Probability

5

Unit Overview

In this unit, you will begin your study of probability. You will learn how to interpret probabilities and how to calculate probabilities in a variety of settings. You will also learn several ways to estimate probabilities.

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- predict
- simulation

Math Terms

- probability experiment
- probability
- equally likely outcomes
- selected at random
- event
- complement
- theoretical probability
- estimated probability
- sample space
- tree diagram
- random digits

ESSENTIAL QUESTIONS

? How is probability used to make decisions in everyday situations?

? How can a probability be estimated?

EMBEDDED ASSESSMENTS

These assessments, following activities 21 and 23, will give you an opportunity to demonstrate your understanding of probability and your ability to calculate and estimate probabilities.

UNIT 5

Getting Ready

Write your answers on notebook paper. Show your work.

1. Express each of the following as a fraction in lowest terms.

a. $\frac{3}{6}$

b. $\frac{2}{8}$

c. $\frac{12}{20}$

d. $\frac{5}{8}$

2. Express each fraction as a decimal. (Where applicable, round your answers to the nearest hundredth.)

a. $\frac{1}{2}$

b. $\frac{1}{3}$

c. $\frac{1}{8}$

d. $\frac{5}{7}$

e. $\frac{5}{8}$

f. $\frac{12}{15}$

g. $\frac{22}{40}$

h. $\frac{37}{50}$

3. Express each decimal as a percentage.

a. 0.19

b. 0.47

c. 0.032

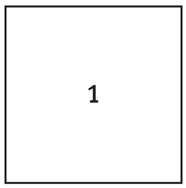
d. 0.05

e. 0.169

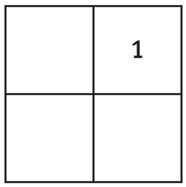
f. 0.95

4. What fraction of the area of the squares shown below corresponds to the section labeled 1?

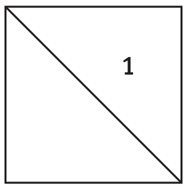
a.



b.



c.



Exploring Probability

Spinner Games

Lesson 20-1 Making Predictions

My Notes

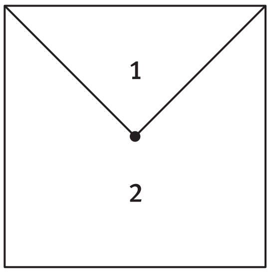
Learning Targets:

- Reason about the likelihood of winning a game based on a probability experiment.
- Provide support for winning strategies of a game based on a probability experiment.

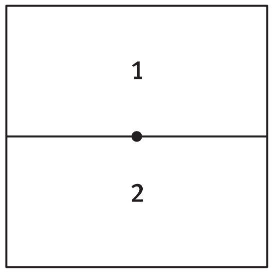
SUGGESTED LEARNING STRATEGIES: Mark the Text, Think-Pair-Share, Create Representations, Predict and Confirm, Look for a Pattern, Summarizing, Paraphrasing

Suppose that you and a partner play a game using the two spinners below.

Spinner 1



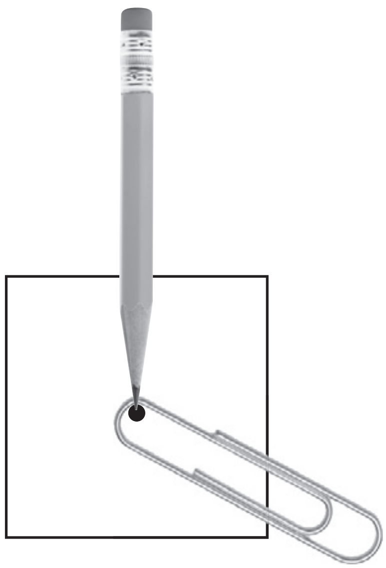
Spinner 2



Use a paper clip and a pencil to make the spinner work, by following these steps:

- Place the paper clip on the spinner so that the center dot is just inside the long loop of the paper clip.
- Put the pencil point on the dot.
- Flick the paper clip with your finger.
- After the paper clip has stopped spinning, look to see where the center of the outer loop of the paper clip falls and note the number for this section. If the center of the outer loop of the paper clip falls on a boundary line, spin again.

Try this a few times with each of Spinners 1 and 2.



My Notes

For this game, you and your partner will each spin one of the spinners. Whoever spins the greater number wins. If you both spin the same number, you each spin again until someone wins.

1. If you could decide which spinner you will use and which spinner your partner will use, how would you assign the spinners?

My spinner will be Spinner ______.

My partner's spinner will be Spinner ______.

2. Explain your reasoning in assigning the spinners. Why did you assign them this way?

3. If you play the game once using the spinner you chose in Item 1, are you sure to win?

4. Based on your answer to Item 3, do you want to change your spinner? Explain your reasoning.

ACADEMIC VOCABULARY

The word ***predict*** means to make a reasonable guess about something that will happen.

5. Reason abstractly. If you play the game 20 times using the spinner you chose in Item 1, which of the following would you ***predict***?

a. You would win more often than you would lose.
b. You would win and lose about the same percentage of the time.
c. You would lose more often than you would win.

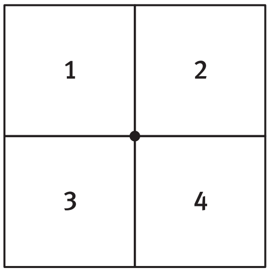
Explain why you made this prediction.

6. Suppose you play the game 20 times using the spinner you chose in Item 1, about how many times do you think you would win? Explain your answer.

My Notes

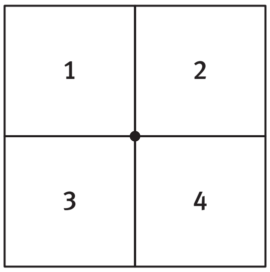
Check Your Understanding

7. You play a game in which two players each spin the spinner below. The player who lands on the greatest number wins. This game is an example of a *fair* game. Why do you think this is called a fair game?

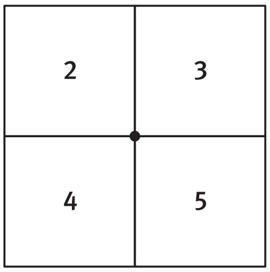


8. Construct viable arguments. You will play a game where two players each spin one of the two spinners below. The player who lands on the greatest number wins. Which spinner would you choose and why?

Spinner 1



Spinner 2

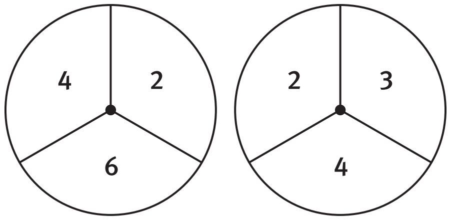


9. Describe a situation in which the person using Spinner 1 would win over the person using Spinner 2.

My Notes

LESSON 20-1 PRACTICE

Spinner A Spinner B



You and a friend are about to play a game in which each player spins one of the spinners above. The player who lands on the greater number wins.

10. Which spinner would you choose and why?

11. List all of the possible outcomes for spinning a number on each spinner. List the outcomes as ordered pairs, where the first number is the result from Spinner A and the second is the result from Spinner B.

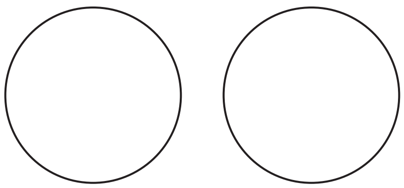
12. Play the game. Fill in the table below with W if Spinner A wins, T if the two spinners show the same number, and L if Spinner B wins.

		Spinner B Results		
		2	3	4
Spinner A Results	2			
	4			
	6			

Do the results in the table support your answers to Items 10 and 11? Explain.

13. Model with mathematics. Create two spinners to play a game, using the circles below. If you use the same rules as those above, explain whether either spinner gives a player a winning advantage and why.

