

# Ratio and Proportion

3

## Unit Overview

In this unit, you will use pictures, graphs, tables, and verbal descriptions to study unit rates, rate of change, and proportions. You will solve problems involving scale, percentage, and proportional relationships.

## Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

## Academic Vocabulary

- tip

## Math Terms

- ratio
- rate
- unit rate
- proportion
- cross products
- conversion factor
- constant of proportionality
- constant ratio
- constant rate of change
- relative size
- scale drawing
- percent
- percent equation
- discount
- markup
- interest
- percent error

## ESSENTIAL QUESTIONS



How are ratios, unit rates, and proportions used to describe and solve real-world problems?



How can representations, numbers, words, tables, and graphs be used to solve problems?

## EMBEDDED ASSESSMENTS

These assessments, following Activities 9, 10, and 12, will give you an opportunity to demonstrate how you can use ratios and rates to solve mathematical and real-world problems involving proportional relationships.

### Embedded Assessment 1:

Ratios, Proportions, and Proportional Reasoning p. 99

### Embedded Assessment 2:

Proportional Relationships and Scale p. 113

### Embedded Assessment 3:

Percents and Proportions p. 133

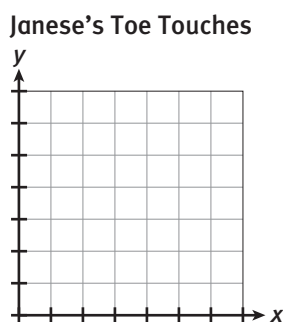
# Getting Ready

Write your answers on notebook paper.  
Show your work.

- Janese can complete 7 toe touches in 10 seconds. Write a ratio of Janese's toe touches to seconds in three ways.
- Complete the following table representing Janese's toe touches.

Janese's Toe-Touching Record							
Time (in seconds)	0	10	20	30	40	50	60
Toe Touches							

- Use the grid below to graph Janese's toe touches. Label the horizontal and vertical axes. Provide a scale on the horizontal and vertical axes.



- Write an algebraic expression for each the following.
  - The cost of each ticket, if  $x$  tickets cost \$106.25
  - The cost of  $g$  gallons of gas if each gallon costs \$3.67
  - Five less than 3 times a number

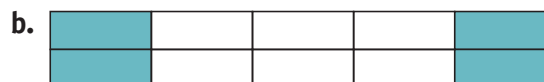
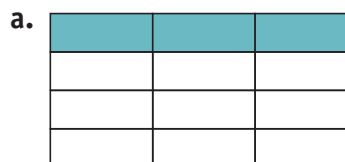
- Solve each of the following equations.

- $2x + 5 = 8$
- $16 + 3x = 28$

- Copy and complete this table to show equivalent values.

%	Decimal	Fraction
25%		
		$\frac{1}{2}$
	0.15	

- What percent of the figures are shaded?



- Explain how to determine which of the following values is the greater and tell which expression has the larger value.

$$\frac{1}{3} \text{ of } 60$$

$$25\% \text{ of } 60$$

# Ratio and Proportion

## Strange, But True

### Lesson 8-1 Ratio and Unit Rates

#### ACTIVITY 8

#### Learning Targets:

- Express relationships using ratios.
- Find unit rates.

**SUGGESTED LEARNING STRATEGIES:** Graphic Organizer, Paraphrasing, Note Taking, Sharing and Responding, Discussion Groups

Interesting sports facts and statistics are abundant. For example, there are only *two* days of the year in which there are *no* professional sports games (MLB, NBA, NHL, or NFL): the day before and the day after the Major League Baseball (MLB) All-Star Game.

You can write a **ratio** to compare the number of days without any professional sports events each year with the total number of days in a year. There are three ways to write a ratio to express the relationship between two quantities:

$a$  to  $b$

$a : b$

$\frac{a}{b}$

1. Write a ratio, in all three forms, to compare the days without games to the days in a year.
2. What ratio compares days with games to days in a year? Write it three ways.
3. What ratio can you write that compares days with games to days without them? Write this ratio three ways, too.

The ratios above all compare two *like* units—*days* and *days*. A ratio that compares two *different* kinds of units is called a **rate**. One common rate in sports is miles per hour (mi/h or mph), as in car racing.

4. List some other sports statistics commonly given as rates.

You can use basketball free throws to explore rates. In a group of four, make 12 paper basketballs. Place a wastebasket about six feet from a “free-throw line.” Record how many “baskets” each of you makes within the time listed in the table. Have one member of the group keep time. Then work together to answer Items 5-8 on the next page. If you do not know exact words to use during discussion, use synonyms or request assistance from group members. If you need to, use non-verbal cues such as raising your hand for help.

#### My Notes

#### CONNECT TO SPORTS

NBA = National Basketball Association

NHL = National Hockey League

NFL = National Football League

#### MATH TERMS

A **ratio** is a comparison of two quantities. You can write a ratio as a fraction, using the word “to,” or using a colon.

A **rate** is a ratio that compares two *different* units, such as distance and time, or a ratio that compares two different things measured with the same unit, such as cups of water and cups of frozen orange juice concentrate.

#### DISCUSSION GROUP TIPS

As you interact with your group in solving problems, you may hear math terms and other words that may be new to you. Ask for clarification of their meaning, and make notes to help you learn and use vocabulary heard during classroom instruction and interactions.

My Notes

Team Member	Baskets Made	Time (in seconds)	Rate
1		30	
2		15	
3		20	
4		10	

5. What units are you comparing in this free-throw activity?

6. **Reason quantitatively.** Examine all results. What can you say about the relationship between baskets made and time allowed?

MATH TERMS

A **unit rate** is a rate expressed in terms of 1 unit.

In some cases, you find unit rates by *multiplying*. In other cases, you find them by *dividing*.

When the second term of a rate is 1, the rate is called a **unit rate**. Miles *per* hour is a kind of unit rate. So is price *per* pound.

Suppose that you and your friends attend a basketball game. You buy a block of 8 tickets for \$192. You want to know the price per ticket. That price can be expressed as a unit rate.

7. What is that rate? How did you figure it out?

Now look back at your made-basket rates. To see who the best shooter was, you can express each rate as a unit rate.

*Think:* 60 seconds = 1 minute. Let made-baskets per minute be your unit rate.

Suppose you made 7 baskets in 30 seconds. Use mental math to find how many you made in 60 seconds.

$$\frac{7}{30} = \frac{7(2)}{30(2)} = \frac{14 \text{ baskets}}{60 \text{ seconds}} = \frac{14 \text{ baskets}}{1 \text{ minute}}$$

So, your unit rate is 14 baskets per minute.

8. a. How can you find the one-minute unit rate for the baskets team member 1 made in 30 seconds? Explain your reasoning.

b. How can you find the unit rate for the baskets that team member 3 made in 20 seconds? Explain your reasoning.

**Check Your Understanding**

9. There are nine position players on a baseball team:  
3 outfielders, 4 infielders, 1 pitcher, 1 catcher.  
Write a ratio in simplest form to express each relationship.
  - a. infielders to number of players
  - b. outfielders to infielders
  - c. number of players to number of outfielders
  - d. number of players other than infielders and outfielders to number of players
  - e. number of pitchers to number of catchers
10. Write the rate. Then find the unit rate.
  - a. \$162 for 9 tickets
  - b. 15 baskets in 5 minutes
  - c. 84 yards in 14 running attempts
  - d. 24 strikeouts in 12 innings

**LESSON 8-1 PRACTICE**

11. Jed made 2 free throws in 5 seconds. How many would he make in one minute?
12. Rita ran 5 miles in 48 minutes. What was her time per mile?
13. In a typical Wimbledon tennis tournament, 42,000 balls are used and 650 matches are played. About how many balls are used per match?
14. Reggie Jackson played in major league baseball for 21 years. Although this slugger was known best for his home runs, he holds the major league record for strikeouts: 2,597. What was his approximate rate of strikeouts per year?
15. In skateboarding, an *ollie* is a move in which the athlete pops the skateboard into the air, making it appear that the board and skateboarder are attached. At the 2009 X Games, one skateboarder completed 34 ollies in 30 seconds.
  - a. If he could keep up that rate, what would be his rate of ollies per minute?
  - b. What would be his rate per hour?
16. **Reason quantitatively.** A major-league baseball player was able to hit 70 home runs in 162 games. Create a unit rate for this information.

My Notes



My Notes

MATH TERMS

A **proportion** is an equation stating that two ratios are equivalent.

READING MATH

Read the proportion  $\frac{2}{5} = \frac{4}{10}$  as "the ratio 2 to 5 equals the ratio 4 to 10" or as "2 is to 5 as 4 is to 10."

Learning Targets:

- Determine whether quantities are in a proportional relationship.
- Solve problems involving proportional relationships.

**SUGGESTED LEARNING STRATEGIES:** Close Reading, Marking the Text, Predict and Confirm, Note Taking, Create Representations

As you read the following scenario, mark the text to identify key information and parts of sentences that help you make meaning from the text.

The fastest time for running a mile while balancing a baseball bat on a finger is 7 min 5 s. This record was set by Ashrita Furman on June 20, 2009. At that rate of speed, Meg predicts that it would take 1,275 seconds, or 21 min 15 s, for Ashrita to run 3 mi. Is she right?

You can write a proportion to find out. A **proportion** is an equation. It consists of two equivalent ratios.

Example:  $\frac{2}{5} = \frac{4}{10}$

To determine if Meg is correct, let  $n$  = the time it will take Ashrita to run 3 miles balancing a baseball bat.

First, convert 7 min 5 seconds to seconds: 425 seconds.

$$\frac{\text{time}}{\text{distance}} \rightarrow = \frac{425}{1} = \frac{1275}{3} \leftarrow \frac{\text{time}}{\text{distance}}$$

When two ratios are equal, their **cross-products** are equal.

For any proportion  $\frac{a}{b} = \frac{c}{d}$ ,  $ad = bc \leftarrow$  cross-products

1. Using what you know about proportions, use the proportion above involving Ashrita's speed and distance data and find the cross-products. Are the cross products equal?
2. Is Meg's prediction correct?
3. **Reason quantitatively.** Are there other ways to determine if Meg is right? Explain.
4. **Construct viable arguments.** Suppose a fast-running juggler beat Ashrita's record by half a minute. Could that person, continuing at that new world-record rate of speed, run 2 mi while juggling in 13 min 10 s? Use a proportion to find out and explain your reasoning.

## Lesson 8-2

### Identifying and Solving Proportions

## ACTIVITY 8

continued

5. A three-toed sloth can cover a mile in 0.15 of an hour. Use proportions and sloth speed to complete the table for the distances shown.

Distance (mi)	1	2	5	12
Time (h)				

You can use proportions to solve problems about ratios and rates.

### Example

Roger Bannister was the first person to break the four-minute mile. On May 6, 1954, his time was 3 minutes, 59.4 seconds. Bannister's first quarter-mile time was 57.5 seconds. Use a proportion to find his time if he had kept up this pace.

**Step 1:** Write a proportion.

Let  $n$  = time to run the entire race.

Use 0.25 for the first quarter-mile Bannister ran.

$$\frac{\text{time (sec)}}{\text{distance (mi)}} \rightarrow \frac{57.5}{0.25} = \frac{n}{1} \leftarrow \frac{\text{time (sec)}}{\text{distance (mi)}}$$

**Step 2:** Solve the proportion using cross-products:

$$0.25 \times n = 1 \times 57.5$$

$$0.25 n = 57.5$$

**Step 3:** Solve the equation to find  $n$ .

$$n = 57.5 \div 0.25 \leftarrow \text{Think: Divide both sides by 0.25.}$$

$$n = 230 \text{ s}$$

$$n = 3 \text{ min } 50 \text{ s} \leftarrow \text{write as minutes and seconds}$$

**Solution:** Had Bannister kept up his quarter-mile pace, he would have run the mile in 230 sec, or 3 min 50 sec.

**a. Model with mathematics.** Why do you divide *both* sides of the equation by 0.25?

**b. Reason abstractly and quantitatively.** What other proportions could you have written to solve the problem?

### Try These

Solve each proportion.

a.  $\frac{n}{1} = \frac{2.45}{0.35}$

b.  $\frac{n}{5} = \frac{23.4}{2}$

c.  $\frac{3}{n} = \frac{5.4}{7.2}$

### My Notes

### MATH TIP

Recall that in any proportion, cross-products will be equal. For the proportion

$$\frac{a}{b} = \frac{c}{d}$$

$$ad = bc$$

When you write a proportion, be sure to set up the ratios in a consistent way according to the units associated with the numbers.

My Notes

Check Your Understanding

6. Use cross products to determine if the ratios are equivalent.

- a.  $\frac{3}{4}, \frac{6}{8}$       b.  $\frac{8}{5}, \frac{24}{16}$       c.  $\frac{70}{60}, \frac{6}{7}$       d.  $\frac{1.3}{7.8}, \frac{3}{18}$   
 e.  $\frac{4}{7}, \frac{10}{17.5}$       f.  $\frac{9}{4}, \frac{2.1}{1.4}$       g.  $\frac{3}{0.8}, \frac{21}{5.6}$       h.  $\frac{0.3}{2}, \frac{0.03}{20}$

7. **Make use of structure.** Write a proportion for each situation. Then solve.

