \frac{17}{3}, -91.55, -45$



4. Reason abstractly. Tell whether each statement is *true* or *false*. Explain why any false statements are false.

a. If n is an integer, then n is a whole number.

b. There are no rational numbers that are also whole numbers.

c. All rational numbers are integers.

d. A number cannot be both a whole number and a rational number.

My Notes

Check Your Understanding

5. Place a checkmark in the box for any set of which the given number is a member.

Number	Whole Number	Integer	Rational Number
0.25			
3.14159			
-12			
0			
$-0.333\ldots$			
$5\frac{9}{10}$			
29,116			
$-2\frac{1}{89}$			

6. Tell whether each statement is *never*, *sometimes* or *always* true.
a. An integer is a whole number.
b. A whole number is a rational number.
c. A rational number is a whole number.

LESSON 4-1 PRACTICE

7. Name all the sets of which the given set is a subset.
a. the set of whole numbers
b. the set of positive integers
c. the set of negative rational numbers
d. the set of natural numbers

8. Explain why 2 is a rational number.

9. **Reason abstractly.** Why does the definition of rational number state that b, the denominator of the rational number $\frac{a}{b}$, cannot equal 0?

10. **Construct viable arguments.** A rational number is defined as a ratio of two integers. Given that a ratio is a fraction, how can a decimal be a rational number?

11. Explain why the set of mixed numbers is not a subset of the set of integers.

Lesson 4-2
Adding Rational Numbers

My Notes

Learning Targets:
- Add two or more rational numbers.
- Use properties of addition to add rational numbers.
- Solve real-world problems by adding two or more rational numbers.

SUGGESTED LEARNING STRATEGIES: KWL Chart, Think Aloud, Create Representations

When you add rational numbers, use the same rules for determining signs as you used to add integers.

Example A

Julia needed to do some repainting around her pool so she drained $4\frac{1}{2}$ feet of water. After painting, she added $1\frac{2}{3}$ feet of water. How far below its original level did she leave the water in order to let the paint dry?

Step 1: $-4\frac{1}{2}+1\frac{2}{3}=-\frac{9}{2}+\frac{5}{3}$ Write the mixed numbers as improper fractions.

Step 2: $=-\frac{27}{6}+\frac{10}{6}$ Write the fractions with a ***common denominator***.

Step 3: $=-\frac{17}{6}$ Add using the rules for adding integers.

Step 4: $=-2\frac{5}{6}$ Write the improper fraction as mixed number.

Solution: Julia left the water $2\frac{5}{6}$ feet below its original level.

MATH TERMS

A **common denominator** is a common multiple of two or more denominators.

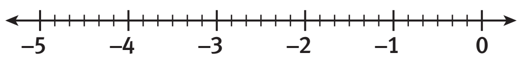
Try These A

Find each sum.

a. $-5\frac{5}{6}+2\frac{1}{4}$ **b.** $14.62 + (-19.3)$

c. Make sense of problems. Explain how the sum was found in Example A, Step 3.

1. Model with mathematics. Show how the final water level can be found using a number line.



My Notes

Example B

The water level in the Blue River was already 1.75 meters below normal when a drought caused the level to fall an additional 2.5 meters. What was the water level after the drought?

Step 1: $-1.75 - 2.5 = |-1.75| + |-2.5|$ Add using the rules for adding integers.

Step 2: $= 1.75 + 2.5$ Write the absolute values.

Step 3: $= 4.25$ Add.

Step 4: $= -4.25$ Use the sign of the addends.

Solution: The water was 4.25 meters below normal after the drought.

Try These B

a. $-420.5 - 98.6$

b. $-\frac{4}{15} + \left(-\frac{3}{10}\right)$

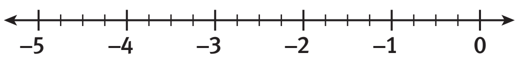
CONNECT TO SCIENCE

A *drought* is a long period of unusually low rainfall, especially one that causes extensive damage to crops.

2. Explain Step 1 of Example B.

3. How do you know that the final water level was below normal rather than above normal?

4. **Model with mathematics.** Show how the final water level can be found using a number line.