Spinner C Spinner D



Lesson 20-2
Investigating Chance Processes

My Notes

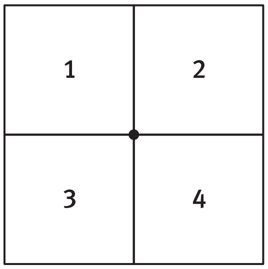
Learning Targets:

- Collect data about chance processes in frequency tables or lists.
- Determine probabilities for outcomes in a probability experiment.
- Describe the results of an investigation and support the conclusions.

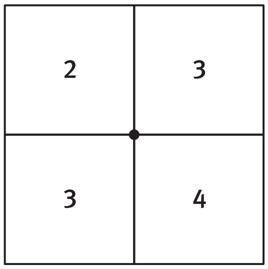
SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Look for a Pattern, Interactive Word Wall

Suppose you and a partner plan to play a game using two of the three spinners shown below. Each person will spin his or her spinner, and the person that lands on the greater number will win.

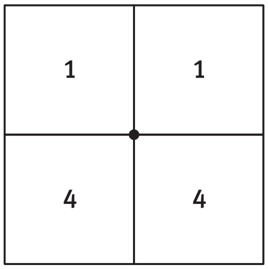
Spinner 3



Spinner 4



Spinner 5



1. Your partner picks a spinner first and picks Spinner 3. Which would you then choose, Spinner 4 or Spinner 5? Explain the reason for your choice.

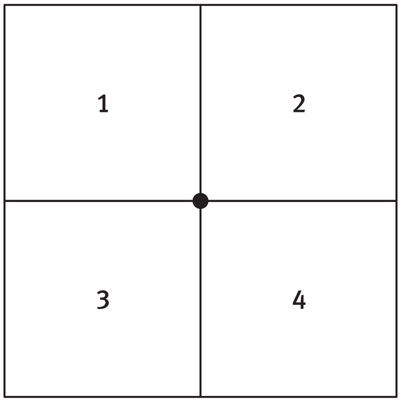
2. Use appropriate tools strategically. If you could choose first, which of the three spinners would you choose, and why? As you justify your choice of spinner, use precise mathematical language to describe your reasoning.

My Notes

3. Now let's do some spinning. Spin Spinner 6 below 20 times, and record the results in the table. Remember that if the paper clip lands on a line, you should just spin again.

Spin	Spin Result	Spin	Spin Result
1		11	
2		12	
3		13	
4		14	
5		15	
6		16	
7		17	
8		18	
9		19	
10		20	

Spinner 6



4. Count the number of spins that resulted in each number. Record the results in the table below.

Spin Result	Count
1	
2	
3	
4	

MATH TIP

To complete the table, count the number of 1's in the table above, and record the number under *Count*. Then do the same for the number of 2's, the number of 3's, and the number of 4's.

My Notes

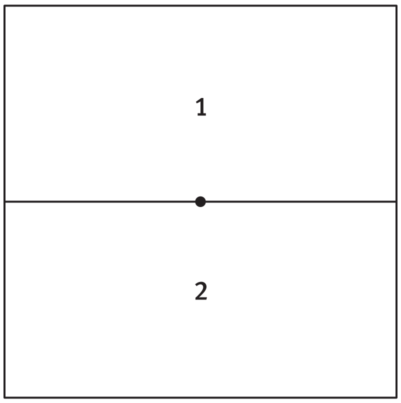
5. Do you have exactly five 1's, five 2's, five 3's, and five 4's? Does your answer surprise you?

6. Compare your counts from Item 4 to the counts of another student in your class. Did you both get the same counts? Does this surprise you?

Spinning a spinner and observing the result is an example of a ***probability experiment***. Even though you may be able to say what the possible outcomes are (such as 1, 2, 3, or 4 for Spinner 6), you can't be sure which outcome will actually occur on any spin.

7. Think about the probability experiment of spinning Spinner 7, shown below. What are the possible outcomes of this experiment?

Spinner 7



8. Even though you can't be certain what the outcome of any particular spin will be, if you were to spin Spinner 7 many times, about what percentage of the time do you think the outcome 1 would occur? Explain your reasoning.

MATH TERMS

A **probability experiment** is the process of observing an outcome when there is chance involved. This means that before you do the experiment, you can't be sure what the outcome will be.

My Notes

MATH TERMS

The **probability** of an outcome of a probability experiment is a number between 0 and 1 that tells you what fraction of the time you would expect the outcome to occur.

WRITING MATH

Probability Notation
The probability of an outcome is written in symbols as

P(outcome), where the *P* stands for probability.

For example, the probability that the outcome of a spin is 1 can be written as

P(spin results in a 1), or simply, as *P*(1).

9. The ***probability*** of spinning a 1 with Spinner 7 is $\frac{1}{2}$, or 0.5, when written as a decimal. How does this probability relate to your answer to Item 8?

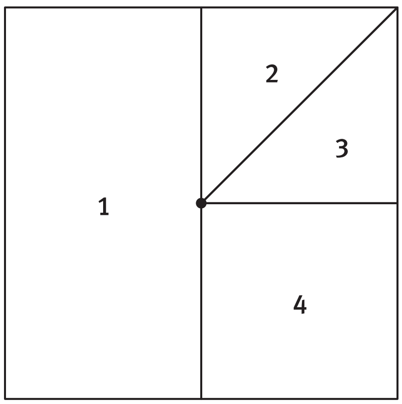
10. For Spinner 7, what is $P(2)$?

11. **Reason abstractly.** Since $P(1) = \frac{1}{2}$ for Spinner 7, if you spin this spinner many times you would expect $\frac{1}{2}$ or 50% of the spin results to be 1. Does this mean that if you spin ten times, you will get exactly five 1's? Explain why or why not.

12. Now think about the probability experiment of spinning Spinner 8 below. For Spinner 8, what are the following probabilities? Use clear and precise mathematical language to explain your reasoning. Remember to use complete sentences, including transitions and words such as *and*, *or*, *since*, *for example*, *therefore*, *because of*, to make connections between your thoughts.

$P(1) =$ $P(2) =$ $P(3) =$ $P(4) =$

Spinner 8

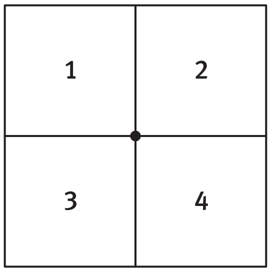


My Notes

Check Your Understanding

13. You and your friend are playing a game with the spinner below. Here are the rules:

- The first player spins. Then the second player spins.
- The second player wins if he or she lands on the same number as the first player did. If not, the first player wins.

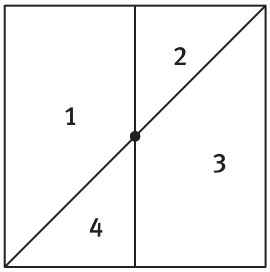


a. If your friend spins first and spins a 1, is it more likely that you will win? Why or why not?

b. Which is more likely in this game, landing on the same number as the first player or not landing on the same number?

c. Who has the better chance of winning the game?

14. Consider spinning the following spinner.



For this spinner, what are the following probabilities?

a. $P(1)$

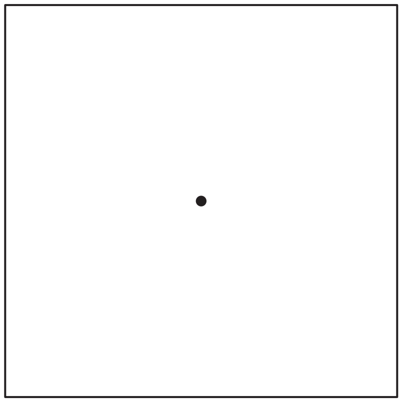
b. $P(2)$

c. $P(3)$

d. $P(4)$

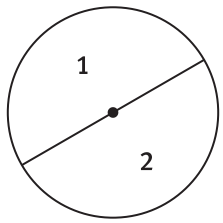
My Notes

15. Using the blank spinner, design a spinner for which
$P(1) = 25\%$
$P(2) = 12.5\%$
$P(3) = 12.5\%$
$P(4) = 25\%$
$P(5) = 25\%$



LESSON 20-2 PRACTICE

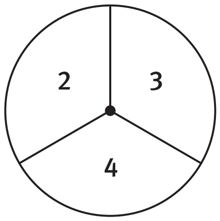
16. You and your friend are playing a game with the spinner below.
- The first player spins. Then the second player spins.
- The second player wins if he or she lands on the same number as the first player does. If not, the first player wins.



a. If your friend spins a 2, is it more likely that your spin will match your friend's spin, or is it more likely that your spin will not match your friend's spin?
b. When you play this game, is *a match* or *not a match* more likely?
c. **Make sense of problems.** Is this considered to be a fair game? Why or why not?