- a. 336 dimples on one golf ball; 2016 dimples in  $n$  balls  
 b. 3 miles in 2.8 minutes; 33.3 miles in  $x$  minutes  
 c. 25 yards in  $2\frac{1}{2}$  seconds; 100 yards in  $y$  seconds  
 d. 480 heartbeats in 4 minutes;  $z$  heartbeats in 1 minute

8. A zebra can run at a speed of 40 mph. Complete the table using this information.

Time (h)	1	0.25	0.5	1.75
Distance (mi)				

LESSON 8-2 PRACTICE

Solve by writing and solving a proportion.

9. There are 20 stitches per panel on a soccer ball. A soccer ball has 32 leather panels. How many stitches, in all, are on a soccer ball?
10. Jed took 2 free throws in 5 seconds. Alex took 4 free throws in 12 seconds. Did the two shoot free throws at the same rate? Explain.
11. The ratio of girls to boys on a soccer team is 2 to 3. If there are 25 players on the team, how many are girls?
12. **Model with mathematics.** A package of tickets for 4 home games costs \$180. What proportion can you write to find what a 12-game package costs if all individual tickets have the same price?
13. Carlos completed 7 of 10 passes. Ty completed 21 of 30 passes. Compare their pass-completion rates.
14. Carla's team won 3 of its 5 games. Elena's team won games at the same rate and won 12 games. How many games did Elena's team play?
15. Greta completed a mile race in 5 minutes. Inez ran a mile in which each quarter-mile split was 1 min 20 seconds. Which of the two girls had the faster time? How much faster?



### Learning Targets

- Convert between measurement. Use unit rates and proportions for conversions

**SUGGESTED LEARNING STRATEGIES:** Visualization, Think Aloud, Discussion Groups, Sharing and Responding, Create a Plan, Identify a Subtask, Note Taking

Some problems involving measurements will require you to convert between customary and metric units of measure.

### Example

A tennis court is 78 feet in length and for singles play is 27 feet in width. How many meters wide is the tennis court?

**Step 1:** Start by converting feet to yards:  $\frac{3 \text{ ft}}{1 \text{ yard}} = \frac{27 \text{ feet}}{x \text{ yards}}$

**Step 2:** Use cross-products to solve the proportion:

$$27 \cdot 1 = 3x$$

$$27 = 3x$$

$$9 = x$$

So, there are 9 yards in 27 feet

**Step 3:** Next, convert 9 yards to meters:

$$\frac{9 \text{ yd}}{x \text{ m}} = \frac{1 \text{ yd}}{0.9144 \text{ m}}$$

$$1x = 9(0.9144)$$

$$x = 8.2296 \text{ meters}$$

$$x \approx 8.23 \text{ meters}$$

**Solution:** The tennis court is 8.23 meters wide.

### Try These

- a. Attend to precision.** Find the length of the tennis court above in meters. Be sure to include units.

1. How do can you tell that a proportion involving conversions has been set up correctly?
2. The conversion factor for converting meters to yards is  $\frac{1}{0.9144}$ , and the conversion factor for converting yards to meters is  $\frac{0.9144}{1}$ . Use the conversion chart in the My Notes column to find the conversion factors for converting grams to ounces and converting liters to quarts.

### My Notes

### MATH TIP

In general, conversions between customary and metric systems result in approximate measurements.

The symbol  $\approx$  means "is approximately equal to."

### MATH TIP

**Conversion factors** for some common customary and metric measures:

$$1 \text{ yd} \approx 0.9144 \text{ m}$$

$$1 \text{ m} \approx 1.094 \text{ yd}$$

$$1 \text{ in.} \approx 2.54 \text{ cm}$$

$$1 \text{ mi} \approx 1.61 \text{ km}$$

$$1.06 \text{ qt} \approx 1 \text{ L}$$

$$1 \text{ oz} \approx 28.4 \text{ g}$$

$$1 \text{ lb} \approx 0.4536 \text{ kg}$$

$$2.2 \text{ lb} \approx 1 \text{ kg}$$

$$1 \text{ cu ft (ft}^3\text{)} \approx 0.0283\text{m}^3$$

My Notes

Five hundred years ago, the toy that we now call a yo-yo was bigger and used as a weapon in the Philippines. Each weighed about 4 pounds and was attached to a 20-ft cord.

3. About how much did one of those killer yo-yos weigh, in kilograms? Find out by writing and solving a proportion using a conversion factor as one of the ratios. Use a calculator to speed computation.

Check Your Understanding

4. **Use appropriate tools strategically.** Convert rounding your answers to the nearest tenth when necessary.
  - a. 8 in.  $\approx$  \_\_\_\_ cm    b. \_\_\_\_ mi  $\approx$  20 km    c. 16 cm  $\approx$  \_\_\_\_ in.
  - d. \_\_\_\_ L  $\approx$  50 qt    e. \_\_\_\_ km  $\approx$  100 mi    f. 60 g  $\approx$  \_\_\_\_ oz
  - g. 44 lb  $\approx$  \_\_\_\_ kg    h. 500 g  $\approx$  \_\_\_\_ lb    i. 1.5 oz  $\approx$  \_\_\_\_ g
5. Write a short note to your teacher explaining how you would estimate the number of kilometers in 19 miles.

LESSON 8-3 PRACTICE

6. The 50-km walk is the longest track event at the Olympics. To the nearest mile, about how long is the race in miles?
7. The Tour de France bicycle race is not only challenging; at 2,300 miles, it is long! In kilometers, about how long is the race?
8. The fastest ball game in the world may well be Jai-Alai. In it, players use a scoop attached to their hand to throw a small hard ball as fast as 188 mph at a granite wall. To the nearest tenth of a kilometer, about how fast is that speed in km/h?
9. **Reason abstractly and quantitatively.** A baseball used in major league games weighs at least 5 oz and not more than 5.25 oz. About what is that range measured in grams? Explain your reasoning.
10. About 50 years ago, the Yankees' Mickey Mantle was one of baseball's great sluggers. He is credited with hitting the longest homerun ever. It traveled a distance of 643 feet. How many kilometers did the ball travel, rounded to the nearest hundredth?
11. **Reason abstractly and quantitatively.** How many km/h equals 880 ft/min? Explain how you solved this problem.
12. **Make sense of problems.** Ed can run a mile in 6 min 30 sec. Ned can run a kilometer in 4 min. Who runs at a faster rate? Explain.

### ACTIVITY 8 PRACTICE

Write your answers on notebook paper. Show your work.

#### Lesson 8-1

There are five position players on a starting basketball team: 2 guards, 2 forwards, 1 center.

Write a ratio in simplest form to express each relationship.

- centers to forwards
- forwards to guards
- guards to players on the team
- guards to players on the court
- players who are not centers to players on the court

For Items 6–9, determine the rate and the unit rate.

- \$279 for 9 tickets
- \$18 for 6 volleyballs
- 4 fouls in 20 minutes
- 36 strikeouts in 54 innings

A total of 180 students and 35 chaperones are going on a field trip to the Smithsonian Institution in Washington, D.C.

Write each ratio in simplest form.

- the ratio of students to chaperones
- the ratio of chaperones to students
- the ratio of students to people on the trip.

Determine the unit rate. Use mental math when you can.

- 6 golf balls for \$15
- 2 dozen tennis balls for \$36
- 4 lb meat for \$18
- 24 tickets for \$480

Determine if each pair of ratios are equivalent.

- $\frac{8}{5}, \frac{24}{20}$
- $\frac{0.5}{10}, \frac{5}{100}$
- $\frac{1.3}{5.2}, \frac{12}{48}$

Density is the ratio of mass to volume. A 3-liter jug of honey has a mass of 4.5 kg.

20. Write the density of honey as a ratio in three different ways.

21. Write the density of honey as a unit rate.

#### Lesson 8-2

Solve.

22. In 2002, Takaru Kobayashi ate 50 hot dogs in 12 minutes! At that rate, and assuming that he wouldn't explode, how many dogs could Takaru eat in an hour?

23. If  $\frac{3}{4}$ -cup of packed brown sugar is needed for one batch of chocolate chip cookies, how much packed brown sugar is needed for five batches?

A.  $\frac{15}{100}$  cup

B.  $3\frac{3}{4}$  cups

C. 3 cups

D.  $5\frac{3}{4}$  cups

24. **Make use of structure.** If a person walks  $\frac{1}{2}$  mile in  $\frac{1}{4}$  hour, how far does that person walk in  $1\frac{3}{4}$  hours at that rate?

A.  $\frac{1}{8}$  of a mile

B.  $\frac{7}{8}$  of a mile

C. 5 miles

D.  $3\frac{1}{2}$  miles

**Solve by writing and solving a proportion.**

25. One recipe for pancakes says to use  $1\frac{1}{2}$  cup of mix to make 7 pancakes. How much mix is needed to make 35 pancakes?
26. At the local pizza parlor, game tickets can be traded for small toys. The rate is 10 tickets for 4 small toys. If Meg won 55 tickets playing skeeball, for how many small toys can she trade her tickets?
27. The ratio of boys to girls on a swimming team is 4 to 3. The team has 35 members. How many are girls?
28. Jay made 8 of 10 free throws. Kim made 25 of 45. Who made free throws at the better rate? How do you know?

Troy is going to Spain and needs to convert his dollars to Euros. He knows that when he goes, \$5.00 is equivalent to about 3.45 Euros.

29. Find the unit rate of Euros per dollar
30. How many Euros will he get for \$125?
31. About how many more or fewer Euros would Troy get for \$125 if the exchange rate had changed to 0.75 Euros per dollar?

### Lesson 8-3

**Convert. Round your answers to the nearest hundredth, as needed.**

32. 6 in.  $\approx$  \_\_\_\_ cm
33. 500 g  $\approx$  \_\_\_\_ oz
34. 24 lb  $\approx$  \_\_\_\_ kg
35. \_\_\_\_ mi  $\approx$  40 km
36. \_\_\_\_ oz = 170.4 g

**Solve. Use the conversion factors provided on page 85. As needed, round answers to the nearest hundredth.**

37. How many ounces are in 80 grams?
38. What might weigh 20 kg: a small car, a tablet, a heavy suitcase, or a watermelon?
39. A recipe calls for 8 oz of raisins. The raisins come in 100-gram packages. How many packages do you need to buy?
40. A golf ball weighs about 45.9 grams. About how many ounces would a dozen golf balls weigh?
41. A regulation volleyball can weigh anywhere from 260 grams to 280 grams. In ounces, what is the least a volleyball can weigh?
42. The most a bowling ball can weigh is 7,258 grams. What is the most it can weigh when measured in pounds?
43. Lisa can run a mile in 7 minutes. At that rate of speed, how long would it take her to run 2 kilometers?
44. Jen can run a mile in 8 minutes. Which is the most reasonable time for her to run a 10-km race: 1.6 min, 5 min, 50 min, or 500 min?

An official rugby ball can weigh anywhere from 383 grams to 439 grams.

45. What is the least one of these balls can weigh, measured in ounces?

### MATHEMATICAL PRACTICES

#### Make Sense of Problems

46. The record for the most Major League Baseball career innings pitched is held by Cy Young, with 7,356 innings. If the average length of an inning is 19 minutes, how many minutes did Young play in Major League games? How many hours is this?

# Proportional Reasoning

## Scrutinizing Coins

### Lesson 9-1 Equations Representing Proportional Relationships

#### ACTIVITY 9

#### Learning Targets:

- Given representations of proportional relationships, represent constant rates of change with equations of the form  $y = kx$ .
- Determine the meaning of points on a graph of a proportional relationship.
- Solve problems involving proportional relationships.

**SUGGESTED LEARNING STRATEGIES:** Shared Reading, Marking the Text, Summarizing, Use Manipulatives, Look for a Pattern, Predict and Confirm, Discussion Groups

Ratios and proportions are used to solve all kinds of problems in the real world. For example, ratios and proportions are used in cooking to double recipes, by travelers to find distances on maps, and by architects to make scale models.

Work with your group to explore the proportional relationship between the number of pennies in a stack and their heights in millimeters. You will need a centimeter ruler and 25 pennies. As you work with your group, you may hear math terms or other words that are unfamiliar. Record words that are frequently used in your math notebook. Ask for clarification of their meaning and make notes to help you remember how they are used.

- Without using your pennies or ruler, predict the height of a stack of 150 pennies, and explain why you made this prediction. Be sure to include units in your prediction.

- Attend to precision.** Explore this finding by measuring and recording the height of a stack of each number of pennies in the table below.

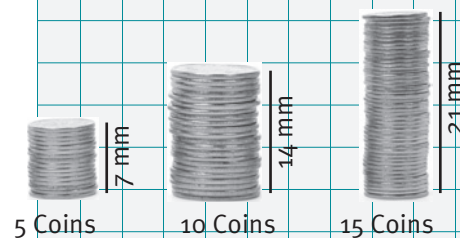
Number of Pennies	10	15	20	25
Height of Stack (mm)				

- Write a ratio in fraction form that relates the number of pennies to the height of a stack.
  - 10 pennies
  - 15 pennies
  - 20 pennies
  - 25 pennies
- Write a ratio that relates the number of pennies in each stack at the right to the height of the stack.

#### My Notes

#### DISCUSSION GROUP TIPS

As you explore and discuss the relationship between the number of pennies in a stack and their heights, demonstrate listening comprehension of what each group member says by taking notes on their contributions. Ask and answer questions to clearly aid understanding of all group members' ideas.





My Notes

5. What do you notice about the ratios you wrote in Item 3?

6. Use the ratio you found in Item 4 and proportional reasoning to complete the table below.

Number of Pennies	10	15	20	25
Height of Stack (mm)				

7. Use your table in Item 6 to answer the following.

a. Write two ratios in fraction form relating the number of pennies to the height of the stacks.

b. Write these ratios as an equation.

c. Is your equation a proportion? Explain why or why not.