My Notes

Check Your Understanding

5. Find each sum.

a. $\frac{5}{12}+\left(-\frac{11}{12}\right)$

b. $3\frac{3}{8}+2\frac{1}{4}$

c. $-\frac{7}{15}+\left(-\frac{4}{5}\right)$

d. $-3.49+7.22$

e. $12.5+(-21.32)$

f. $-36.91+(-16.7)$

g. $\frac{1}{6}+\left(-\frac{11}{12}\right)+\frac{2}{3}$

h. $29+(-15.7)+(-31.05)$

i. Describe a possible real-world context for the expression in Item 5b.

j. Describe a possible real-world context for the expression in Item 5d.

6. a. Describe how to use the Commutative Property of Addition to simplify finding this sum:

$$\frac{9}{20}+(-2.45)+\left(-\frac{3}{5}\right)+6.7$$

b. Use the Commutative Property to find the sum.

LESSON 4-2 PRACTICE

7. Olympic swimming pools are rectangles measuring 164.042 feet in length and 82.021 feet in width. What is the perimeter of an Olympic pool?

8. Starting at sea level, a kingfisher flew to an elevation of $37\frac{1}{4}$ feet. Spotting a fish below, the bird descended $41\frac{5}{6}$ feet and caught the fish.

a. Write a numerical expression involving addition that you can use to find the elevation of the fish.

b. What was the elevation of the fish?

9. The lowest temperature ever recorded on Earth's surface was −128.5°F. The highest temperature was 262.5°F higher than the lowest.

a. Write a numerical expression involving addition that you can use to find the highest temperature.

b. What was the highest temperature ever recorded?

10. Make sense of problems. Justify Step 1 in the following evaluation:

Step 1: $-2.79+\left((-3.91)-5\frac{1}{2}\right)=\left(-2.79+(-3.91)\right)-5\frac{1}{2}$

Step 2: $=-6.7-5.5$

Step 3: $=-12.2$

My Notes

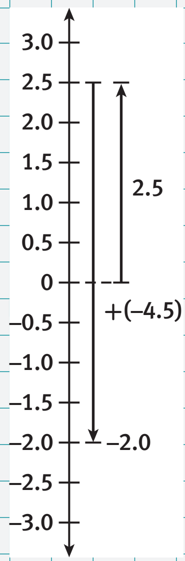
Learning Targets:

- Subtract rational numbers.
- Apply the fact that for all rational numbers a and b, $a - b = a + (-b)$, to add and subtract rational numbers.
- Solve real-world problems by subtracting rational numbers and possibly by adding rational numbers as well.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Think-Pair-Share

Recall that you can subtract an integer by adding its opposite. The number line at the right illustrates $2.5 + (-4.5)$ and shows that the same rule applies to subtracting rational numbers: $2.5 - 4.5 = -2$.

- To subtract a rational number, add its opposite.



Example

As the Yellowstone River flows through Yellowstone National Park, it breaks into two waterfalls. At the Upper Falls, the river drops 33.22 meters. At the Lower Falls, it drops 93.88 meters. Find the river's total change in elevation as it passes the two falls.

Subtract: $-33.22 - 93.88$

Step 1: To -33.22, add the opposite of 93.8 — $-33.22 - 93.88 = -33.22 + (-93.88)$

Step 2: The signs are the same so find the sum of the absolute values. — $|-33.22| + |-93.88| = 33.22 + 93.88 = 127.1$

Step 3: Use the sign of the addends: — -127.1

Solution: The river's total change of elevation is -127.1 meters.

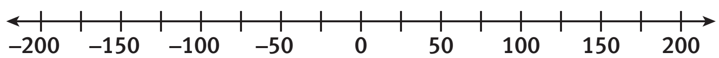
CONNECT TO HISTORY

Established in northwestern Wyoming in 1872, Yellowstone National Park was America's first national park.

Try These

a. $-4.13 - (-5.46)$ **b.** $\frac{5}{12} - \frac{7}{8}$

1. **Model with mathematics.** Draw arrows on the number line below to show the changes in the river's elevation at the Upper Falls and Lower Falls.



My Notes

Check Your Understanding

2. Write each subtraction problem as an addition problem.

a. $\frac{2}{3} - \frac{4}{5}$

b. $-0.08 - 3.62$

c. $-7\frac{3}{8} - \left(-2\frac{1}{4}\right)$

d. $527.4 - (-748.62)$

3. Subtract.

a. $\frac{9}{13} - \frac{11}{13}$

b. $-57.49 - (-35.7)$

c. $7\frac{3}{10} - \left(-4\frac{1}{5}\right)$

d. $-319.12 - 88.16$

LESSON 4-3 PRACTICE

4. Yosemite Falls in Yosemite National Park drops in three separate sections.

Name	Height (m)
Upper Falls	435.86
Middle Cascades	205.74
Lower Falls	97.54

a. Write a numerical expression you can use to find the total change in elevation.

b. What order of operation rule must you use to evaluate the expression?

c. What is the total change in elevation from the top of the falls to the bottom?