My Notes

17. You and your friend are playing a game with the spinner below.
- The first player spins. Then the second player spins.
- The second player wins if he or she lands on the same number as the first player does. If not, the first player wins.



a. List the possible outcomes using ordered pairs. The first number in the ordered pair is the number the first player spins.

b. Complete the table below. Fill in the ordered pairs for the spin results in the first column. In the second column, write *M* if the numbers match and *N* if the numbers do not match.

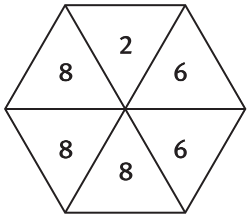
Ordered Pairs	M or N

c. **Construct viable arguments.** If you want to win this game, should you play so that the spins will match or so that the spins will not match? How do you know?

d. Is this a fair game? Why or why not?

My Notes

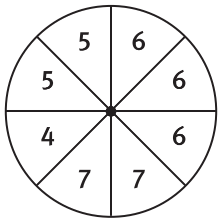
18. Consider the following spinner.



For this spinner, what are the following probabilities?

$P(2) =$
$P(5) =$
$P(6) =$
$P(8) =$

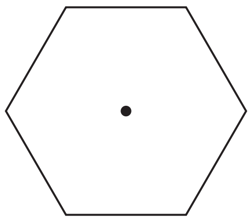
19. Consider the following spinner.



For this spinner, what are the following probabilities?

$P(4) =$
$P(5) =$
$P(6) =$
$P(7) =$
$P(\text{even number}) =$

20. Using the blank spinner, design a spinner for which:



$P(1) = \frac{1}{6}$

$P(3) = \frac{1}{3}$

$P(7) = \frac{1}{12}$

$P(\text{odd number}) = 1$

My Notes

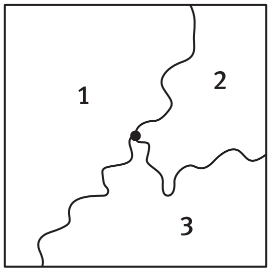
Learning Targets:

- Interpret a probability as the fraction of the number of times that an outcome occurs when a probability experiment is repeated many times.
- Estimate probabilities of outcomes in probability experiments.

SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Predict and Confirm, Look for a Pattern

1. Below is a "crazy" spinner. What can you tell about $P(1)$ just by looking at the spinner? Explain.

Spinner 9



2. Do you think that $P(1)$ is closer to 0, 0.25, 0.50, 0.75, or 1.0? Explain your answer.

3. Make sense of problems. Even though you don't know the exact value of $P(1)$ for crazy Spinner 9, can you think of a way to estimate this probability?

CONNECT TO GEOMETRY

In geometry, you learned to find the area of different shapes. You can use areas to determine probability. The greater the area you want, the greater the chance (or probability) of landing in that area. Think about how the area you want compares to the entire area on the spinner.

My Notes

4. Spin crazy Spinner 9 twenty times and record the results in the table below.

Spin	Spin Result
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	

Spin	Spin Result
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

5. How many of your 20 spins resulted in a 1? What fraction of the spins resulted in a 1?

6. If you had to estimate $P(1)$ for crazy Spinner 9 based on your twenty spins, what would your estimate be?

Estimate of $P(1) =$

7. Combine your spin results with those of another student in your class so that you now have outcomes from 40 spins. Based on the 40 spins, estimate $P(1)$ for crazy Spinner 9.

Estimate of $P(1) =$

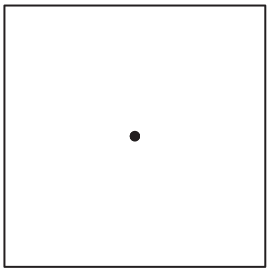
My Notes

8. Which of the two probability estimates do you think is a better estimate of $P(1)$ for crazy Spinner 9, the one from Item 6 or the one from Item 7? Explain your choice.

9. How could you get an even better estimate of this probability?

Check Your Understanding

10. Using the blank spinner below, design a "crazy" spinner with four sections. Label the four sections 2, 4, 6 and 8. (By a "crazy" spinner, we mean one for which the probabilities of the different possible spinner results are not obvious from looking at the spinner.)



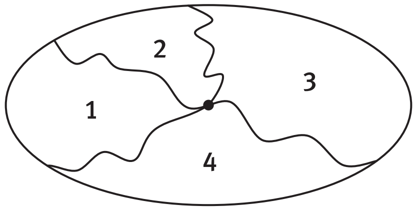
11. Spin your spinner 25 times, and use the results to estimate the following probabilities:

Estimate of P(spin result is 2)
Estimate of P(spin result is 8)

My Notes

LESSON 20-3 PRACTICE

12. Consider the crazy spinner below.



What can you tell about *P*(4) just by looking at the spinner? Explain your reasoning.

13. Should a reasonable estimate of the value of *P*(4) be closest to 0, 0.25, 0.50, 0.75, or 1. Why do you think this?

14. Suppose Jamie spins the crazy spinner from Item 12 sixteen times and records the results as shown in the table below. How many of the spins resulted in a 4? What fraction of the spins resulted in a 4?

Spin	Spin Result
1	1
2	2
3	4
4	1
5	1
6	3
7	2
8	3

Spin	Spin Result
9	3
10	3
11	1
12	2
13	3
14	1
15	3
16	4

15. Suppose Ibdi had seven 4's in 16 spins and Dar had six 4's in 16 spins. Estimate *P*(4) based on the combined results of Jamie, Ibdi, and Dar.

16. What do you feel is the best estimate for *P*(4)? How could you produce a better estimate of this probability?

Lesson 20-4
Making Decisions

My Notes

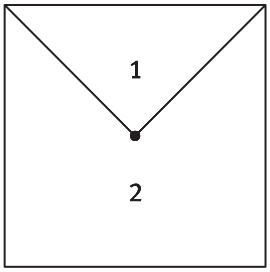
Learning Targets:

- Make decisions based on probabilities.
- Expect variation in results from chance processes.
- Write about chance processes and justify conclusions based on probability experiments.

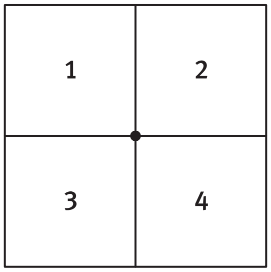
SUGGESTED LEARNING STRATEGIES: Create Representations, Think-Pair-Share, Predict and Confirm, Look for a Pattern

Consider the four spinners below.

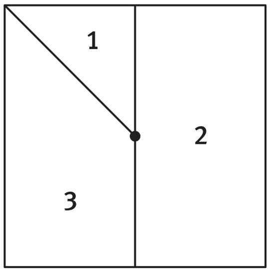
Spinner 10



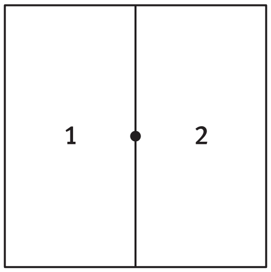
Spinner 11



Spinner 12



Spinner 13



1. Order the four spinners starting with the one that has the greatest probability of spinning 1, and ending with the one that has the least probability of spinning 1.