8. When quantities are proportional, they have a **constant rate of change**.

a. What is the rate of change of the stack of coins in Item 4?

b. Explain what the rate of change in Item 4 means.

9. **Make use of structure.** How could you find the height of a stack of 60 pennies without having 60 pennies to measure? Determine a reasonable estimate of the height and explain your method.

10. Now suppose you wanted to find the height of a stack of 372 pennies. Determine a reasonable estimate and explain your method.

11. Compare and contrast your methods for answering Items 9 and 10.

MATH TERMS

A **rate of change** is the ratio of output values to corresponding input values. If the rate of change remains the same throughout a problem situation, it is a **constant rate of change**.

## Lesson 9-1

### Equations Representing Proportional Relationships

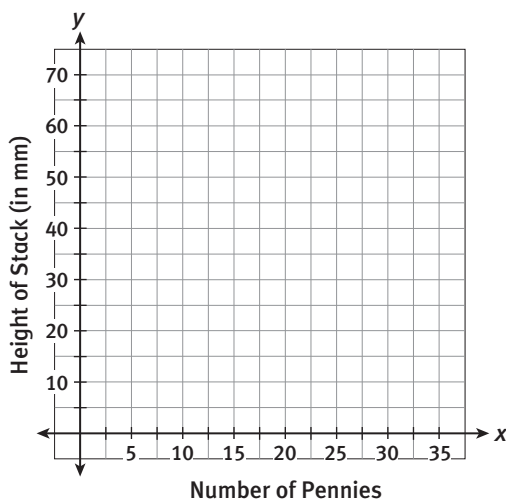
#### ACTIVITY 9

continued

12. Why might the value you determined for height in Items 9 and 10 be different from the actual measured height of a stack of 60 pennies or 372 pennies?
13. Write and solve a proportion to determine the number of pennies,  $x$ , in a stack that is 100 mm high. Use numbers, words, or both to explain your method.

The proportional relationship between the number of pennies in a stack and the height of the stack that you recorded in the table in Item 7 can also be represented in a graph. The graph will help you predict the height of a stack of pennies.

14. Graph the data from Item 7 onto the graph.



15. What does a point  $(x, y)$  on the graph mean for this situation?
16. **Construct viable arguments.** Does it make sense to include the point  $(0, 0)$  on your graph? Explain. If yes, plot  $(0, 0)$  on your graph.

My Notes

MATH TIP

Remember to think about whether or not you should connect the points on your graph.

WRITING MATH

Another way to write a proportional relationship is as an *equation* of the form  $y = kx$ , where the constant of rate of change is  $k$ .

17. If the points on your graph were connected what would the graph look like?
18. How does the graph in Item 14 show a constant rate of change?
19. Does it make sense to include the point  $\left(\frac{1}{2}, \frac{5}{14}\right)$  on your graph? Explain.
20. Use your graph to predict the height of a stack with only one penny in it. Explain your method.
21. What does it mean for the ratio of number of pennies to height of the stack of pennies to be in the ratio 1:1.4?
22. Find the height of a stack of 30 pennies.
  - a. Use the graph. Explain your reasoning.
  - b. Using the height of 1 penny that you found in question 21. Explain your reasoning.
23. What equation could you write to find the height  $y$  in millimeters of any number of pennies  $x$ ?
24. Use your equation in Item 24 to find the height of a stack of 35 pennies. Confirm this solution using your graph.

## Lesson 9-1

### Equations Representing Proportional Relationships

## ACTIVITY 9

continued

### Check Your Understanding

- 25. Model with mathematics.** Look back at your original prediction for the height of a stack of 150 pennies.
- Use a proportion to revise your original prediction. Explain your reasoning.
  - Use the equation you wrote in Item 24 to revise your original prediction. Justify your reasoning.
  - Explain how you could use your graph to revise your original prediction.

### LESSON 9-1 PRACTICE

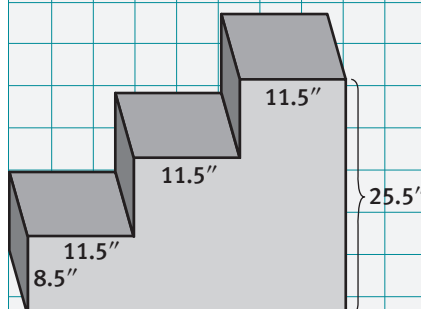
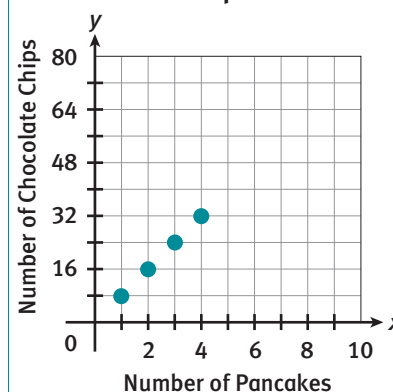
- 26.** Solve the proportion  $\frac{4}{5} = \frac{28}{x}$  using two different methods. Explain each method.
- 27. Construct viable arguments.** Solve  $\frac{x}{42} = \frac{3}{7}$  using two different strategies. Explain each strategy.
- 28.** Is the ratio 4.2:1.5 proportional to the ratio 12.6:4.5? Explain.
- 29.** Is the ratio 35 to 10 proportional to the ratio 7 to 5? Explain.
- 30.** At Lake Middle School, the average ratio of boys to girls in a classroom is 3:2. Use a proportion to predict the number of girls in a classroom that has 15 boys.
- 31.** Complete the ratio table below to show ratios equivalent to 4:18.

48	160		8	
		20		90

- 32.** Use the graph at the right.
- Predict the number of chocolate chips in nine pancakes. Explain.
  - Predict the number of pancakes that would have 48 chocolate chips. Explain.
  - What does the point (1, 8) mean in this situation?
  - Which of the equations below represents this situation?
    - $y = 16x$
    - $y = 8x$
    - $y = x$
    - $y = 48x$
- 33.** Three steps of a staircase are shown here.
- What is the ratio of the width of a step to its height?
  - Explain why the staircase represents a constant rate of change.
  - What does the rate of change mean in the context of a staircase?

My Notes

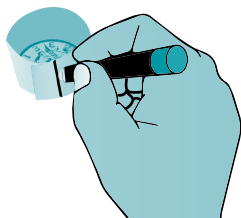
Chocolate Chips in Pancakes



My Notes

MATH TIP

To measure the circumference, wrap a piece of tape around the edge of your coin. Make a mark on the tape to show where the tape begins to overlap.



Unwrap the tape and place it along the edge of your centimeter ruler.

MATH TIP

Find the average of a set of data items by adding the items and then dividing by the number of data items.

Learning Targets:

- Determine the constant of proportionality from a table, graph, equation, or verbal description of a proportional relationship.

**SUGGESTED LEARNING STRATEGIES:** Shared Reading, Marking the Text, Interactive Word Wall, Note Taking, Self Revision/Peer Revision

When two quantities are proportional they have a constant rate of change which is the ratio found between the output values and their corresponding input values.

Work with a partner and use the relationship between the circumference and diameter of a circle to explore finding a **constant of proportionality**. You will need a centimeter ruler, tape, a penny, a nickel, a dime, and a quarter.

- Use your tape and ruler to measure the circumference of each coin to the nearest millimeter. Record the measurement in the table below.

	Penny	Nickel	Dime	Quarter
Circumference (mm)				
Diameter (mm)				

- Use your ruler to measure the diameter of each coin to the nearest millimeter. Record the measurement in the table above.
- For each coin write the ratio of the length of the circumference to the length of the diameter as a fraction and as a decimal to the nearest hundredth.
- Because the ratios are very close to being the same, there appears to be a proportional relationship. Calculate the average of their decimal ratios.
- Suppose you had a coin with a diameter of 30 mm. What would you expect its circumference to be? Explain.
- Model with mathematics.** Write an equation in the form  $y = kx$  using the constant of proportionality you found in Item 4 above to determine the approximate circumference,  $y$ , of a coin with a diameter  $x$ . Explain.



## Lesson 9-2

### Constants of Proportionality

## ACTIVITY 9

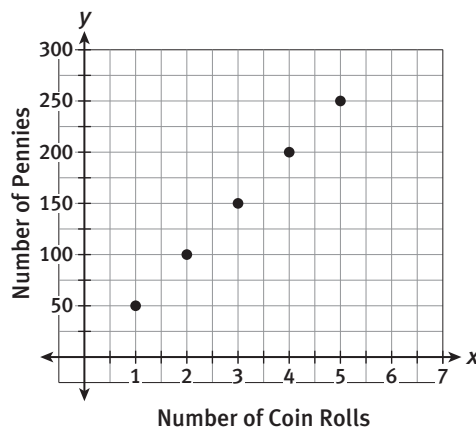
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The factor  $k$  that you multiplied by in Item 6 also represents the **constant rate of change** in the situation.

7. What is the constant rate of change in the equation you wrote?

Graphs can also be used to find a constant of proportionality in proportional relationships.

The graph below shows the number of pennies in a number of standard coin rolls.



8. Plot a point at  $(0, 0)$  and connect the points with a line. What does the point  $(0, 0)$  represent?
9. Create a table showing this information in your My Notes column.
10. Why do the points in the graph lie on a straight line?
11. What is the ratio of number of pennies to the number of coin rolls?
12. Define the variables and write an equation in the form  $y = kx$  for this situation.
13. What is the constant of proportionality in this situation?
14. Describe what the constant of proportionality means in this situation.

My Notes

My Notes

**Check Your Understanding**

Describe how to find the constant of proportionality in each representation below.

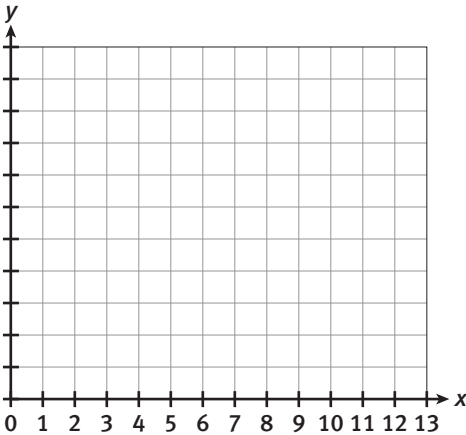
15. A ratio table
16. A graph of a proportional relationship
17. The equation of a proportional relationship

**LESSON 9-2 PRACTICE**

18. There are 40 nickels in every standard coin roll.
  - a. What is the constant of proportionality?
  - b. **Model with mathematics.** Define the variables and write an equation that can be used to show this relationship.
  - c. Create a table of this information.

<b>Number of Coin Rolls</b>						
<b>Number of Nickels</b>						

- d. Represent this information in the graph below.



- e. How many nickels are needed to fill 8 coin rolls? Explain how you determined your answer.

## ACTIVITY 9 PRACTICE

Write your answers on notebook paper.  
Show your work.

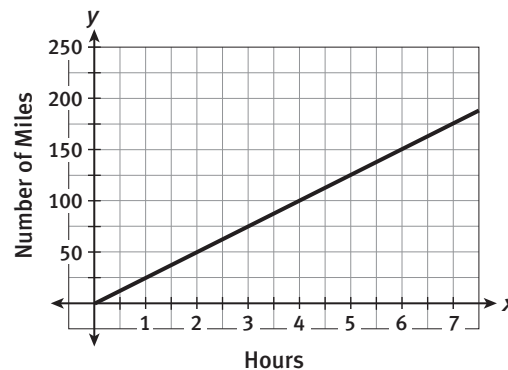
### Lesson 9-1

- Complete the ratio table to show ratios equivalent to 16:10.

2			8	
	36	9		72

- Solve the proportion  $\frac{3}{8} = \frac{21}{x}$  using two different methods. Explain each method.
- Solve  $\frac{x}{48} = \frac{5}{6}$  using two different strategies. Explain each strategy.
- Is the ratio 25 to 16 proportional to the ratio 5 to 4? Explain.
- Are the ratios 2.5:3.5 and 5:7 proportional? Explain.
- Is the ratio 4.2:1.5 proportional to the ratio 12:5? Explain.
- At the library, the average ratio of hardbound books to paperback books on a shelf is 5:3.
  - Use a proportion to predict the number of hardbound books on a shelf that has 75 paperback books.
  - Use a proportion to predict the number of paperback books on a shelf that has 75 hardbound books.

For Items 8–12, use the following graph to make predictions.



- Use the graph to predict the number of miles driven in 8 hours. Choose the correct answer below.
  - 150 miles
  - 175 miles
  - 200 miles
  - 250 miles
- Use the graph to predict the number of hours it would take to drive 162.5 miles. Choose the correct answer below.
  - 15.5 hours
  - 6 hours
  - 6.5 hours
  - 7 hours
- What does the point (0, 0) mean in this situation?
- What does the point (1, 25) mean in this situation?
- Write an equation in  $y = kx$  form to represent this situation.