5. On January 22, 1943, the temperature in Spearfish, South Dakota, fell from 53.6°F to −4°F in just 27 minutes.

a. Write a numerical expression you can use to find the change in temperature.

b. Evaluate your expression.

c. What was the mean change in temperature per minute? Write your answer using bar notation.

6. a. Describe two ways to find the difference $\frac{13}{50} - 4.93$.

b. Which way is better? Explain your reasoning.

7. Construct viable arguments. Is the difference between two rational numbers always as rational number? Why or why not?

8. Jodi is finding the sum $4 + (-6.5)$ on a number line.

a. What is the distance from 4 to the sum?

b. Is the sum to the left or to the right of 4 on the number line? How do you know?

c. What is Jodi's sum?

My Notes

Learning Targets:

- Multiply and divide rational numbers.
- Apply properties of operations to multiply and divide rational numbers.
- Solve real-world problems involving the four operations with rational numbers.

SUGGESTED LEARNING STRATEGIES: Visualization, Create Representations, Think-Pair-Share

Engineers at an underwater oil drilling operation drilled beneath the ocean floor an average of -734.8 meters per day for four consecutive days. What was the total change in elevation from the ocean floor to the deepest point drilled?

To answer this question, you can use addition of rational numbers.

$$(-734.8) + (-734.8) + (-734.8) + (-734.8) = -2{,}939.2$$

You could also multiply $4(-734.8)$.

1. The above methods work if all the numbers are the same, but suppose you want to find the product $7.9(-3.5)$. One way would be to use the following method. Write a reason for each step.

$0 = 7.9(0)$	**a.**	______________
$0 = 7.9(3.5 + (-3.5))$	**b.**	______________
$0 = 7.9(3.5) + 7.9(-3.5)$	**c.**	______________
$0 = 27.65 + 7.9(-3.5)$	**d.**	______________
$-27.65 = 7.9(-3.5)$	**e.**	______________

The last step shows that $7.9(-3.5) = -27.65$. Not only does this give the product, but it establishes an important fact about multiplication of rational numbers:

- The product of two rational numbers having *different* signs is negative.

CONNECT TO HISTORY

The first underwater oil wells were drilled in 1891, in Ohio's Grand Lake St. Mary's, a freshwater lake. The first saltwater wells were drilled five years later, in California's Santa Barbara.

2. Find each product.

a. $-2.5(6.7)$

b. $\frac{2}{3}\left(-\frac{9}{10}\right)$

c. $-\frac{11}{12}\left(\frac{4}{7}\right)$

d. $9.02(-3.4)$

My Notes

You already know that the product of two positive rational numbers is positive. What about the product of two negative rational numbers? You can find out using a method like the one used in Item 1 above for numbers with different signs.

3. **Make sense of problems.** Find the product $-\frac{2}{3}\left(-\frac{5}{7}\right)$. Write a reason for each step.

$0 = -\frac{2}{3}(0)$ **a.** ______________

$0 = -\frac{2}{3}\left(\frac{5}{7} + \left(-\frac{5}{7}\right)\right)$ **b.** ______________

$0 = -\frac{2}{3}\left(\frac{5}{7}\right) + \left(-\frac{2}{3}\left(-\frac{5}{7}\right)\right)$ **c.** ______________

$0 = -\frac{10}{21} + \left(-\frac{2}{3}\left(-\frac{5}{7}\right)\right)$ **d.** ______________

$\frac{10}{21} = -\frac{2}{3}\left(-\frac{5}{7}\right)$ **e.** ______________

The last step shows that $-\frac{2}{3}\left(-\frac{5}{7}\right) = \frac{10}{21}$. You already know that the product of two positive rational numbers is positive.

The multiplication of two negative rational numbers, shown above, establishes this important fact:

- The product of two rational numbers having the *same* sign is positive.

4. **Make use of structure.** Why must the rules for finding the signs when you multiply two integers be the same as the rules for finding the signs when you multiply two rational numbers?