Greatest probability	Spinner ______
↓	Spinner ______
↓	Spinner ______
Least probability	Spinner ______

My Notes

2. Suppose you spin Spinner 12 twenty times and Spinner 13 twenty times. Would it surprise you if you got more 1's with Spinner 12 than with Spinner 13? Explain your thinking.

3. Suppose you spin Spinner 10 twenty times and Spinner 11 twenty times. Would it surprise you if you got more 1's with Spinner 10 than with Spinner 11? Explain your thinking.

4. Now consider Spinner 14 shown below.

Spinner 14

1	2
3	4

Only one of the following sequences resulted from actually spinning Spinner 14 twenty times. Which sequence do you think this was?

Sequence 1: 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
Sequence 2: 2 3 1 3 1 1 3 2 2 3 1 1 3 3 3 2 3 1 3 3
Sequence 3: 1 1 1 3 2 4 3 4 4 3 3 2 4 2 1 4 3 1 4 3

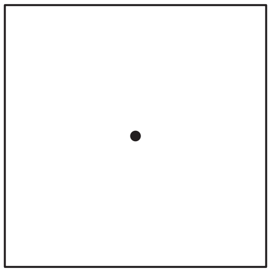
5. For the two choices you did not select in Item 4, explain why you ruled out that choice.

My Notes

6. The following were the outcomes from 20 spins of a spinner.

1 1 2 4 1 4 1 3 2 1 4 4 1 2 1 4 3 1 4 1

Using the blank spinner below, design a spinner that you think might have generated this sequence of outcomes. Explain your reasoning in designing this spinner.



My Notes

MATH TIP

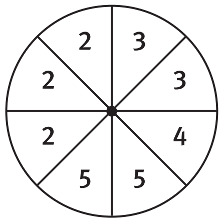
It is often easier to compare decimals than to compare fractions. You can write probabilities in decimal form to compare them.

For example, if the probability of spinning 3 is $\frac{3}{8}$ on one spinner and $\frac{2}{5}$ on another spinner, you can compare the probabilities in decimal form. Since $\frac{3}{8} = 0.375$ and $\frac{2}{5} = 0.4$, compare 0.4 and 0.375 to see that $\frac{2}{5}$ is greater.

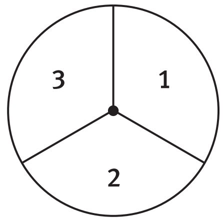
Check Your Understanding

Consider the following spinners.

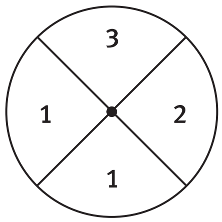
Spinner A



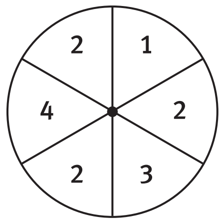
Spinner B



Spinner C



Spinner D



7. Order these four spinners starting with the one that has the greatest probability of spinning 2 and ending with the one that has the least probability of spinning 2.

_______ _______ _______ _______

Greatest probability → → → Least probability

8. Order these four spinners starting with the one that has the greatest probability of spinning 1 and ending with the one that has the least probability of spinning 1.

_______ _______ _______ _______

Greatest probability → → → Least probability

9. Suppose you spin Spinner B twenty times and Spinner C twenty times. Would it surprise you if you got more 2's with Spinner C than with Spinner B? Explain your thinking.

10. **Make sense of problems.** One of the following sequences resulted from spinning Spinner D twenty times. Which sequence do you think that this was? Explain your reasoning.

Sequence 1: 2 2 2 3 1 4 3 4 4 3 3 1 4 1 2 4 3 2 4 3
Sequence 2: 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4 1 2 3 4
Sequence 3: 1 2 2 4 2 4 2 1 1 3 2 2 3 3 3 1 4 2 3 3

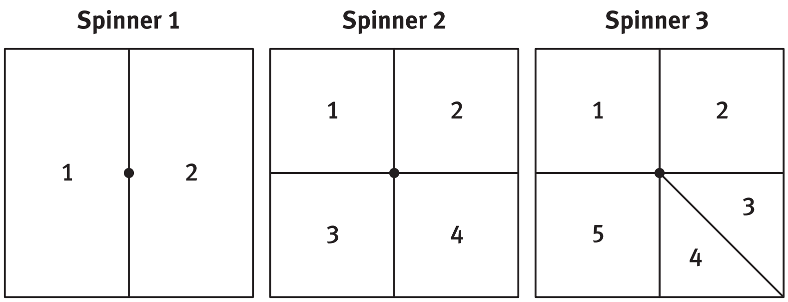
11. The following numbers represent the outcomes for 20 spins of a spinner. On a separate piece of paper, design a spinner that you think might have generated this sequence of outcomes. Explain your reasoning in designing the spinner.

1 2 3 1 4 1 2 1 2 1 1 4 4 1 1 1 1 3 1 2

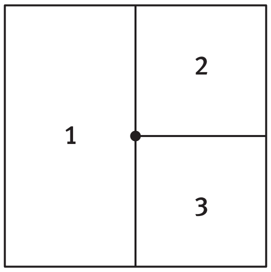
My Notes

LESSON 20-4 PRACTICE

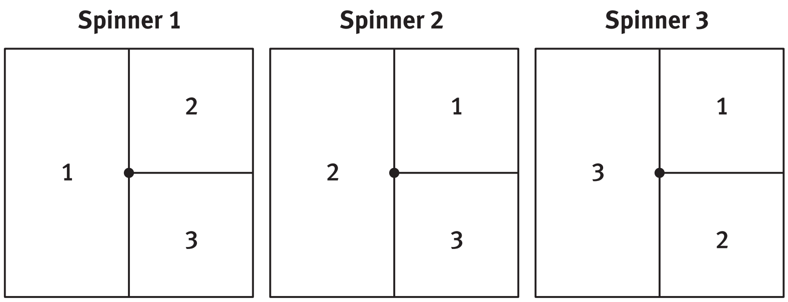
12. For which of the following three spinners is $P(4)$ the greatest?



13. The spinner below is one of the spinners used in a two-spinner game.



If the greater number wins, which of the following spinners is most likely to win when playing against the spinner above?



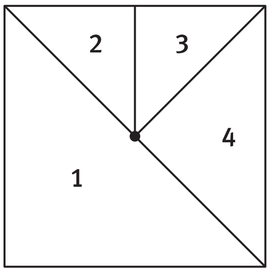
14. What are the following probabilities for the spinner shown below?

a. $P(1)$

b. $P(2)$

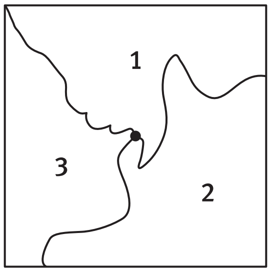
c. $P(3)$

d. $P(4)$



My Notes

15. Estimate $P(1)$ for the spinner below. Explain how you arrived at your estimate.



16. Only one of the following sequences resulted from actually spinning the spinner above twenty times. Which sequence do you think this was?

Sequence 1: 1 2 3 1 2 3 1 2 3 1 2 3 1 2 3

Sequence 2: 2 3 1 3 2 2 3 1 3 2 2 3 2 1 1

Sequence 3: 1 1 1 2 1 3 1 1 1 3 1 2 1 1 2

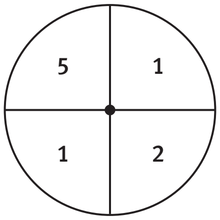
17. **Reason abstractly and quantitatively.** For each choice you did not select in Item 12, explain why you ruled out that choice.

ACTIVITY 20 PRACTICE

**Write your answers on notebook paper.
Show your work.**

Lesson 20-1

1. You plan to play "The Sum Game" in which two players each spin the spinner below one time. Player One wins if the sum of the two numbers is even. Player Two wins if the sum of the two numbers is odd. Is this game is an example of a "fair" game? Explain your reasoning.