# ACTIVITY 9

continued

## Lesson 9-2

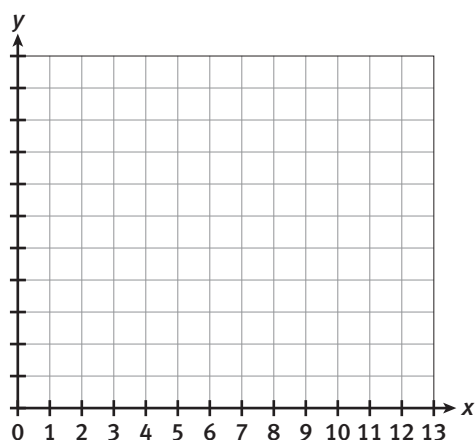
For Items 13–20, use the following information.

A fruit punch uses 1.5 cups of orange juice for every cup of apple juice.

13. What is the constant of proportionality used to find the number of cups of orange juice needed for any amount of apple juice?
14. Define the variables and write an equation that can be used to show this relationship.
15. Create a table of this information.

Apple Juice (cups), $x$					
Orange Juice (cups), $y$					

16. Represent this information in the graph below.



17. How many cups of orange juice are needed for 12 cups of apple juice?

18. How many cups of apple juice are needed for 8 cups of orange juice?
19. What does the point  $(0, 0)$  mean for this situation?
20. What does the point  $(1, 1.5)$  mean for this situation?
21. Therese is on a trip overseas. She uses the table below to determine the conversion rate of her U.S. dollars to British pounds. What is the constant of proportionality?

British Pound, $x$	5	10	15	20	25
U.S. Dollar, $y$	8	16	24	32	40

22. Use the table in Item 21. Write an equation to convert British pounds to U.S. dollars.
23. Use your equation in Item 22 to determine the number of U.S. dollars Therese would spend if she bought an item that cost 110 British pounds.

## MATHEMATICAL PRACTICES

### Model with Mathematics

24. A tire maker produces 20,000 tires to be shipped. They inspect 200 of the tires and find that 16 are defective. How many tires of the 20,000 tires would you expect to be defective?

# Ratios, Proportions, and Proportional Reasoning

## WEIGHING IN ON DIAMONDS

### Embedded Assessment 1

Use after Activity 9

Write your answers on notebook paper. Show your work.

You may have had diamonds in your mouth before. Many dentists' drills are embedded with diamonds. In fact, 18% of your body is made up of carbon, and diamonds are also made of compressed carbon. That must mean you are priceless!

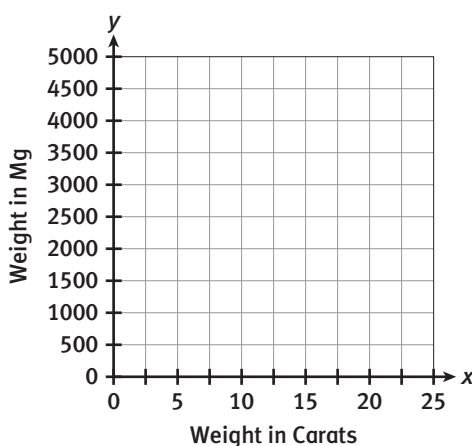
For Items 1–8, use the following information.

Diamonds are weighed in units called carats. Carat weight is based on the diamond's weight in milligrams. The table at the right shows the relationship between carats and milligrams.

1. Write an equation to convert a diamond's weight in carats to its weight in milligrams. Be sure to define your variables.
2. What is the constant of proportionality represented in the table at the right?
3. Complete the last row of the table by using the constant of proportionality.
4. Use your equation to find the weight in milligrams of the Tiffany Yellow Diamond, which weighs 287.42 carats.

Weight in Carats	Weight in Milligrams
$\frac{1}{2}$	100
2	400
4	800
6	1200
10	2000
25	

5. Create a graph of the information in the table.
6. Explain the meaning of the point (0, 0) on your graph.
7. Use your graph to determine the weight in milligrams of a diamond weighing 8 carats.
8. Give the ordered pair for the point on the graph that shows how many milligrams a 1-carat diamond weighs.



Solve.

9. The Cullinan is the largest rough gem-quality diamond ever found. It was 3,106.75 carats. It weighed about 0.62 kg uncut. Recall that 1 kg is equal to 2.2 pounds. What was the uncut Cullinan weight in pounds?
10. How many pounds would a 0.5 kg diamond weigh?
11. The ratio of a diamond's hardness to its specific gravity is 10:3.515, and the ratio of the hardness to specific gravity for a ruby is 9:4.05. Are these ratios in proportion? Explain your answer.



For Items 12–13, use the following information.

The largest diamond is thought to be Lucy, a star consisting of diamonds. Its weight is *10 billion trillion trillion carats*. Lucy is about 50 light-years away from Earth. One light-year is about 5.87 trillion miles, or the distance light travels through space in one year.

12. Use a proportion to determine how many trillion miles away from Earth Lucy is.
13. Write an equation in  $y = kx$  form to represent this situation. Use the equation to check your answer from Item 11.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
<b>Mathematics Knowledge and Thinking</b> (Items 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13)	<ul style="list-style-type: none"> <li>Clear and accurate understanding of ratios, unit rates, and solving proportions.</li> </ul>	<ul style="list-style-type: none"> <li>An understanding of ratios, unit rates, and solving proportions that usually results in correct answers.</li> </ul>	<ul style="list-style-type: none"> <li>An understanding of ratios, unit rates, and solving proportions that sometimes results in correct answers.</li> </ul>	<ul style="list-style-type: none"> <li>Incorrect or incomplete understanding of ratios, unit rates, and solving proportions.</li> </ul>
<b>Problem Solving</b> (Items 4, 7, 9, 10, 12)	<ul style="list-style-type: none"> <li>Accurate interpretation of the solution of a proportion to solve a problem.</li> </ul>	<ul style="list-style-type: none"> <li>Interpretation of the solution of a proportion to solve a problem.</li> </ul>	<ul style="list-style-type: none"> <li>Difficulty interpreting the solution of a proportion to solve a problem.</li> </ul>	<ul style="list-style-type: none"> <li>Incorrect or incomplete interpretation of the solution of a proportion to solve a problem.</li> </ul>
<b>Mathematical Modeling / Representations</b> (Items 1, 5, 6, 7, 8, 13)	<ul style="list-style-type: none"> <li>Accurate representation of a problem situation with a proportional equation or a graph.</li> </ul>	<ul style="list-style-type: none"> <li>A mostly correct representation of a problem situation with a proportional equation or a graph.</li> </ul>	<ul style="list-style-type: none"> <li>Difficulty representing a problem situation with a proportional equation or a graph.</li> </ul>	<ul style="list-style-type: none"> <li>An incorrect or incomplete representation of a problem situation with a proportional equation or a graph.</li> </ul>
<b>Reasoning and Communication</b> (Items 6, 11)	<ul style="list-style-type: none"> <li>Precise use of appropriate math terms and language to explain proportional relationships.</li> </ul>	<ul style="list-style-type: none"> <li>An adequate explanation of solutions using proportional relationships.</li> </ul>	<ul style="list-style-type: none"> <li>A misleading or confusing explanation of solutions using proportional relationships.</li> </ul>	<ul style="list-style-type: none"> <li>An incomplete or inaccurate description of solutions using proportional relationships.</li> </ul>

# Proportional Relationships and Scale

## ACTIVITY 10

### Patriotic Proportions

#### Lesson 10-1 Using Scale Drawings

#### Learning Targets:

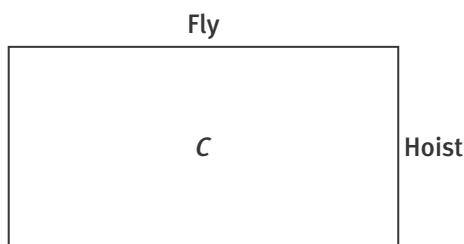
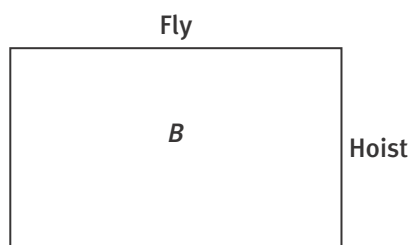
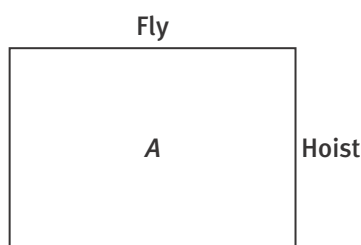
- Represent proportional relationships by equations.
- Determine the constant of proportionality from a table, graph, equation, or verbal description of a proportional relationship.
- Solve problems using scale drawings.

**SUGGESTED LEARNING STRATEGIES:** Close Reading, Marking the Text, Paraphrasing, Critique Reasoning, Create Representations

Martha Rose Kennedy was watching an old black-and-white movie about World War II, and during the parade scene she noticed that the flags seemed to have a “funny shape.” Martha did a little research and learned that according to the *U.S. Code, Title 4, Chapter 1*, the ratio of the **hoist** (height) to the **fly** (width) of the flag should be 1:1.9.

However, in the 1950s, President Dwight D. Eisenhower eased the restrictions on the dimensions of the U.S. flag to accommodate current standard sizes, such as 3 feet  $\times$  5 feet, 4 feet  $\times$  6 feet, and 5 feet  $\times$  8 feet.

1. Without using a ruler, predict which of the following rectangles will have a ratio:  $\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9}$ . Explain how you made your decision.



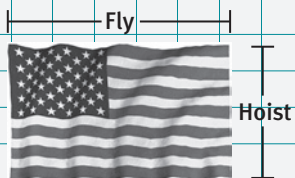
2. a. If  $\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9}$  and the hoist is 1 ft, calculate the fly.

My Notes

WRITING MATH

Another way to write a proportional relationship is as an equation of the form  $y = mx$ , where the constant of proportionality is  $m$ .

Example: The cost of a flag is \$5. If  $x$  = the number of flags purchased, the equation  $y = \$5.00x$  could be used to determine the cost of  $x$  flags.



- b. This relationship can also be shown using the equation  $y = 1.9x$ , where  $y$  = the length of the fly and  $x$  = the length of the hoist. Show how the fly can be found using this equation.
  - c. What is the constant of proportionality in the equation in part b?
  - d. If the ratio remains constant and the hoist of the flag is 2 ft, calculate the fly using the constant of proportionality.
  - e. Check your work from part c by using a proportion to calculate the fly.
3. a. Complete the following table, which displays the correct hoist and fly of the U.S. Flag according to the *U.S. Flag Code*. Assume that the units of measure are the same for the hoist and fly and that  $\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9}$ .

Hoist	Fly
1	
2	
3	
4	
5	
	17
	19
	25
	38

- b. **Use appropriate tools strategically.** How can the constant of proportionality be determined from a table? Explain using the table above as an example.

## Lesson 10-1

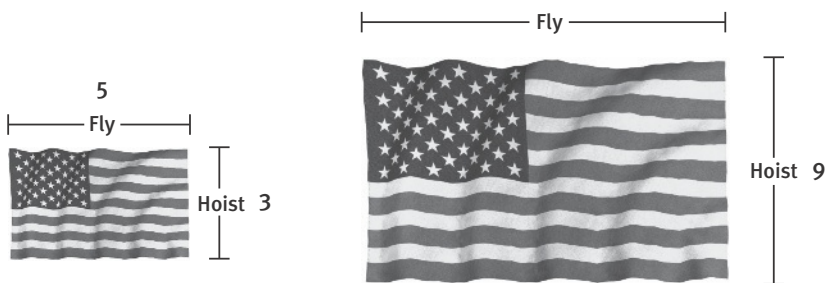
### Using Scale Drawings

## ACTIVITY 10

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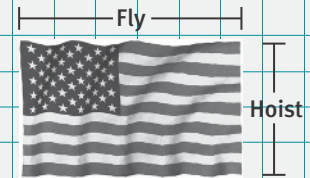
4. Suppose that the proportion used to make a flag is 3:5.
  - a. Use the method of determining the constant of proportionality from Item 3 to determine the constant of proportionality for this flag. Let  $x$  = the length of the hoist.
  - b. Write the equation for determining the length of the hoist or fly.
  - c. If the length of the hoist is 7.5 ft, determine the length of the fly.
  - d. Check your work from part c by using a proportion to calculate the fly.
  - e. Find the fly if the hoist is 1 ft. Show your work.
5. Find the difference in the lengths of a 3:5 flag and the flag measurements in the table in Item 3 whose hoist is 3 ft. Which flag has the longer fly?

Sometimes a scale drawing is used to represent the data in a problem or to show the **relative size** of two items. Use the drawings below to solve.



6. a. Write a proportion to represent the drawing.
- b. Use the proportion to calculate the length of the unknown fly.

### My Notes



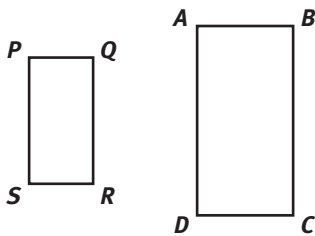
### MATH TERMS

The **relative size** of two items shows how the size of one item is larger or smaller than the other item.

For example: The relative size of a baby is small next to a full grown adult.

### MATH TERMS

A **scale drawing** of a figure is a copy of the drawing with all lengths in the same ratio to the corresponding lengths in the original. If Quad  $ABCD$  is a scale drawing of Quad  $PQRS$  then  $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CD}{RS} = \frac{AD}{PS}$ .



### Check Your Understanding

Martha Rose decides to investigate further by creating a **scale drawing** of the U.S. flag including the thirteen stripes and the blue field for the stars. She chooses the following characteristics for her flag.