5. State the sign of each product.
   - **a.** three negative rational numbers
   - **b.** four positive rational numbers and one negative rational number
   - **c.** three positive rational numbers
   - **d.** one positive and two negative rational numbers
   - **e.** thirteen negative rational numbers
   - **f.** five positive and four negative rational numbers

6. Find each product.

a. $-\frac{11}{12}\left(\frac{4}{7}\right)$ **b.** $9.02(-3.4)$

c. $-2.5(6.7)$ **d.** $\frac{2}{3}\left(-\frac{9}{10}\right)$

You can use inverse operations to find the sign of the quotient of two rational numbers.

7. Use the facts that $7.2(-3.5) = -25.2$ and that $-7.2(-3.5) = 25.2$ to complete these equations:

a. $\frac{-25.2}{7.2} =$ **b.** $\frac{25.2}{-7.2} =$

8. Use the results to complete this statement:

The quotient of two rational numbers with different signs is ________.

9. Use the facts from Item 7 to complete this equation:

$$\frac{-25.2}{-7.2} =$$

10. Use your results and your knowledge of the quotient of two positive numbers to complete this statement:

The quotient of two rational numbers with the same sign is ________.

11. Compare the rules for finding the signs of the products and the signs of the quotients of two rational numbers.

12. A well-drilling crew drilled these distances into Earth's crust on four successive days, beginning at the bottom of the ocean:

$-1{,}574\frac{1}{4}$ feet, $-1{,}289\frac{1}{2}$ feet, $-1{,}719\frac{3}{4}$ feet, $-1{,}400\frac{1}{2}$ feet

What was the mean daily change in elevation of the bottom of the well?

13. Find each quotient.

a. $-60.48 \div 4.8$ **b.** $-\frac{1}{5}\left(-\frac{3}{10}\right)$

c. $\frac{11}{24} \div \left(-\frac{5}{8}\right)$ **d.** $1.376 \div 0.8$

My Notes

MATH TIP

You can use the rules for finding the signs of the sums, differences, products, and quotients of two integers to find the sign of the sum, difference, product, or quotient of any two rational numbers.

My Notes

Check Your Understanding

14. m and n are positive rational numbers.
  a. What is the sign of their product?
  b. What is the sign of their quotient?

15. m and n are negative rational numbers.
  a. What is the sign of their product?
  b. What is the sign of their quotient?

16. m and n are rational numbers with different signs.
  a. What is the sign of their product?
  b. What is the sign of their quotient?

LESSON 4-4 PRACTICE

17. **Make use of structure.** Which of the following fractions are equal to -7?

$\frac{-7}{1}, \frac{7}{1}, \frac{-7}{-1}, -\frac{7}{1}, \frac{-7}{-1}, -\frac{-7}{1}, -\frac{7}{-1}, \frac{7}{-1}, -\frac{-7}{-1},$

18. The low temperatures for one week in Scottsburg, IN are given below. What was the mean daily low temperature for the week?

-7.9°F, -10.3°F, -3.4°F, 2.6°F, 4.9°F, 11.0°F, -2.5°F

19. Margo's grade average in math was 92. Then for seven months, her average dropped an average of five-eighths of a point per month.
  a. Write a rational number expression involving addition that you can evaluate to find her average at the end of seven months.
  b. What was her final average?

20. Given a temperature in degrees Fahrenheit, the formula $C = \frac{5}{9}(F - 32)$ can be used to find the corresponding Celsius temperature. Find the Celsius temperatures corresponding to the following Fahrenheit temperatures.
  a. 113°F **b.** 32°F
  c. -25°F **d.** -40°F

21. **Construct viable arguments.** Two rational numbers are each less than 1. Is their product less than 1? Why or why not? Give examples to support your answer.

ACTIVITY 4 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 4-1

1. Place a checkmark in the box for any set of which the given number is a member.

Number	Whole Number	Integer	Rational Number
-2			
10.5			
0			
9			
0.9812			
$2\frac{15}{17}$			
-68.555			
$-0.787878\ldots$			

2. Which statement is false?
   A. A whole number is always a rational number.
   B. An integer is always a whole number.
   C. A number that can be expressed as a ratio $\frac{a}{b}$, where both a and b are integers and $b \neq 0$, is always a rational number.
   D. A whole number is always an integer.