2. Consider "The Sum Game" in Item 1.

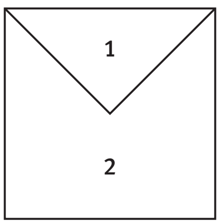
a. Will spinning first give a player an advantage in winning? Explain.

b. If "The Sum Game" is played twice, will each player win exactly once? Explain.

c. If "The Sum Game" is played 100 times, how many wins would you predict for Player One and Player Two? Explain why you think this.

Lesson 20-2

3. Consider the spinner shown below. Each of the following two statements is *incorrect*. For each statement, explain why the statement is incorrect.



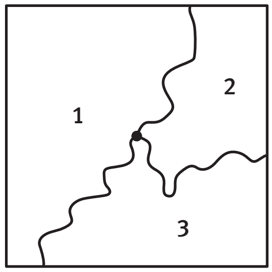
Incorrect Statement 1: If we spin this spinner many times, we would expect the result to be 1 about half the time and 2 about half the time.

Incorrect Statement 2: If we spin this spinner 20 times, we will see five 1's and fifteen 2's.

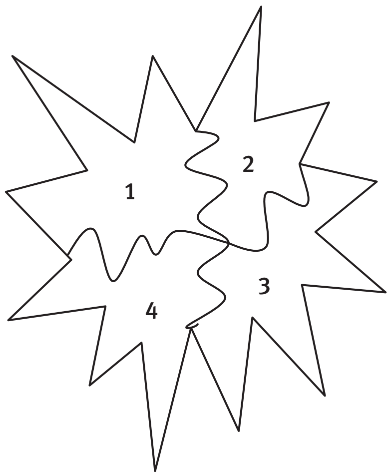
Lesson 20-3

4. Below is a "crazy" spinner.

Even though you cannot tell the exact value of $P(2)$, do you think that this probability is closest to 0, 0.25, 0.5, 0.75 or 1? Explain your reasoning.



5. Consider Spinner B below.



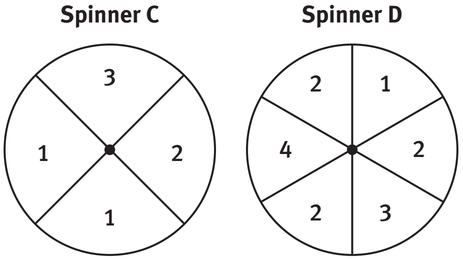
a. Spin Spinner B twenty times and record the results in a frequency table.

b. Estimate $P(2)$ for Spinner B. Explain your reasoning.

c. How could you improve your estimate for $P(2)$?

Lesson 20-4

6. Consider the following spinners.



You plan to play the spinner game where each player chooses a spinner and the player who spins the greater number wins.

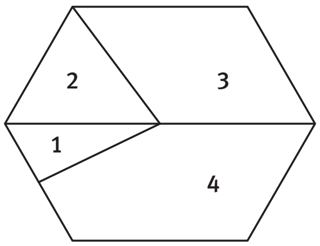
a. Which spinner would you choose to play? Explain why.
b. Play the spinner game 25 times. Keep track of how many times Spinners C and D win and how many ties occur.
c. Should you change your spinner based on these results? Explain.
d. Give the possible outcomes for these two spinners as ordered pairs with the results from Spinner C given first. (Spinner C, Spinner D)
e. Find *P*(Spinner C) and *P*(Spinner D). Explain how you know.
f. Is this a "fair game?" Justify your answer.
g. Which of the three lists below is most likely to represent 27 results of the game for Spinner C versus Spinner D? Justify your choice. (W means that C wins. L means that D wins. T means a tie between C and D.)

List 1: T W L W L W W T W L T W W W T W W L L T W W W W L W W

List 2: T W W W W L L L L T W W W W L L L L T W W W W L L L L

List 3: W W W L L L T W W W L L L T W W W L L L T W W W L L L

7. Consider Spinner E below.



The following numbers represent the results for fifty spins of Spinner E.

4 3 3 2 3   2 1 2 3 1   4 4 4 3 2   1 3 4 1 1

1 3 4 3 2   3 1 4 3 4   4 4 4 2 2   3 3 3 1 4

2 3 1 2 4   4 4 4 4 4

a. What fraction of the time would you expect each of the different results to occur?
b. Create a frequency table for the results of the fifty spins of Spinner E.
c. Calculate the probability for each of the different results based on frequency table.
d. Which result has the greater likelihood when using Spinner E, getting a 1 or getting a 2? Explain your reasoning.

MATHEMATICAL PRACTICES
Construct Viable Arguments

8. Consider the following:

A spinner has the probability $P(3) = \frac{2}{3}$. If this spinner is spun 30 times, 20 of these spins will definitely result in a 3.

Is this correct? Explain why or why not.

Probability

Probability Two Ways

Lesson 21-1 Equally Likely Outcomes

ACTIVITY 21

My Notes

Learning Targets:

- Recognize when a probability experiment has outcomes that are equally likely.
- Calculate probabilities for a probability experiment with equally likely outcomes.
- Know what "selected at random" means.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

In the previous activity, you learned about probability experiments in the context of spinner games. In this activity, you will see other examples of probability experiments.

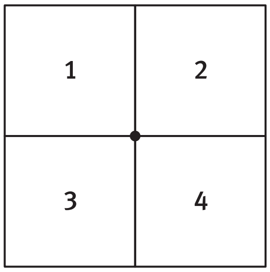
Some probability experiments have outcomes that are ***equally likely***.

MATH TERMS

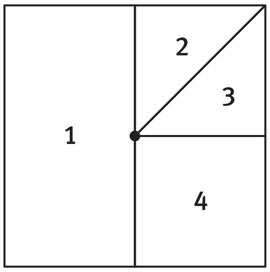
A probability experiment has **equally likely outcomes** if every different outcome has the same chance of occurring.

1. Each of the following spinners has four possible outcomes. Which of these spinners has equally likely outcomes?

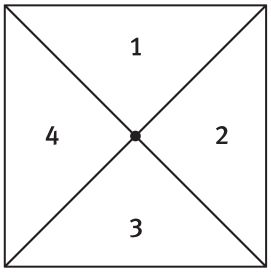
Spinner 1



Spinner 2



Spinner 3



2. What are the different possible outcomes for Spinner 1?

3. For Spinner 3, what are the following probabilities?

$P(1) =$

$P(2) =$

$P(3) =$

$P(4) =$

My Notes

4. What do you notice about the probabilities for the possible outcomes for Spinner 3? How is this probability related to the number of possible outcomes?

5. If a probability experiment has 7 different equally likely outcomes, what is the probability of each outcome?

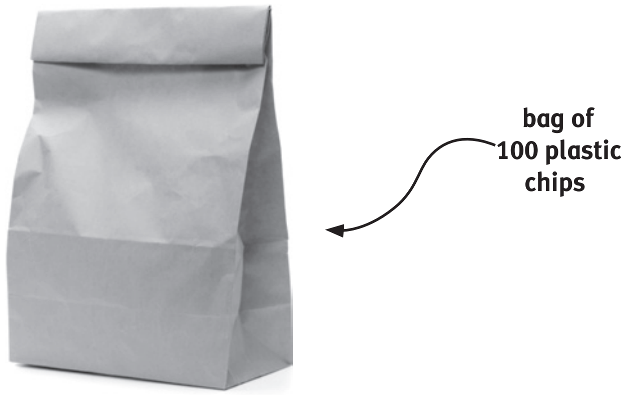
6. Suppose that a probability experiment has k different equally likely outcomes. Write an expression for the probability of each outcome.

MATH TERMS

Selected at random means choosing in a way that gives each member of the group the same chance of being chosen.