- $\frac{\text{height}}{\text{width}} = \frac{3}{5}$
  - There are 7 red stripes and 6 white stripes, all of which have the same height, the hoist.
  - The height of the blue field equals the height of seven stripes.
  - $\frac{\text{height of the blue field}}{\text{width of the blue field}} = \frac{2}{3}$
7. The height of one stripe is what fraction of the height of the entire flag?
  8. The height of the blue field is what fraction of the height of the flag?
  9. **Reason quantitatively.** Since all of the information concerning the dimensions of the flag and its parts are given in terms of the height, Martha decides to begin her scale drawing by choosing 13 cm for the height. Explain why 13 cm is a good choice for the height.

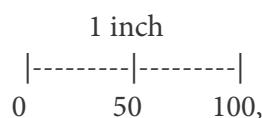
### LESSON 10-1 PRACTICE

10. Write an equation for the following proportional relationships.
  - a.  $\frac{1}{2.5}$
  - b.  $\frac{6}{18}$
  - c.  $\frac{5}{22.5}$
  - d.  $\frac{4}{17}$
11. Determine the constant of proportionality for the following.
  - a. a ratio of 3 to 4
  - b. the point (2, 5) on a graph
  - c. three red to two blue
  - d.  $7 = 5m$
12. The long side of a rectangle is 4 times the side of a square of length 3. What is the length of the side of the rectangle?
13. **Make sense of problems.** Two stripes on the American flag represent what fraction of the height of the flag?



- Given the scale of a map and a distance on a map, find the actual distance.
- Convert scale factors with units to scale factors without units

The Green family set out by car from Boston to visit the Statue of Liberty in New York City, the Liberty Bell in Philadelphia, and the nation's capital in Washington, D.C. They had a map of the Northeast region of the United States. In the corner was a scale that showed this:



which gives the proportion  $\frac{\text{inches}}{\text{miles}} = \frac{1}{100}$ .

1.
  - a. Write the proportion to find the number of miles from Boston to Philadelphia if the distance measures 3.08 inches on the map.
  - b. Solve the proportion for the number of miles.
2. The next stop was Washington, D.C., which was 1.38 inches farther from Boston. What was the total mileage from Boston to Washington, D.C.?
3. **Reason quantitatively.** The distance from Boston to New York is 216 miles.
  - a. If this represents 4 inches on the map, what is the scale used?
  - b. Calculate the number of inches on the map if the scale is  $\frac{\text{inches}}{\text{miles}} = \frac{1}{50}$ ?
  - c. What is the difference in miles travelled over 4 inches using the scales in parts a and b?

**My Notes**

The Green family thought that they would travel to Mount Rushmore for their next trip. Since that is in Keystone, South Dakota, they need to use a different map. The scale of that map is  $\frac{\text{inches}}{\text{miles}} = \frac{1.5}{250}$ .

4. The distance on the map is approximately 11.5 inches. How many miles is this?
5. How many inches represent  $\frac{3}{4}$  of the trip?

After visiting Washington, D.C., Joey Green wanted to make a scale model of the Washington Monument. The actual height is 555 feet.

6. a. Is the scale  $\frac{\text{inches}}{\text{feet}} = \frac{1}{25}$  a reasonable scale for the model?
- b. What is the scale if the height of the model is 10 inches?

The scales that the Green family have been using have different units. If they want to eliminate the units, both need to be the same. For example,  $\frac{1 \text{ inch}}{1 \text{ foot}} = \frac{1 \text{ inch}}{12 \text{ inches}} = \frac{1}{12}$ .

7. As a class project, you are asked to make a scale drawing of your home.
  - a. What unit should be used for a scale drawing of your bedroom?
  - b. What unit should be used for a scale drawing of your yard?
8. If feet were used as the unit for the model of the Washington Monument, what would the scale be if the model is only 10 inches high?

## Lesson 10-2

### Using Maps

## ACTIVITY 10

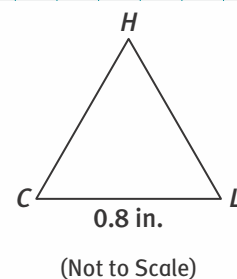
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- 9. Model with mathematics.** On a neighborhood map shown at the right, 1 inch = 5 miles. If the distance from your house ( $H$ ) to the convenience store ( $C$ ) is 3 miles, the distance from your house to the library ( $L$ ) is 6 miles, and the distance from the convenience store to the library is 4 miles, label the path from  $H$  to  $C$  and from  $H$  to  $L$  in inches.

### Check Your Understanding

- 10.** If the scale on a map is  $\frac{\text{inches}}{\text{miles}} = \frac{1}{10}$ , find the following actual distances.
- a. 2 inches      b.  $\frac{1}{2}$  inch      c. 3.5 inches
- 11.** Two towns on a map are  $2\frac{1}{4}$  inches apart. The actual distance between the towns is 45 miles. Which of the following could be the scale on the map?
- a.  $\frac{1}{2}$  inch : 10 miles      b. 1 inch : 5 miles      c. 1 inch : 20 miles

### My Notes



## LESSON 10-2 PRACTICE

- 12.** If the scale on a map is  $\frac{\text{inches}}{\text{miles}} = \frac{1}{50}$ , find the following actual distances.
- a. 2.5 inches      b. 4.75 inches      c. 7.6 inches
- 13.** If the scale on a map is  $\frac{\text{inches}}{\text{feet}} = \frac{2}{10}$ , how would the following lengths be represented?
- a. 15 feet      b. 30 feet      c. 55 feet
- 14.** On a map, the scale is  $\frac{\text{inches}}{\text{miles}} = \frac{1}{75}$ . Going from home to the first destination is 1.5 inches, and then from there to the next destination is 2.25 inches. How many miles were traveled?
- 15. Reason abstractly.** You are making a scale model of the White House using blocks that represent 15 feet. If the length of the White House is 170 feet, can you use these blocks without having a part of the block extend outside the length?
- 16. a.** A scale drawing of your classroom is 3 inches by 5 inches. If one inch represents 6 feet, what is the actual size of your classroom?
- b.** Using another scale, the shorter side of the classroom is 1.5 in. in a scale drawing. What is the length of the longer side of the classroom in the same scale drawing?

My Notes

**Learning Targets:**

- Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing.
- Reproduce a scale drawing at a different scale.

**SUGGESTED LEARNING STRATEGIES:** Close Reading, Visualization, Create a Plan, Create Representations, Look for a Pattern

The Constitution of the United States is a document with dimensions  $23\frac{5}{8} \times 28\frac{3}{4}$  inches.

The Declaration of Independence is  $24\frac{1}{2} \times 29\frac{3}{4}$  inches.

Recall that the scale of a figure “redraws” the figure with a new size.

1. **a.** If the Declaration of Independence is reproduced at half scale, what would be its dimensions?

- b. Attend to precision.** What is the area of this document? (Round to the nearest hundredth.)

- c.** What would be the dimensions of the Constitution reproduced at  $\frac{1}{4}$  scale?

2. A student version of the Constitution fits on a piece of paper that is  $8\frac{1}{2} \times 11$  inches. Which of the following is the largest scale that can be used so that it fits on the paper?

- A.**  $\frac{1}{8}$       **B.**  $\frac{1}{4}$       **C.**  $\frac{3}{8}$       **D.**  $\frac{1}{2}$

**MATH TIP**

To find the scale you will need to use division.

## Lesson 10-3

### Make Scale Drawings

## ACTIVITY 10

continued

3. a. How would you determine the largest scale to use for the Declaration of Independence to allow it to fit on an  $8\frac{1}{2} \times 11$  inch piece of paper?

- b. What is the largest scale that can be used?

The New England Patriots are a professional football team. Some information about football fields is given below.

- The dimensions are 160 ft by 360 ft.
- The end zone is 160 ft by 30 ft.
- The upright (goal post) is 10 ft off the ground.
- The width of the upright is 18 ft, 6 in.

4. a. **Make sense of problems.** What are the dimensions of a field that is drawn  $\frac{1}{8}$  scale?

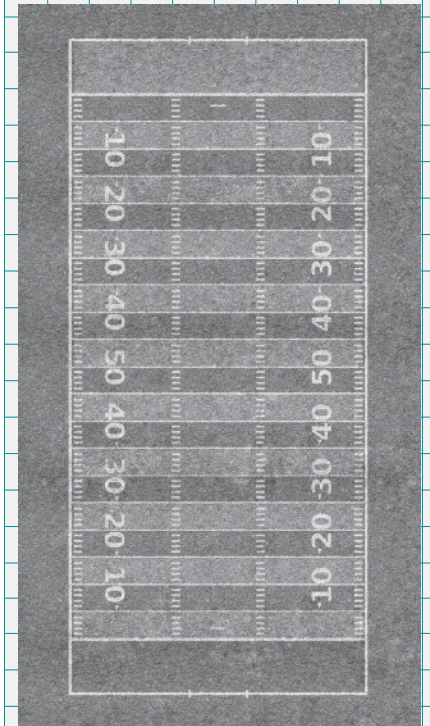
- b. The area of a full-sized field is how many times larger than the area of a field at  $\frac{1}{8}$  scale?

- c. Sketch and label a diagram of a football field reproduced at  $1\frac{1}{4}$  scale.

5. For a backyard game of football you make an upright that is 5 ft off the ground and 9 ft 3 in. high. What scale did you use?

6. **Express regularity in repeated reasoning.** A football field scaled to  $\frac{1}{4}$  its size is then scaled to  $\frac{1}{4}$  of its size again. What are the dimensions of this field?

### My Notes



My Notes

Check Your Understanding

7. The dimensions of a professional basketball court are 50 ft by 94 ft. A game maker is producing a video basketball game and the court must be reproduced to fit on a computer screen that is 13 inches by  $10\frac{1}{2}$  inches. Which of the following scales is the greatest they can use to fit on the screen?  
A.  $\frac{1}{2}$       B.  $\frac{1}{8}$       C.  $\frac{1}{100}$       D.  $\frac{1}{150}$
8. A book cover is 8 inches by  $10\frac{1}{4}$  inches. What are the dimensions of the cover when it was reproduced for a catalog picture using a  $\frac{1}{12}$  scale?
9. A car model box has " $\frac{1}{8}$  scale" printed on the outside of the box. If the actual car is 178 inches long, what is the length of the model?

LESSON 10-3 PRACTICE

10. A document is 20 in. by 34 in. What are the dimensions of documents using the following scales?  
a.  $\frac{1}{4}$       b.  $\frac{1}{2}$       c.  $\frac{3}{4}$       d.  $1\frac{1}{2}$
11. a. A teacher wants a large poster of the Declaration of Independence that is three times its actual size. What are the dimensions of the poster?  
b. What is the area of the poster?
12. **Reason quantitatively.** A copy of a document that was originally 24 in. by 36 in. is now 8 in. by 12 in. What scale was used for the reduction?
13. a. A giant paper football field is made for the floor of the gym. The length of the gym is 90 ft. Using this same scale, determine the width of the paper football field.  
b. What is the area of the paper field?  
c. Compare the area of the paper to the area of an actual field.  
d. Using this scale, what would the height and width of the upright be?
14. A document that is 30 in.  $\times$  40 in. is redrawn at  $1\frac{1}{2}$  scale and redrawn again at  $\frac{1}{2}$  scale. What are the final dimensions?

Football Field Measurements

- The dimensions are 160 ft by 360 ft.
- The end zone is 160 ft by 30 ft.
- The upright (goal post) is 10 ft off the ground.
- The width of the upright is 18 ft 6 in.



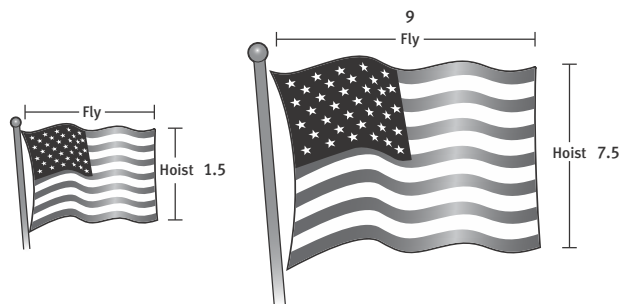
### ACTIVITY 10 PRACTICE

Write your answers on notebook paper.

Show your work.

#### Lesson 10-1

- The ratio of the hoist to the fly of an American Flag is  $\frac{\text{hoist}}{\text{fly}} = \frac{1}{1.9}$ .
  - Using this ratio, determine the fly of a flag that has a hoist of 3 feet.
  - If the ratio is changed to 3:5, determine the fly that has a hoist of 4.5 feet.
- In the equation  $y = 1.9x$ , 1.9 represents the constant of proportionality.
  - Find the constant of proportionality for the ratio  $\frac{2}{5}$ .
  - Find the constant of proportionality for the ratio  $\frac{4}{7}$ .
- Use the following drawings to determine the missing dimension.



#### Lesson 10-2

- The scale on a map has the proportion  $\frac{\text{inches}}{\text{miles}} = \frac{1}{50}$ .
  - How many miles is 3.25 inches on the map?
  - How many miles is 5.4 inches on the map?
- If 3 inches on a map covers 225 miles, what is the scale of inches to miles?
  - If 4.5 inches on a map covers 270 miles, what is the scale of inches to miles?
- The Statue of Liberty is 305 feet tall. What scale would be used to make a model 20 inches high?
- A sketch of the Roosevelt Room in the White House is drawn to  $\frac{1}{15}$  scale. The sketch shows a room that is 3 feet by 4.5 feet. What are the actual dimensions of the room?