3. Give an example of each.
   a. an integer that is not a whole number
   b. a rational number that is not an integer
   c. a rational number that is not a whole number

4. Explain how you know that each number is a rational number.
   a. $3\frac{8}{9}$ **b.** -25
   c. 1.479 **d.** -6.01

5. Is 0 a rational number? Why or why not?

Lesson 4-2

6. Find the value of each expression.
   a. $\frac{13}{16} + \left(-\frac{3}{4}\right)$ **b.** $7\frac{2}{3} + 6\frac{1}{4}$
   c. $-\frac{7}{20} + \left(-\frac{2}{5}\right)$ **d.** $-6.98 + 2.75$
   e. $\frac{2}{3} + \left(-\frac{5}{8}\right) + \left(-\frac{1}{6}\right)$
   f. $29 + (-15.7) + (-31.05)$
   g. Describe a possible real-world context for the expression in item 6a.
   h. Describe a possible real-world context for the expression in item 6d.

7. Bette had \$452.13 in her checking account. She wrote checks for \$53.15 and \$117.48.
   a. Write an expression involving addition that you can evaluate to find the amount that remained in Bette's account.
   b. Evaluate the expression.

8. Which property is illustrated by the following equation?
   $\frac{3}{5} + \left(-\frac{7}{8}\right) + \frac{4}{5} = \frac{3}{5} + \frac{4}{5} + \left(-\frac{7}{8}\right)$
   A. Commutative Property of Addition
   B. Addition Property of Equality
   C. Associative Property of Addition
   D. Identity Property of Addition

9. The lowest point on Earth's surface is the shore of the Dead Sea, elevation $-1{,}344.99$ meters. The highest point, the summit of Mount Everest, is 30,380.42 meters above the Dead Sea. What is the elevation at the summit of Mount Everest?

10. Find each sum.
    a. $\frac{7}{20} + (-4.8) + \left(-\frac{4}{5}\right) + 4.9$
    b. $5.6 - 1\frac{3}{8} + (-3.9) + 2\frac{3}{4}$

Lesson 4-3

11. Write each subtraction problem as an addition problem.

a. $\frac{7}{8} - \frac{9}{10}$ **b.** $-6.39 - 10.4$

c. $5\frac{5}{9} - \left(-8\frac{3}{5}\right)$ **d.** $0.45 - (-1.3)$

12. Find the value of each expression.

a. $\frac{5}{12} - \frac{2}{3}$ **b.** $-2.81 - (-1.77)$

c. $12\frac{9}{16} - \left(-13\frac{1}{24}\right)$ **d.** $-46.03 - 21.7$

e. $-9.77 - 14.52 - (-61.2)$

f. $\frac{5}{6} - \frac{7}{9} - \frac{1}{2}$

13. The elevation of the deepest point in the Pacific Ocean is $-11{,}033$ meters. The elevation of the deepest point in the Atlantic Ocean is $-8{,}648$ meters.

a. Write a subtraction expression you can use to find how much deeper the Pacific Ocean's deepest point is than that of the Atlantic Ocean's.

b. Evaluate your expression.

14. Greg borrowed \$100 from his parents. After he did some chores, they reduced the amount of his debt by \$25.

a. Let -100 represent the amount Greg owed his parents before he did chores. Write a subtraction expression you can use to find the amount Greg still owes his parents.

b. Evaluate your expression.

15. Is there a Commutative Property of Subtraction for rational numbers? Why or why not? Use examples to support your answer.

Lesson 4-4

16. Find each product or quotient.

a. $-\frac{5}{9}\left(\frac{3}{10}\right)$ **b.** $0.55(-2.6)$

c. $-25.28 \div 3.2$ **d.** $-\frac{3}{8} \div \left(-\frac{9}{16}\right)$

e. $-0.4(0.7)$ **f.** $52\left(-\frac{7}{13}\right)$

g. $1\frac{3}{4} \div \left(-4\frac{3}{8}\right)$ **h.** $2.4 \div 48$

i. $(1.8)\left(-1\frac{2}{5}\right)$ **j.** $(-9.6) \div \left(-3\frac{1}{5}\right)$

17. A glacier that was 1,076 meters thick changed in thickness at an average rate of -22.7 meters per year for 7 years.

a. Write an addition expression you can use to find the glacier's thickness after 7 years.

b. Evaluate your expression.

18. In golf, a player's score on each hole is always an integer. The more negative the score, the better it is. A golfer's combined score for the 18 holes is -5. The golfer scored -2 on each of several holes. On all the other holes the golfer scored a combined total of $+1$. On how many holes did the golfer score -2?