For example, if you shake a bag containing 100 chips to mix up all of the chips and then put your hand in the bag and pull out a chip without looking in the bag, that would be one way of selecting a chip at random from the bag.

One type of probability experiment that has equally likely outcomes is ***selecting at random*** from a group of objects or people. For example, suppose that a brown paper bag holds 100 plastic chips and that each chip has a different number on it. The chips are numbered from 1 to 100.



Suppose that you can win points in a game based on the number on the chip when you select a chip at random from the bag.

7. In the probability experiment that consists of selecting a chip at random from the bag of 100 numbered chips, what are the different possible outcomes for the number you might get? Are these outcomes equally likely?

8. What is the probability of each of the different outcomes?

My Notes

9. What is the probability that you get the chip numbered 61 when you select a chip at random from the bag?

10. Suppose that you can earn points if you get an even number when you select a chip at random from the bag of 100 chips. What do you think the probability of getting an even number is? Explain how you arrived at this probability.

When all of the outcomes are equally likely, the probability of an event A is

$$P(A) = \frac{\text{number of outcomes in the event}}{\text{number of possible outcomes}}$$

Use this result to answer the following items.

11. Express regularity in repeated reasoning. If the outcomes of a probability experiment are equally likely, it is easy to calculate the probability of an ***event***, such as the event that you select an even number. For the probability experiment of selecting a chip at random from the bag of 100 numbered chips, calculate the following probabilities:

a. The probability that the number selected is less than 10.

b. The probability that the number selected is greater than 75.

c. The probability that the number selected is equal to your age.

d. The probability that the number selected is **not** equal to your age.

e. The probability that the number selected is 75 or less.

MATH TERMS

An **event** in math is a collection of outcomes in a probability experiment.

MATH TIP

For a probability experiment with equally likely outcomes, the probability of an event E, written $P(E)$, is calculated by dividing the number of outcomes in E by the total number of possible outcomes.

Only use this method to calculate a probability if the outcomes are equally likely!

My Notes

MATH TERMS

The **complement** of an event *A* represents all the outcomes from a probability experiment not included in the outcomes of the event *A*. The *complement of event A* is written as *A′*.

Consider each pair of events below.

The number selected is equal to your age and the number selected is **not** equal to your age.

The number selected is greater than 75 and the number is 75 or less.

These paired events are ***complements*** of one another since when considered together they represent every possible outcome of the probability experiment.

12. Give the complement of each event and find its probability.

a. The number selected is an even number.

b. The number selected is less than 10.

Suppose that you can earn game points that can be redeemed for prizes if any of the following events occur when you select a chip at random.

Event	Description
E	The selected number is even.
T	The selected number is less than 10.
S	The selected number is greater than 75.
A	The selected number is equal to your age.

The number of points earned might be 10, 20, 30 or 40.

13. Do you think the greatest number of points should be assigned to the event with the greatest probability or the event with the least probability? Explain why you think this.

My Notes

14. If 10 points are assigned to the event with the greatest probability, 20 points to the event with the next greatest probability and so on, how many points would be assigned to each of the four events?

Event	Description	Points Assigned
E	The selected number is even.	
T	The selected number is less than 10.	
S	The selected number is greater than 75.	
A	The selected number is equal to your age.	

15. Suppose you will select one chip at random.

a. What is the probability that you will win 10 points?

b. Make sense of problems. If two of the events happen at the same time, then you get the points assigned to *both* events. For example, if you pick 76, then S and E both happen, and so you get 30 points. What is the probability that you will win 40 points? (*Hint:* There is more than one way to win 40 points.)

c. List all the outcomes that would result in you winning 30 points. What is the probability that you will win 30 points?

d. Is there a way for someone in your class to win 50 points? If so, how could this happen? If not, explain why this is not possible.

My Notes

Check Your Understanding

16. If a probability experiment has 12 equally likely outcomes, what is the probability of each outcome?

17. A probability experiment will consist of selecting one student at random from your math class. Is the probability that the selected student is female greater than, equal to, or less than the probability that the selected student is male? Explain your reasoning.

18. A brown paper bag contains 50 plastic chips. Each chip has a different number and the chips are numbered from 1 to 50.

a. For a probability experiment where one chip is selected at random from the bag, what is the probability of each of the following events?

Event	Description
B	The selected number is between 20 and 35 (not including 20 and not including 35).
L	The selected number is less than your age on your last birthday.
T	The selected number is a multiple of 3.

$P(B) =$
$P(L) =$
$P(T) =$

b. Write a description of each of the complementary events.

B'
L'
T'

c. Find the probability for each of the complementary events.

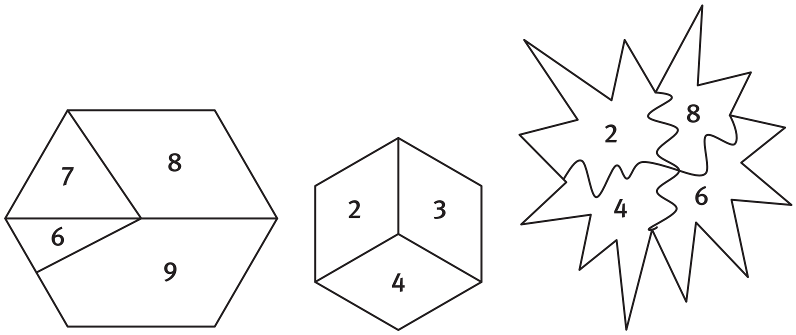
$P(B') =$
$P(L') =$
$P(T') =$

d. Write a statement about the relationship between the probability of an event and the probability of the complement of the event. Give examples to support your statement.

My Notes

LESSON 21-1 PRACTICE

19. Consider the spinners below.



Spinner 4   Spinner 5   Spinner 6

a. Which spinner has equally likely outcomes?
b. If a spinner has six equally likely outcomes, what is the probability of each outcome?

Consider a bag of white marbles, each with a number from 1 to 80 printed on it. After selecting a marble from the bag and replacing it, the bag is shaken to mix up the marbles so that each time a marble is pulled from the bag it is selected at random.

20. Suppose that you win points in a game based on the number of the marble that is chosen.
a. What are the possible outcomes for the number you might get?
b. Are the outcomes equally likely? If so, why is this important? If not, is there a way to estimate the probabilities?
c. What is the probability of each of these different outcomes?
d. What is the probability that a number greater than 60 will be selected?

21. Suppose that you earn points based on the ones digit that appears on the marble chosen at random.
a. What are the possible outcomes for this probability experiment?
b. Are the outcomes equally likely? If so, what is the probability of each outcome? If not, which outcomes are most likely to occur?

My Notes

22. Consider the following events.

Event	Description
O	The selected number is odd.
S	The selected number is more than seventy.
D	The selected number is equal to today's date.
T	The selected number is less than twenty.

a. If the number of points to be earned for each event is 10, 20, 30, 40, with the least points being assigned to the event with the greatest probability, then how should the points be assigned?

O _______ *S* _______ *D* _______ *T* _______

b. What is the probability that you will win 20 points?

23. Suppose a game is to be played with the marble bag where points are awarded for the following events.

a. If two of the events happen at the same time, then you get the points assigned to both events. What is the probability that you win 40 points?

b. List the possible outcomes for earning 25 points.

c. What is the greatest number of points that can be earned for the selection of a single number and what selection would allow that to happen?

d. State the complement of events *O* and *S*.

e. Find the $P(D')$ and $P(T')$.

My Notes

Learning Targets:

- Calculate theoretical probabilities for a probability experiment.
- Estimate probabilities by observing outcomes of a probability experiment.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

Now think about a different probability experiment. Suppose that a brown paper bag contains 40 chocolate candies that are all the same size and shape. Thirty of these candies are milk chocolate and 10 are dark chocolate.

1. Explain what it means when we say that a candy will be selected from the bag at random.

2. For this probability experiment, what is the probability of the event M, where M is the event that a milk chocolate candy is selected? How did you calculate this probability?