**Lesson 10-3**

8. To make a large poster of the Bill of Rights to hang on the classroom wall, a poster that is 22 inches by 46 inches is copied. If the classroom version is to be  $3\frac{1}{2}$  times scale, what are the dimensions of the new document?
9. Complete the following table.

Scale	Length	Width
1	14	8
1.5		
2		
2.5		

**MATHEMATICAL PRACTICES**

**Reason Quantitatively**

10. **a.** Using the data in the table for Item 9, what is the area of the item with scale 1.5?
- b.** What is the area of the item with scale 2.5?

Write your answers on notebook paper. Show your work.

A competitive youth soccer team is preparing for a soccer tournament.

1. The coach uses a scale drawing of the soccer field, shown in the diagram, to review plays with the team. The diagram uses a scale of  $\frac{1}{4}$  in.:30 ft
  - a. Explain how to use the scale and the scale drawing to find the actual dimensions of the soccer field.
  - b. What are the actual length and the width of the soccer field?
  - c. What is the actual area of the soccer field?
2. The shaded box indicates the goal box. How long is the actual goal box?
3. The center circle is not included in the diagram. On the field, the center circle has a diameter of 60 feet. How long would the diameter of the center circle be if it were included on the scale drawing? Explain your thinking.
4. The coach wants to make a larger version of the scale drawing to distribute to team members.
  - a. Use the scale  $\frac{1}{2}$  in.:15 ft to reproduce the scale drawing. Explain your thinking.
  - b. The actual width of the goal box is 18 feet. Include the goal box, to scale, in the new scale drawing.
5. The soccer team must travel to the tournament. On a map, the tournament is 6.5 centimeters away. The map scale is 2 cm = 25 mi.
  - a. What is the actual distance represented by 1 cm on the map
  - b. How far will the team have to travel to the tournament?



Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
<b>Mathematics Knowledge and Thinking</b> (Items 1a-c, 2, 3, 4a-b, 5a-b)	<ul style="list-style-type: none"> <li>Effective understanding and accuracy in using proportional relationships and scale to relate dimensions of real objects and dimensions in scale drawings.</li> <li>Correctly determining a scale factor.</li> </ul>	<ul style="list-style-type: none"> <li>Using proportional relationships and scale to relate dimensions of real objects and dimensions in scale drawings with few errors.</li> <li>Determining a scale factor.</li> </ul>	<ul style="list-style-type: none"> <li>Using proportional relationships and scale to relate dimensions of real objects and dimensions in scale drawings with multiple errors.</li> <li>Errors in determining a scale factor.</li> </ul>	<ul style="list-style-type: none"> <li>Incorrect or incomplete understanding of using proportional relationships and scale to relate dimensions of real objects and dimensions in scale drawings.</li> <li>No understanding of determining a scale factor.</li> </ul>
<b>Problem Solving</b> (Items 1b-c, 2, 3, 5b)	<ul style="list-style-type: none"> <li>An appropriate and efficient strategy that results in a correct answer.</li> </ul>	<ul style="list-style-type: none"> <li>A strategy that may include unnecessary steps but results in a correct answer.</li> </ul>	<ul style="list-style-type: none"> <li>A strategy that results in some incorrect answers.</li> </ul>	<ul style="list-style-type: none"> <li>No clear strategy when solving problems.</li> </ul>
<b>Mathematical Modeling / Representations</b> (Item 1a-c, 2, 3, 4a-b, 5b)	<ul style="list-style-type: none"> <li>Clear and accurate understanding of writing a proportion to solve a problem involving scale.</li> <li>Creating a clear and accurate scale drawing.</li> </ul>	<ul style="list-style-type: none"> <li>Writing a proportion to solve a problem involving scale.</li> <li>Creating a scale drawing that is largely correct.</li> </ul>	<ul style="list-style-type: none"> <li>Difficulty writing a proportion to solve a problem involving scale.</li> <li>Difficulty creating a scale drawing.</li> </ul>	<ul style="list-style-type: none"> <li>Little or no understanding of writing a proportion to solve a problem involving scale.</li> <li>An inaccurate or incomplete scale drawing.</li> </ul>
<b>Reasoning and Communication</b> (Item 1a, 3, 4a)	<ul style="list-style-type: none"> <li>Precise use of math terms and language to explain scale and scale drawings.</li> </ul>	<ul style="list-style-type: none"> <li>An adequate explanation of scale and scale drawings.</li> </ul>	<ul style="list-style-type: none"> <li>A misleading or confusing explanation of scale and scale drawings.</li> </ul>	<ul style="list-style-type: none"> <li>An incomplete or inaccurate explanation of scale and scale drawings.</li> </ul>

# Percent Problems

## ACTIVITY 11

### Well, There is More Than One Way Lesson 11-1 Basic Percent Problems

#### Learning Targets:

- Find a percent of a number.
- Find the percent that one number is of another.
- Given the percent and the whole, find the part.

**SUGGESTED LEARNING STRATEGIES:** Shared Reading, Note Taking, Think-Pair-Share, Simplify the Problem, Look for a Pattern

There are several methods you can use to solve **percent** problems.

One way to solve such problems is to use a proportion like the one below.

$$\text{percent} = \frac{\text{number}}{\text{hundred}} \text{ and } \frac{\text{number}}{100} = \frac{\text{part}}{\text{whole}}$$

#### Example A

The percent of moviegoers under age 30 is 31%. There were a total of 1,290,000,000 moviegoers in 2011. What number of moviegoers in 2011 were under age 30?

**Step 1:** Use the given information to set up a proportion.

$$\frac{31}{100} = \frac{m}{1,290,000,000}$$

The fraction, 31 over 100, represents the percent of moviegoers under 30.

The variable  $m$  represents the number of moviegoers under 30. This is the part of the total number of moviegoers, or the whole.

The 1,290,000,000 under the  $m$  represents the total number of moviegoers in 2011.

**Step 2:** Cross-multiply and write the cross products as an equation.

$$31 \cdot 1,290,000,000 = 100 \cdot m$$

**Step 3:** Solve the equation for  $m$ , the number of moviegoers under age 30.

$$\begin{aligned} 31 \cdot 1,290,000,000 &= 100 \cdot m \\ m &= \frac{31 \cdot 1,290,000,000}{100} \\ m &= 399,900,000 \end{aligned}$$

**Solution:** The number of moviegoers under 30 was 399,900,000.

#### My Notes

#### MATH TERMS

A **percent** is a ratio that compares a number to 100 and uses the % symbol.

**My Notes**

If we go back to the original proportion  $\frac{\text{number}}{100} = \frac{\text{part}}{\text{whole}}$  and consider that  $\frac{\text{number}}{100}$  can be written as decimal, we now have the equation:  
 percent (as a decimal) =  $\frac{\text{part}}{\text{whole}}$

In Example A, we wanted to know the **part**, so we can solve the above equation for **part** by multiplying both sides by the **whole**.

$$\text{whole} \cdot \text{percent (as a decimal)} = \cancel{\text{whole}} \cdot \frac{\text{part}}{\cancel{\text{whole}}}$$

The two wholes on the right hand side cancel each other out, and the result is the **percent equation**:

$$\text{whole} \cdot \text{percent (as a decimal)} = \text{part}$$

Substitute the information from the movie situation in the percent equation.

$$\begin{aligned} 1,290,000,000 \cdot 0.31 &= \text{part} \\ 399,900,000 &= \text{part} \end{aligned}$$

The percent equation can also be used when the percent and the part are known, and you want to find the whole.

**Example B**

We know that 68% of a number is 204 but want to know the total that 204 is part of.

**Step 1:** Convert the percent to a decimal.

$$68\% = 0.68$$

**Step 2:** Set up the percent equation using the information you know.

$$\begin{aligned} \text{percent} \cdot \text{whole} &= \text{part} \\ 0.68x &= 204 \end{aligned}$$

**Step 3:** Solve the equation for  $x$  by dividing both sides by 0.68.

$$\frac{0.68x}{0.68} = \frac{204}{0.68} \quad x = \frac{204}{0.68} = 300$$

**Solution:** 68% of 300 is 204. Check using the percent equation.

$$0.68 \cdot 300 = 204$$



## Lesson 11-1

### Basic Percent Problems

## ACTIVITY 11

continued

The percent equation has been used to solve for the part and for the whole. Lastly, use the equation to solve for the percent.

### Example C

What percent is 208 of 320?

**Step 1:** Set up the percent equation using the given information.

$$\text{percent} \cdot \text{whole} = \text{part}$$

$$p \cdot 320 = 208$$

**Step 2:** Solve the equation for  $p$  by dividing both sides by 320.

$$\frac{p \cdot 320}{320} = \frac{208}{320}$$

$$p = \frac{208}{320}$$

$$p = 0.65 = 65\%$$

**Solution:** 208 is 65% of 320.

### Try These A-B-C

- What percent is 49 of 50?
- 30 is 6% of what number?
- 32% of 250 is what number?

### Check Your Understanding

- Women make up 5% of all film directors. There are 122,500 film directors working. How many women are directing films?
- There were 548 films released in 2011. Of these, approximately 8.75% were rated PG (parental guidance suggested). How many PG movies were released in 2011?
- 35% of the total length of the screen at the IMAX is 51.8 feet. What is the total length of the screen?
- 7.5% of the box office receipts on a Friday night are \$270. What are the total box office receipts?
- Make sense of problems.** The movie *Avatar* is the number one movie in U.S. history, earning \$760,507,625. The second place movie is *Titanic*, earning \$600,788,188. What percent is \$600,788,188 of \$760,507,625?
- Attend to precision.** There are 143 people in an audience. Out of this number, 63 are female. What percent of the people in the audience are male?

**My Notes**

**LESSON 11-1 PRACTICE**

**Round your answers to nearest hundredth, if needed.**

7. In a group of 75 fourth graders, 20% do not like hot chocolate. How many students like hot chocolate?
8. The center on the basketball team scored 19 of the team's 98 points. What percent of the points did he score?
9. Ed spent 8.5% of his savings on lunch, which cost \$5.25. How much did he have in savings before lunch?
10. The school band sold T-shirts to fund a trip to play in a parade. They collected \$570, and the band made 34% of that amount. How much money do they have for their trip?
11. **Construct viable arguments.** Is 10% of 10% of 100 the same as 20% of 100?
12. A ticket agency charges a 9.5% fee on all tickets sold. If a ticket costs \$40, what is the fee?
13. **Reason quantitatively.** The student population of a school consists of 300 girls and 500 boys. If 53% of the girls play a sport, and 39% of the boys play a sport, who plays more sports?
14. In a bag of marbles, 12% were red, 14% were blue, and the rest were white. If the bag has 250 marbles, how many were red or blue?
15. Fifty-six out of 128 students went on a trip during vacation. What percent of students went on trips?
16. The school chorus has 52 students, which represents 26% of the seventh graders. How many students are in the seventh grade?

**Learning Targets:**

- Solve problems about sales tax, tips, and commissions.

**SUGGESTED LEARNING STRATEGIES:** Close Reading, Think Aloud, Summarizing, Create a Plan, Identify a Subtask

A sales tax is the amount added to the cost of an item. Sales taxes are most often a percent of the purchase price. Sales tax can be determined using the percent equation. A **tip** is calculated the same way.

**Example A**

In Massachusetts, the sales tax is 6.25%. If someone buys a sweater for \$24.95, what is the total cost, including sales tax?

**Step 1:** First, determine the sales tax on the item using the formula  
percent • whole = part

**Step 2:** Substitute the decimal form of the percent into the equation and determine the amount of the sales tax.  
 $0.0625 \cdot \$24.95 = \$1.56$

**Step 3:** Add the sales tax to the original amount of the sweater to determine the total cost.  
 $\$24.95 + \$1.56 = \$26.51$

**Solution:** The total cost is \$26.51.

**Example B**

A real estate agent earns a commission when he or she sells a home. The commission is 5%. If the agent sells a \$400,000, a \$250,000, and a \$300,000 house in one month, what is the commission for the month?

**Step 1:** Add to find the total value of the homes.  
 $\$400,000 + \$250,000 + \$300,000 = \$950,000$

**Step 2:** Determine the commission using the formula  
percent • whole = part  
 $0.05 \cdot \$950,000 = \$47,500$

**Solution:** The agent gets paid \$47,500 in commissions.

**Try These A-B**

- A salesperson at a car dealership has a salary of \$1,200 per week plus a 2% commission on sales. If a salesperson had sales of \$135,000 in one week, what was he paid that week?
- Two businesswomen are having lunch at a restaurant. Their bill came to \$47.85, and they want to leave an 18% tip. What is the amount of the tip?

My Notes

**ACADEMIC VOCABULARY**

A **tip** is an optional payment to someone who provides good service. This is in addition to the cost of the bill.

**CONNECT TO BUSINESS**

A **commission** is a fee paid for services and is usually a percent of the total cost.

My Notes

Check Your Understanding

1. A skier needs to buy new ski poles during a ski trip to Utah. The price of the poles is \$24 and the sales tax is 4.7%. What is the total cost of the poles, rounded to the nearest cent?
2. Josephina likes the service she receives at her favorite café and wants to leave a 20% tip. Her bill is \$22.58. What tip should she leave?
3. Film studios make a commission from every ticket sold. The movie tickets at Carbob Theatres cost \$8.50. The studios earn a 40% commission. What amount of the movie ticket do they earn?
4. Nick is selling software and earns 12% of his sales as a commission. If he sells a total of \$668 in software this week, how much is his commission?