19. Naief is finding the sum $-7 + 4\frac{3}{4}$ on a number line.

a. What is the distance from -7 to the sum?

b. Is the sum to the left or right of -7 on the number line? How do you know?

c. What is Naief's sum?

MATHEMATICAL PRACTICES
Reason Abstractly and Quantitatively

20. In the 3×3 array below, the product of the rational numbers in each row, in each column, and in each diagonal is the same number. The numbers in four of the squares are given. Find the remaining five numbers.

		-0.4
-3.6	-0.8	0.6

Rational Number Operations and Multiplying and Dividing Integers

TOP TO BOTTOM

Embedded Assessment 2
Use after Activity 4

Write your answers on a separate sheet of paper. Show your work.

The diagram at the right shows the approximate elevations of the tops and bottoms of the layers of the atmosphere (the envelope of gas above the Earth) and the zones of the ocean.

1. a. Write a subtraction expression you can use to find the difference between the elevation at the top of the exosphere and the deepest point of the ocean.
b. Write your expression as an addition expression.
c. Evaluate the expression.

2. a. How many times as thick as the ocean's epipelagic zone is the hadalpelagic zone?
b. Explain how you found the answer.

3. An airplane flew over the ocean at an elevation 7.9 kilometers below the top of the troposphere. A wheel came off and fell a total of 16.9 kilometers.
a. In which ocean zone did the wheel come to rest?
b. How far above the elevation of the deepest point in the ocean was the wheel when it stopped?

As you move upward through the lowest three layers of the atmosphere, the air grows thinner and thinner. This causes air temperatures to grow colder and colder. An average temperature at the bottom of the troposphere might be 65°F. The temperature at the top of the mesosphere might be 250°F colder than that.

4. Find the colder temperature at the top of the mesosphere.

In the thermosphere, the heat of the sun overcomes the thinness of the air and causes temperatures to rise dramatically. The hottest temperature at the top of the thermosphere can be 3700°F hotter than the temperature you found in Item 4.

5. a. Find the hotter temperature at the top of the thermosphere.
b. How many times as hot as the temperature at the top of the mesosphere is the temperature at the top of the thermosphere?

In 2012, film director James Cameron descended to the bottom of the Mariana Trench, the deepest point of the ocean, in a submarine called the *Deepsea Challenger.* The descent took 2 hours and 36 minutes.

6. a. Write the depth of the Mariana Trench and Cameron's descent time as mixed numbers.
b. Use the mixed numbers to find the average rate of descent of the *Deepsea Challenger.* Show your work. Round your answer to the nearest tenth.
c. The submarine ascended to the ocean surface in 70 minutes. Use any method you choose to find the average rate of ascent. Round your answer to the nearest tenth.

Elevations of Atmosphere Layers and Ocean Zones

10,000 km --------------------
Exosphere
600 km ------------------------
Thermosphere
90 km ------------------------
Mesosphere
50 km ------------------------
Stratosphere
20 km ------------------------
Troposphere
0 km ------------ SEA LEVEL
Epipelagic
–0.2 km ----------------------
Mesopelagic
–1 km ------------------------
Bathypelagic
–4 km ------------------------
Abyssopelagic
–6 km ------------------------
Hadalpelagic
–10.9 km ---DEEPEST POINT OF THE OCEAN

Rational Number Operations and Multiplying and Dividing Integers

TOP TO BOTTOM

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1a-c, 2a-b, 3a-b, 4, 5a-b, 6a-c)	• A clear and accurate understanding of operations with rational numbers and integers.	• Operations with rational numbers and integers that are usually correct.	• Operations with rational numbers and integers that are sometimes correct.	• Incorrect or incomplete computation of operations with rational numbers and integers.
Problem Solving (Items 1a-c, 2a-b, 3a-b, 4, 5a-b, 6a-c)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps that result in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1a-b, 2a, 3a-b, 4, 5a-b, 6a-c)	• Clear and accurately written expressions involving operations with rational numbers and integers that result in a correct answer.	• Some difficulty in writing the best expression for operations on rational numbers and integers, but with correct answers.	• Errors in writing expressions for operations on rational numbers and integers.	• Inaccurately written or missing expressions for operations on rational numbers and integers.
Reasoning and Communication (Item 2b)	• Precise use of appropriate math terms and language when explaining the process of dividing integers.	• An adequate explanation of the process of dividing integers.	• A misleading or confusing explanation of the process of dividing integers.	• Incomplete or inaccurate explanation of the process of dividing integers.