This probability is an example of a ***theoretical probability***.

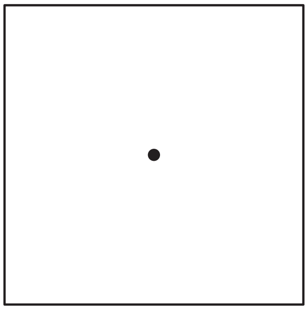
3. If we are just interested in whether the selected candy is milk chocolate or dark chocolate, are the outcomes milk chocolate and dark chocolate equally likely?

MATH TERMS

A **theoretical probability** is one that you can calculate from the description of a probability experiment.

My Notes

4. Use the blank spinner below to design a spinner with two regions that could represent the probability experiment of selecting a candy at random from the bag and noting if it is milk chocolate or dark chocolate. Label the two regions *Milk* and *Dark*.



5. With a partner, discuss how spinning this spinner is like selecting a candy at random from the bag and noting whether the candy is milk chocolate or dark chocolate. Then write a few sentences explaining how spinning is like selecting from the bag.

6. Using a paper clip, as you did in the previous activity, record the outcomes (milk or dark) for 20 spins of the spinner. This is like making 20 selections from the bag.

Spin outcomes:

7. If someone didn't know what the spinner looks like, but saw your outcomes from Item 6, what would that person estimate $P(M)$ to be? How does this ***estimated probability*** compare to the actual (theoretical) probability calculated in Item 2?

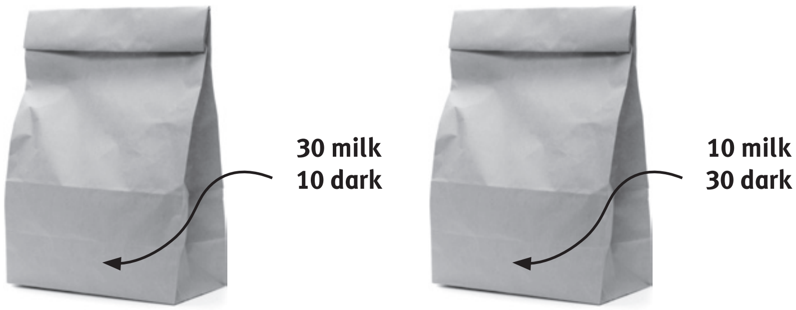
8. Combine your spin results with those of another student in your class so that you now have outcomes from 40 random selections. What is the estimated value of $P(M)$ for these combined results?

Total number of milk chocolate out of 40:

Estimated $P(M) =$

Is this estimated probability closer to the actual probability of 0.75 than your earlier estimate?

9. Suppose there are two paper bags and that each bag has 40 chocolate candies. One bag has 30 milk chocolate candies and 10 dark chocolate candies, and the other bag has 10 milk chocolate candies and 30 dark chocolate candies.



Someone selects one of the bags above at random and gives it to you, but you don't know which bag you have been given.

Bag?

My Notes

MATH TERMS

An **estimated probability** is one that is calculated by observing the outcome of a probability experiment many times.

Estimated probabilities are sometimes also called empirical probabilities.

My Notes

You select one candy at random from this bag, and it is a milk chocolate candy.

a. If you had to guess if this bag is Bag 1 or Bag 2, what would you say? Why did you pick the bag you did?

b. How likely do you think it is that you picked the correct bag?
   1. There is about a 50-50 chance.
   2. There is better than a 50-50 chance.
   3. It is certain that I picked the correct bag.

c. Explain your reasoning.

d. Discuss your reponses to parts a-c in pairs or small groups. As you share ideas in your group, ask group members or your teacher for clarification of any language, terms, or concepts you do not understand.

10. Suppose that you put the selected candy back in the bag, mix up the candies in the bag, and select a candy at random. This candy is milk chocolate. How would you answer the two items above (part a and part b from Item 9) in light of this additional information?

My Notes

Check Your Understanding

A brown paper bag contains 100 plastic chips. Of these chips, 30 are red, 50 are green, and 20 are blue. A chip will be selected at random from this bag. Use this information to answer Items 11–14 below.

11. Calculate the probability that the selected chip is green.

Is the probability that you just calculated a theoretical probability or an estimated probability?

12. A friend reasons that because there are 3 different colors of chips in the bag, the probability of selecting a blue chip is $\frac{1}{3}$. Explain why this is incorrect.

13. **Make sense of problems.** Suppose you can add blue chips to the bag. If you wanted the probability of selecting a blue chip to be $\frac{1}{3}$, how many blue chips should you add to the bag?

14. Once you have added enough blue chips to the bag so that the probability of selecting a blue chip is $\frac{1}{3}$, explain why the three events described below are still not equally likely.

Event	Description
R	The selected chip is red.
G	The selected chip is green.
B	The selected chip is blue.

My Notes

LESSON 21-2 PRACTICE

A paper bag contains 100 chips. Some of these chips are labeled Win and the others are labeled Lose. You will select a chip at random from this bag and note whether the chip says win or lose. Then, put that chip back in the bag, mix up the chips, and select at random from the bag again. Repeat 30 times.

15. Suppose the results of these 30 selections are shown in the table below, in which W represents Win and L represents Lose.

L	L	W	L	W	W	L	L	L	L
W	L	L	L	L	W	L	W	W	L
L	L	W	L	L	L	L	L	L	L

a. Use the 30 observed outcomes to estimate the probability of selecting a chip that says Win. (Write your answer as a decimal, rounded to two decimal places.)

b. Is the probability you just calculated a theoretical probability or an estimated probability?

c. The actual number of Win chips in the bag is a multiple of 5. How many Win chips do you think are in the bag? Explain how you arrived at your answer.

16. Design and draw a spinner that has two regions labeled Win and Lose. Create the spinner to represent the probability experiment of selecting a chip at random from the bag and noting if it says Win or Lose.

17. Use the spinner you drew to generate 30 outcomes. Record them and produce totals.

18. Calculate $P(\text{W})$ based on your outcomes. Is the probability that you calculated an estimated or theoretical probability?

19. **Construct viable arguments.** Do you think that the spinner generates results similar to selecting chips at random from the bag? Provide reasoning and evidence to support your answer.

My Notes

Learning Targets:

- Compare theoretical probabilities and estimated probabilities.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing

The students in Ms. Bailey's seventh grade class completed a survey with these five questions:

a. Are you male or female?
b. Are you right-handed or left-handed?
c. About how many text messages did you send yesterday?
d. What is your favorite subject in school?
e. If you could have one super power, which of the following would you choose?

Invisibility Super strength Able to read minds
Able to fly Able to freeze time

Review the survey questions to ensure that you understand their meaning. A summary of the survey results is on page 267. Remove that page and use it to answer the following items.

1. In the probability experiment that consists of selecting a student at random from Ms. Bailey's class, how many possible outcomes are there?

2. Are the possible outcomes equally likely? Explain why or why not.

3. If a student is selected at random from this class, what is the probability that the student selected is female? How did you calculate this probability?

4. Is the probability you calculated in Item 3 a theoretical probability or an estimated probability?

My Notes

WRITING MATH

You could use the notation of events and probability to write the probability notation for events in these questions.

For example,

If *F* is the event that the selected student is female in Item 3, you would write *P*(*F*) for the probability.

If *S* is the event that the selected student's favorite subject is science, then Item 5 asks you to calculate *P*(*F* and *S*).

5. If a student is selected at random from this class, what is the probability that the student selected is female and a student whose favorite subject is science? How did you calculate this probability?

6. Is the probability that the selected student is a female whose favorite subject is science less than, equal to, or greater than the probability that the selected student is female? Explain why this makes sense.

7. If a student is selected at random from this class, what is the probability that the student selected sent more than 20 text messages yesterday? How did you calculate this probability?

8. If a student is selected at random from this class, what is the probability that the student selected is a right-handed male? How did you calculate this probability?