LESSON 11-2 PRACTICE

5. Bobby delivers newspapers in his neighborhood. In addition to a weekly salary, he earns tips from the people he delivers to. If he delivers \$200 worth of papers each week and earns 17% in tips, how much does he make from tips each week?
6. The sales tax on a piece of furniture that cost \$450 was \$28.13. What was the percent sales tax?
7. An art dealer makes a 17.5% commission on every painting sold. If a painting sold for \$1,500, what was the commission?
8. **Construct viable arguments.** Sam lives in Massachusetts and wants to buy a television that costs \$2,000. The sales tax in Maine is only 5%, which is less than the 6.25% in Massachusetts. Dave tells Sam to just buy the television in Massachusetts. If gas costs \$30 to get to Maine, who is correct? Explain.
9. Art supplies in Washington, D.C., cost \$28. If the sales tax is 6%, what was the total cost of supplies?
10. A family went out to dinner and the bill was \$113. If they want to leave a 19% tip, how much tip should they leave?
11. A salesperson earning a 22.5% commission takes home \$1,200 as a commission. What was the dollar value of the items she sold?
12. If a new CD player in North Carolina costs \$225 and the sales tax is 4.75%, how much will a customer save during a tax-free holiday, when tax does not need to be paid?
13. **Reason quantitatively.** Joe leaves \$3 as a tip for a meal that cost \$22. If a good tip is between 15% and 20%, is this a good tip?

## ACTIVITY 11 PRACTICE

Write your answers on notebook paper. Show your work.

## Lesson 11-1

1. Find the percent of the number.
  - a. 35% of 750
  - b. 24% of 480
  - c. 105% of 400
  - d. 54% of 220
2. Solve for the number.
  - a. 75 is 20 percent of what number?
  - b. 82 is 70 percent of what number?
  - c. 160 is 120% of what number?
  - d. 18 is 9% of what number?
3. Find the percent.
  - a. 32 is what percent of 160?
  - b. 15 is what percent of 165?
  - c. 120 is what percent of 90?
  - d. 36 is what percent of 144?
4. Thirteen out of twenty-one people surveyed prefer fresh fruit over canned fruit. What percent of people prefer fresh fruit?
  - A. 38.1%
  - B. 162%
  - C. 61.9%
  - D. 161.9%
5. A survey at Lake Middle School shows that only 8% of the student population walks to school each day. If 1,200 students attend the school, about how many walk to school? Explain your reasoning.
6. The population of a school is 400 students. Next year it is expected to be about 120% of what it is now. Choose next year's student population from the answers below.
  - A. 320 students
  - B. 80 students
  - C. 400 students
  - D. 480 students
7. On Monday night, 21% of the television shows are comedy shows. There are 13 comedy shows on Monday night. How many total shows are on Monday night? Explain your reasoning.
8. The soccer team made a total of 36 goals this season and 22 goals last season. What percent of this year's total did the team make last year?
9. Out of 139 seventh graders, 86 voted math as their favorite subject. What percent is this?

**Lesson 11-2**

10. A realtor sells a house for \$460,000. If she earns 3% of the house's price as a commission, how much is her commission?
11. Nicole made 24% of her daily salary in tips, receiving \$28. What is her daily salary?
12. Kevin bought some souvenirs in Missouri for \$13.50. The sales tax is 4.225%. What is the total cost of the souvenirs?
13. Terry has a goal of making \$300 in commissions for the week. If her commission is based on 18% of sales, how much does she need to sell to make her goal?
14. Karl bought a pair of jeans for \$25 at a store in California, where the sales tax is 6.5%. He was charged \$1.45 in sales tax. Is this correct? Explain why or why not.
15. Five friends went out to dinner, and the bill came to \$85. They wanted to leave a 17% tip because they thought the waiter did a good job. If they split the bill evenly, how much does each person need to pay?

**MATHEMATICAL PRACTICES****Construct Viable Arguments**

16. Max's bill at a restaurant is \$8.67. He uses the following method to figure out a tip of about 15%.

Round the amount of the bill to the nearest ten cents. Move the decimal point one place to the left. Then find half of the result. Add the last two answers, and round to the nearest five cents.

How much will the tip be for this bill? Why does Max's method work?



# More Percent Problems

## ACTIVITY 12

### Like Animals? Have I Got a Job for You!

#### Lesson 12-1 Percent Increase and Decrease

##### Learning Targets:

- Solve problems about percent increase, percent decrease, markups, and discounts.

**SUGGESTED LEARNING STRATEGIES:** Shared Reading, Summarizing, Group Presentation, Think-Pair-Share, Create a Plan

Veterinarians care for the health of animals. They diagnose, treat, or research animal diseases in homes, zoos, and laboratories. Although most veterinarians work in clinics, others travel to farms, work outdoors, or work in labs.

There were 61,400 veterinarians practicing in the United States in 2010.

The U.S. government predicts that the need for veterinarians will rise in the future. In fact, they predict that 22,000 more veterinarians will be needed by 2013.

1. a. To find the **percent increase** predicted for veterinarians from 2010 to 2013, first find the difference in the number of veterinarians from 2010 to 2013.

- b. What was the original number of veterinarians?

- c. Use the equation to determine the percent increase.

2. What would the **percent decrease** be if there were to be 10,000 fewer veterinarians in 2013?

3. A pet album that usually sells for \$12.50 was put out on sale for \$8.50. What is the percent decrease?

4. The price of a set of feeding bowls for dogs had been \$9.25, but was increased to \$11.10. What was the percent increase?

##### My Notes

##### MATH TIP

**Percent increase or decrease** can be found using an equation.

$$\left( \frac{\text{difference}}{\text{original amount}} \right) \cdot 100$$

= % of change

My Notes

Check Your Understanding

5. Dr. Detwiler saw 500 dogs. In 2010, 21.4% were considered to be obese, and in 2011, 20.6% were considered to be obese. To find the **percent decrease** from 2010 to 2011, students Marley and Leland used two different methods.

Marley		Leland	
$500 \cdot 0.214 = 107$ $500 \cdot 0.206 = 103$ $107 - 103 = 4$ $\left(\frac{4}{107}\right) \cdot 100$ $= 3.74\%$		$21.4 - 20.6 = 0.8$ $\left(\frac{0.8}{21.4}\right) \cdot 100 = 3.74\%$	

- a. Use the empty space in the table to explain what each student did.  
 b. Explain which method makes better sense to you and why.

LESSON 12-1 PRACTICE

- Dr. North Piegan's practice sees 125 cats. In 2010, 24.8% of cats were classified as obese, compared to 21.6 percent in 2011. Find the percent decrease in obese cats in one year.
- Dr. Piegan also treated 440 dogs. The percentage of obese dogs increased from 52.5 percent in 2010 to 55.0 percent in 2011. What was the percent increase?
- Susie's dog weighed 32 pounds in 2010. At the end of 2011, the dog weighed 43.5 pounds. What was the percent increase in weight from 2010 to 2011?
- Reason quantitatively.** Pet Food, Inc. created a new package size that was 25% larger than the old package. If the old package contained 20 pounds, how much does the new package contain?
- Sunil bought a new crate for his dog. The old crate was 8 square feet, and the new crate was 12 square feet. What is the percent increase in space?
- Pet Store, Inc., found that they were selling more guinea pigs and fewer birds. In 2010, they had 42 birds in stock, and in 2011, they decided to only keep 34 birds in stock. What was the percent decrease in inventory from 2010 to 2011?
- Jay has a pet grooming business. In 2010, he groomed 200 dogs. In 2011, he groomed 450 dogs. What was the percent increase in the number of dogs groomed?
- In 2006, there were 72 million dogs in U.S. homes. That number declined by 2 million in 2010. What was the percent decline in the number of dogs in U.S. homes?

**Learning Targets:**

- Solve problems about percent increase, percent decrease, markups, and discounts.

**SUGGESTED LEARNING STRATEGIES:** Shared Reading, Interactive Word Wall, Summarize, Think/Pair/Share, Share and Respond, Note Taking, Create a Plan, Identify a Subtask

Veterinarians markup medicine and supplies so that they can earn a salary and pay their bills. A **markup** is an amount added to the original cost of an item to find the selling price. Items can also be **marked down** by decreasing the original price. Markdowns are also considered to be **discounts**.

**Example A**

A pet owner bought a leash at the veterinarian's office. The leash cost the vet \$2.25 to buy from the supplier. The vet marked the price up 45% before selling it to the pet owner. What is the price the pet owner paid?

**Step 1:** To determine the amount of the markup, use the percent equation.

$$\$2.25 \cdot 0.45 = \$1.01$$

**Step 2:** Add this amount to the original amount to find the final cost.

$$\$2.25 + \$1.01 = \$3.26$$

**Solution:** The final cost is \$3.26.

**Example B**

Suppose the vet wanted to make \$0.75 profit on each leash. What is the percent markup applied?

Use the percent increase equation.

$$\left( \frac{\text{difference}}{\text{original amount}} \right) \cdot 100 = \text{percent markup}$$

$$\left( \frac{0.75}{2.25} \right) \cdot 100 = 33.33\%$$

**Solution:** The markup is 33.33%.

**Try These A-B**

- A dog food company sells its canned dog food to stores for \$0.35 per can. One of the stores sells the can of dog food for \$0.79. What percent markup did the store use?
- A bird food company sells its box of birdseed to the pet shop for \$1.89. The pet store uses a 55% markup. What is the pet store price for the box of birdseed?

**My Notes**

**MATH TERMS**

A **discount** is a reduction in price.

~~\$10~~  
\$8

Here the discount is \$2.

Sometimes discounts are a percent, such as a 10% discount.

**GROUP DISCUSSION TIPS**

Read the problem scenarios and look for key information. Organize the information and describe the math concepts you will use to solve the problems.

My Notes

Check Your Understanding

1. A cat's catnip toy is on sale for 10% off the regular price of \$3.30. Find the sale price.
2. Dr. Star Blanket gives a 20% discount if his customers pay cash for their office visits. Determine the cost of a \$65 office visit if the customer pays cash.
3. A horse that is ill needs medication. The medication costs the veterinarian \$17.50 to buy. He marked up the medication 54.2% before selling it to the customer. Find the final selling price of the horse's medication.

LESSON 12-2 PRACTICE

4. Kim purchases pet insurance for her cat. In 2011, she paid \$40 per month. She switched to discounted insurance she bought online for \$34 dollars a month. What was the percent discount?
5. A new dog leash with an attached flashlight was on sale for 20% off. If the original price was \$16.50, what is the discounted price?
6. **Make sense of problems.** A sale on dog food that costs \$10 a bag says buy 1, get one  $\frac{1}{2}$  off. What is the true percent discount on the dog food?
7. A veterinarian purchases some medication for \$19.50 and wants to make a \$2 profit. What is the percent markup on this medication?
8. The grooming department at a local pet store is running a sale of 35% off grooming. If the regular cost is \$47.75, what is the sale price?
9. A doggie day care charges \$21.50 per day. If you leave your dog 5 days a week, the discount is 20%. What is the cost of leaving your dog for 5 days in a week?
10. A bag of birdseed that normally costs \$22.25 is marked up to \$25.30. What is the percent markup?
11. **Reason quantitatively.** A gerbil cage, priced at \$18, is discounted 20% for one week, and then this price is marked up 20%. Is the price back to \$18 Explain.

**Learning Targets:**

- Solve problems about interest.

**SUGGESTED LEARNING STRATEGIES:** Close Reading, Marking the Text, Summarizing, Interactive Word Wall, Create a Plan

Starting a veterinary clinic often includes taking out loans that the veterinarians need to pay back. Paying back a loan includes **interest**.

**Example**

Dr. Blevins is a first-year veterinarian and plans to rent space for an office for her new hospital. She will take out a loan of \$94,000 over 7 years at 6.25% interest to remodel the office. How much will she pay in interest on this loan?

The problem is asking for the simple interest, so use the formula for finding interest:

$$\text{Interest} = P \cdot r \cdot t$$

where  $p$  is the principal, or the amount of money borrowed,  $r$  represents the rate of interest charged, and  $t$  is the time in years.

Substitute in the values and solve for  $I$ .

$$I = P \cdot r \cdot t$$

$$I = \$94,000 \cdot 0.0625 \cdot 7$$

$$I = \$41,125$$

**Solution:** The interest paid on the loan will be \$41,125.

**Try These**

Work with your group to find the amount of simple interest to the nearest cent. As needed, refer to the Glossary to review the meaning of key terms. Use your understandings of the terms in group discussions to confirm your knowledge and correct use of math language.

- \$12,000 at 2.5% for 3 years
- \$15,000 at 4% for 6 months

**My Notes****MATH TIP**

The variables in the equation represent:

$I$  = interest

$P$  = principal

$r$  = rate

$t$  = time

**MATH TIP**

If time is given in another unit, such as months or days, it needs to be converted to years when using the interest formula.

My Notes

**Check Your Understanding**

1. Dr. Blevins will have to take out another loan for other costs, such as equipment, furniture, products, and advertising. The loan will be for \$25,900 over 5 years at 6.25%. Find what the interest will be on the loan for these other costs.
2. **Attend to precision.** The local shelter decided to build an addition for cats. They borrowed \$6300 from a local bank at a rate of 8%. The high school held a fundraiser that allowed the shelter to pay off the loan after only 9 months. How much interest did the shelter pay on this loan?

**LESSON 12-3 PRACTICE**

3. **a.** Pet owners may have to take out a loan when their animal has a major operation. Buddy needs to have surgery on his leg. The cost of the surgery is \$2500. His owner will get a loan for this amount over 2 years at a rate of 7.5%. How much will the interest be on this loan?  
**b.** What is the total cost of the surgery including the loan and the interest?
4. The average cost of an education to become a veterinarian is \$147,656, which includes veterinary school. To cover these costs, most students take out loans. What would the interest on a loan be if the rate is 3.25% and the term is 15 years?
5. **Reason abstractly.** How is interest similar to sales tax and markups? How is it different?
6. Dr. Jones took out a loan for a van that could be equipped with veterinarian supplies for visits to animal shelters. The van cost \$22,500 and the loan was for 4 years. If the interest rate was 4.5%, how much will be paid?
7. An X-ray machine for animals costs \$125,000. If the loan is for 10 years, and the interest paid is \$65,625, what is the interest rate on the loan?
8. **Make sense of problems.** Dr. Jones would like a computer system for the office to help with management of the clinic. The cost is \$15,000, and the interest rate is 4.6%. The amount of interest to be paid is \$3,450. How many months is the loan for?



**Learning Targets:**

- Solve problems about percent error.

**SUGGESTED LEARNING STRATEGIES:** Shared Reading, Discussion Groups, Interactive Word Wall, Create a Plan, Identify a Subtask

Veterinarians sometimes have to give animals transfusions to help them recover from illness or accidents. Cats have their own unique blood types. Cats are either type A, type B, or, rarely, type AB.

There are about 600,000 cats living in Las Vegas, Nevada. Local veterinarians assume that 90% of these cats have type A blood, a total of 540,000 cats.

It is found that actually 95% of cats have type A blood, or 570,000 cats.

**Example**

What is the **percent error** between the veterinarians' earlier estimate and the actual number?

**Step 1:** Find the absolute value of the difference between the estimate and the actual number.

$$\begin{aligned} 540,000 - 570,000 &= -30,000 \\ |-30,000| &= 30,000 \end{aligned}$$

**Step 2:** Divide this difference by the actual number.

$$\frac{30,000}{570,000} = 0.0526$$

**Step 3:** Multiply this value by 100 to make it a percent.

$$0.0526 \cdot 100 = 5.26\%$$

**Solution:** The percent error is 5.26%.

**Try These**

Work with your group.

- A vet's assistant weighs a cat at 5.2 kg. The cat's actual weight is 4.8 kg. What is the percent error between the two weights?
- The vet estimated that the facility had kept 23 dogs in their overnight boarding. The actual number of dogs kept overnight was 18. What is the percent error between the vet's estimate and the actual number?

**My Notes****CONNECT TO MEDICINE**

A **transfusion** is the transfer of whole blood or blood products from one individual to another.

**MATH TERMS**

**Percent error** is the amount of error between the assumed value and the actual value.

**GROUP DISCUSSION TIPS**

Summarize the information needed to create a reasonable solution. Be sure to describe the math concepts your group will use to solve.

My Notes

Check Your Understanding

1. An adult Labrador retriever weighing 86 pounds came into the veterinarian's office for treatment. The vet estimated that 45 pounds of the dog's weight was fluid. When he did the math, he found that 51.6 pounds of the dog's weight was fluid. What was the percent error between the vet's estimate and the actual amount?
2. On a busy day, a vet thought the practice saw 28 cats. The actual number was 24 cats. What is the percent error between the vet's estimate and the actual amount?

LESSON 12-4 PRACTICE

3. Sally estimated that the cost of a new fish tank would be \$45. The actual cost was \$37.79. What was the percent error between Sally's estimate and the actual amount?
4. Marco estimated that he took his dog on a 2.5-mile walk. When he drove the same route in his car, he found that it was 2.65 miles. What is the percent error between Marco's estimate and the actual mileage?
5. A new horse owner estimated that the horse would eat 30 pounds of food a day. The horse was training for a race, so it actually ate 42 pounds per day. What is the percent error between the owner's estimate and the actual amount?
6. Kyle brought his guinea pig to the vet to see what was wrong with it. He thought the visit would cost \$25, but the actual cost was \$33. What is the percent error between what Kyle thought it would cost, and the actual cost?
7. **Reason quantitatively.** Pat helps run an animal shelter. The actual number of animals brought in during a particular month was 150. The percent error was 12%. If Pat estimated on the high side (more than 150), how many animals did Pat think were brought to the shelter?
8. Shea's mother wanted to get him a doctor's bag for a graduation present. She thought it would cost \$125, but the actual cost was \$95.99. What is the percent error between her estimate and the actual cost?
9. The average number of prescriptions written by Dr. Jones on a daily basis is 12. The assistant thought that number was 17. What is the percent error between the actual amount and the estimate by the assistant?

## ACTIVITY 12 PRACTICE

Write your answers on notebook paper.

Show your work.

### Lesson 12-1

- The last time Tia bought pet food, she purchased 25 pounds. This time she needed 38 pounds. What is the percent increase in the amount of food she needed?
- The number of people and dogs taking an obedience class increased from 1400 students to 1600 students. Choose the percent increase in enrollment from the choices below.
  - 14.3%
  - 87.5%
  - 114.3%
  - 187.5%
- If the number of veterinarians in California increased by 12.5%, and the original number was 1,215, what is the new number of veterinarians?
- The value of the building owned by two veterinarians went from \$125,000 to \$117,500. What is the percent decrease in value?

### Lesson 12-2

- What is the discount on a stethoscope that is marked down from \$300 to \$250?
- A pet shop pays \$40 for a dog bed, and then sells it to a customer for \$75. What is percent markup?
- A supply company marks up their doctor scrubs by 40%. Find the doctor's cost for a set of scrubs that the supply company buys for \$34.
- Noah wants to buy a purebred golden retriever puppy that he sees for \$499. By the time he goes to buy it, the price has increased to \$599. Find the percent markup in the price of the puppy.
  - If the price increases another 5%, what is the final price of the puppy?

- A fancy new litter box is sold at a pet store for \$47.50. It is on sale for a discount of 15%. What is the final sale price?
  - Would it be better or worse for the store if it had simply lowered the original price by \$7.10? Explain your answer.
- The clownfish Kay wanted to buy for \$12 last week has been marked up 8%. What is the new price of the fish?

### Lesson 12-3

- Angie borrowed \$1,485 to purchase a new laptop for the veterinarian's assistant. The interest on the loan was 7.75%. How much interest will Angie pay if the loan is for 2 years?
- Mark takes out a loan for \$30,000 to add on a new exam room for his practice. If the bank charges 5.4% interest, how much interest will he pay in 5 years?
- Dr. Jones took out a loan for a new medical instrument cleaning system. The interest paid on the loan was \$650. The loan was for 24 months, and the interest rate was 2.75%. How much money did Dr. Jones borrow?
- Dr. Lee went to a bank to get a loan for some new furniture for the waiting area at her clinic. She wanted to borrow \$5,000 to be paid back over 3 years. Last week the interest rate was 4.25%, but this week the interest rate went up to 4.50%. How much additional interest will Dr. Lee pay because she waited the extra week?
- A vet borrows \$12,000 for 4 years. The total interest paid was \$875. What was the interest rate on the loan?
- A veterinary student takes a loan to pay for the last year of school. The loan is for \$25,000 at 3.25% to be paid back over 10 years. How much interest will be paid?
  - If the student chose to pay back the loan in 7 years instead, how much interest would be saved?

**Lesson 12-4**

17. The local zoo contracts with one of the local veterinary clinics for pet checkups and sick visits. The zoo manager thought the vet made more than 375 visits to the zoo in one year, but the actual number was 412. What was the percent error between the actual number of visits and the number of visits the manager thought the vet made?
18. A veterinary student estimated the weight of a snapping turtle to be 147 pounds. The actual weight was 175 pounds. What is the percent error between the actual weight of the turtle and the estimated weight?
19. It was estimated that the United States would need 6200 more veterinarians in 2012. The percent error was 10%. If the estimate was low, how many were actually needed?
20. The aquarium estimated that they would need 520 cubic feet of water for a new exhibit. They got more fish than expected and actually needed 620 cubic feet of water. What was the percent error between the actual amount needed and the estimate?
21. It was estimated that a giraffe would be 15.5 feet tall when fully grown. Its actual height was 16.25 feet. What was the percent error between the actual and estimated values of height?
22. The percent of women in veterinary school is 78%. If you had guessed 50%, what would the percent error be?
23. Judy thought she spent \$140 on pet accessories during the year. When she looked at all her receipts, she found she actually spent \$112. What was the percent error between the actual amount she spent and her estimate?

**MATHEMATICAL PRACTICES****Reason Abstractly and Quantitatively**

24. Which two of the following situations would give the same amount of interest? Explain.
  - A. \$6,000 borrowed for 4 years at 4.6%
  - B. \$15,000 borrowed for 3 years at 5%
  - C. \$12,000 borrowed for 8 years at 2.3%
  - D. \$7,500 borrowed for 6 years at 5%

**Write your answers on notebook paper. Show your work.**

Facebook and eBay are two popular Web services with many users. Facebook is a social media site where members connect with others. eBay is a marketing site where people buy and sell merchandise.

There were 845 million Facebook users at the end of 2011, which was up from 608 million users at the end of 2010.

1. What percent is 608 million of 845 million?
2. What is the percent increase in users between 2010 and 2011?
3. 51.5% of all users access Facebook on mobile devices. How many users used mobile devices in 2011?
4. Facebook is now used by one in every 13 people on Earth. What percent is this?

eBay allows people to sell any item for what others will pay for it.

5. The price of a digital camera sold in stores is \$129. It is sold on eBay for \$65. What is the percent decrease from the store's price to the online price?
6. What percent discount would this be in a store?

Some people have others sell items for them on eBay and then give them a part of the profit as a commission.

7. One forklift sells every 4 hours on eBay. If a forklift sells for \$2,999 and the seller receives a 12% commission, how much does the seller receive for the sale?
8. One family spent \$2,000 to start an online business selling goods. They now make \$1,240,000 each year. If they made a one-time investment of half of their yearly earnings in a savings account earning 4% simple interest, how much would the account have in it at the end of 5 years?
9. Sales tax is added to the price of items sold on eBay. If someone in Arizona buys an item online for \$22.50 and the state sales tax rate is 6.6%, what is the final selling price?
10. Businesses use many terms in their daily operation. Separate the terms below into groups. Each group should contain words that are related in some way. Label each group to show how the words are related.

percent increase  
commission  
tip  
fee

markup  
sales tax  
interest  
principal

percent decrease  
discount  
markdown  
rate



Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
<b>Mathematics Knowledge and Thinking</b> (Items 1, 2, 3, 4, 5, 6, 7, 8, 9, 10)	<ul style="list-style-type: none"> <li>Effective understanding and accuracy in calculating percents, percent increase, percent decrease, and finding a part given a percent.</li> <li>Effective understanding and accuracy in calculating sales tax, commissions, and interest.</li> </ul>	<ul style="list-style-type: none"> <li>Few if any errors in calculating percents, percent increase, percent decrease, and finding a part given a percent.</li> <li>Few if any errors in calculating sales tax, commissions, and interest.</li> </ul>	<ul style="list-style-type: none"> <li>Multiple errors in calculating percents, percent increase, percent decrease, and finding a part given a percent.</li> <li>Multiple errors in calculating sales tax, commissions, and interest.</li> </ul>	<ul style="list-style-type: none"> <li>Incorrect or incomplete understanding of calculating percents, percent increase, percent decrease, and finding a part given a percent.</li> <li>Incorrect or incomplete understanding of calculating sales tax, commissions, and interest.</li> </ul>
<b>Problem Solving</b> (Items 2, 5, 7, 8, 9)	<ul style="list-style-type: none"> <li>An appropriate and efficient strategy that results in a correct answer.</li> <li>Accurate interpretation of a percent to solve a problem.</li> </ul>	<ul style="list-style-type: none"> <li>A strategy that may include unnecessary steps but results in a correct answer.</li> <li>Interpretation of a percent to solve a problem.</li> </ul>	<ul style="list-style-type: none"> <li>A strategy that results in some incorrect answers.</li> <li>Difficulty interpreting a percent to solve a problem.</li> </ul>	<ul style="list-style-type: none"> <li>No clear strategy when solving problems.</li> <li>Incorrect interpretation of a percent to solve a problem.</li> </ul>
<b>Mathematical Modeling / Representations</b> (Item 2, 4, 5, 7, 8, 9)	<ul style="list-style-type: none"> <li>Clear and accurate interpretation of a percent problem to write and solve an equation.</li> </ul>	<ul style="list-style-type: none"> <li>Interpreting a percent problem to write and solve an equation</li> </ul>	<ul style="list-style-type: none"> <li>Difficulty interpreting a percent problem to write and solve an equation</li> </ul>	<ul style="list-style-type: none"> <li>Incorrect or incomplete interpretation of a percent problem to write and solve an equation</li> </ul>
<b>Reasoning and Communication</b> (Item 2, 5, 6, 7, 8, 9, 10)	<ul style="list-style-type: none"> <li>Effective understanding and command of terms relating to percents.</li> </ul>	<ul style="list-style-type: none"> <li>An adequate knowledge of terms relating to percents.</li> </ul>	<ul style="list-style-type: none"> <li>Difficulty understanding and distinguishing terms relating to percents.</li> </ul>	<ul style="list-style-type: none"> <li>An incomplete or inaccurate understanding of terms relating to percents.</li> </ul>