9. Answer this question without calculating the probability that the randomly chosen student is male: Which is greater, the probability that the randomly chosen student is male, or the probability that the randomly chosen student is a right-handed male? Explain your reasoning.

10. **Make sense of problems.** Write Item 7 using events and probability notation.

Survey Data from Ms. Bailey's Seventh Grade Class

Student	Male or Female?	Right or Left-Handed?	Favorite Subject?	Texts Sent?	Superpower?
1	Male	Right	Math	34	Read minds
2	Female	Left	English	20	Fly
3	Female	Right	English	0	Fly
4	Female	Right	Art	1	Fly
5	Male	Right	Science	34	Read minds
6	Female	Right	English	3	Fly
7	Male	Right	History	0	Freeze time
8	Female	Right	English	0	Freeze time
9	Female	Right	Science	300	Fly
10	Male	Right	PE	0	Read minds
11	Male	Right	English	34	Read minds
12	Male	Right	Music	237	Fly
13	Male	Right	PE	100	Fly
14	Male	Right	Science	0	Freeze time
15	Female	Left	English	200	Read minds
16	Female	Left	English	40	Fly
17	Male	Right	Math	34	Read minds
18	Female	Right	English	30	Invisibility
19	Female	Right	Science	200	Fly
20	Female	Right	English	0	Fly
21	Male	Right	History	34	Read minds
22	Male	Left	PE	94	Fly
23	Female	Right	English	2	Fly
24	Male	Right	English	35	Freeze time
25	Female	Right	Science	10	Fly
26	Female	Right	English	15	Freeze time
27	Female	Right	English	20	Fly
28	Female	Right	Art	2	Fly
29	Female	Right	English	0	Freeze time
30	Female	Right	Science	2	Read minds
31	Male	Right	Science	34	Read minds
32	Female	Right	Math	0	Fly
33	Female	Right	Art	0	Fly
34	Female	Left	English	8	Fly

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Check Your Understanding

11. Write three probability questions for the probability experiment of selecting a student at random from Ms. Bailey's class that could be answered using the survey data provided.

Question 1:

Question 2:

Question 3:

12. Trade these probability questions with another student in your class. Answer the questions that the other student wrote while that person answers your questions. Then, working together, check each other's work to make sure the probabilities calculated are correct.

13. Considering the results for the thirty-four students on the survey and what you have learned about calculating probabilities, predict (without calculating) how the following probabilities will compare using $<, >, =$.

a. Compare P(Male and Flying) with P(Flying)

b. Compare P(English) with P(English and less than 20 texts)

c. Compare P(Male and Left) with P(Male and Left and Math)

d. Compare P(Female and Right) with P(Right and Female)

14. Express regularity in repeated reasoning. Calculate the probabilities for the comparisons in the previous item to confirm your predictions. Write a conclusion based on patterns you have noticed that would help you when making future comparisons of this type.

LESSON 21-3 PRACTICE

Recall the probability experiment of selecting a student at random from Ms. Bailey's seventh grade class. Some events are defined below.

Event	Description
M	Student selected is male
R	Student selected is right-handed
S	Student selected chose science as favorite subject
T	Student selected sent more than thirty text messages
F	Student selected chose freeze time as super power

Use the survey data to find the following probabilities.

15. $P(R) =$

16. $P(S) =$

17. $P(T) =$

18. $P(F) =$

19. $P(S') =$

20. $P(T') =$

21. $P(F') =$

22. $P(M \text{ and } S) =$

23. $P(R \text{ and } S) =$

24. $P(M \text{ and } T) =$

25. $P(S \text{ and } T) =$

26. $P(R \text{ and } F) =$

27. $P(T \text{ and } F) =$

ACTIVITY 21 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 21-1

1. Suppose that a person has a bag of marbles. Explain what you could conclude about what is in the bag for each of the following statements.
   a. The probability of selecting a green marble when a marble is selected at random is 1.
   b. The probability of selecting a green marble when a marble is selected at random is 0.

Use the information below to answer Items 2–4.

A box contains 50 balls that are numbered from 1 to 50 and each ball has a different number. The balls numbered 1 to 22 are red. The balls numbered 23 to 40 are white. The balls numbered 41 to 50 are blue and white striped. One ball will be selected at random from this box.

2. For the probability experiment of selecting a ball at random from this box, how many different outcomes are there?

3. Are the outcomes equally likely? How do you know this?

4. Find the following probabilities:
   a. *P*(striped)
   b. *P*(a number greater than 27)
   c. *P*(red and has an even number)
   d. Are the probabilities you calculated theoretical probabilities or estimated probabilities?

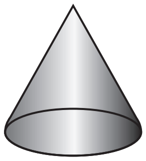
5. From 1999 to 2008, the United States Mint produced commemorative quarters for each state in the United States. Each year, quarters for five different states were circulated to the public, and many people collected the coins. Steven wanted to collect these coins, so his uncle gave him a handful of the state quarters. Steven made this list of the coins that he was given.

State	Year
Tennessee	2002
Tennessee	2002
Ohio	2002
Alabama	2003
Maine	2003
Florida	2004
Florida	2004
Florida	2004
Texas	2004
Texas	2004
California	2005
California	2005
Kansas	2005

Steven will put all the coins in his pocket and will pull one quarter out at random. Find the following probabilities.
   a. The probability that he selects a Texas quarter
   b. The probability that he selects a quarter minted in 2002
   c. The probability that he selects a quarter that has a state name that begins with a letter that is in the first half of the alphabet

Lesson 21-2

6. Sam tossed a cone 25 times and recorded whether it landed on its base or on its side.



On its base	On its side
𝍸 |||	𝍸 𝍸 𝍸 ||

What is the estimated probability that the cone will land on its base when tossed?

7. A bag contains 80 marbles. If the probability of drawing a yellow marble is $\frac{3}{5}$ and the probability of drawing a red marble is $\frac{2}{5}$, how many marbles of each color are in the bag?

8. A bag contains 100 chips. Each of the chips in the bag is either purple or yellow. Suppose you select a chip from the bag at random and record the color. Then you put the chip back in the bag and select again. You repeat this until you have observed 50 outcomes. Of these 50 outcomes, 17 were yellow and 33 were purple.

a. What is the estimated probability of selecting a purple chip when a chip is selected at random from this bag?

b. Suppose that you know that the bag was one of two bags—Bag 1 or Bag 2. Bag 1 has 30 yellow chips and 70 purple chips. Bag 2 has 50 yellow chips and 50 purple chips. Based on the outcomes you observed, which bag do you think you were selecting from—Bag 1 or Bag 2? Explain your reasoning.

Lesson 21-3

Recall the probability experiment of selecting a student at random from Ms. Bailey's seventh grade class. Use the survey data sheet to help you answer Items 9 and 10.

9. What is an example of an event that would have a probability greater than 0.5?

10. What is an example of an event that would have a probability less than 0.25?

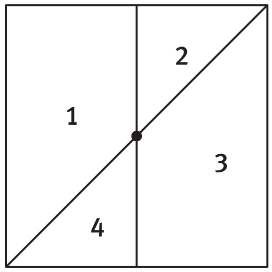
MATHEMATICAL PRACTICES
Attend to Precision

11. Explain the difference between a theoretical probability and an estimated probability. Select a probability experiment to provide examples to support your answer.

Finding Probabilities
SPINNING SPINNERS AND RANDOM PICKS

Write your answers on notebook paper or grid paper. Show your work.

1. Only one of the following sequences resulted from actually spinning the spinner below 20 times. Which sequence do you think this was?



Sequence 1: 4 3 2 1 4 3 2 1 4 3 2 1 4 3 2 1 4 3 2 1

Sequence 2: 4 1 3 4 2 3 4 3 1 4 1 4 1 2 2 2 4 2 1 3

Sequence 3: 4 3 3 3 2 1 1 1 4 3 3 3 2 1 1 1 4 3 3 3

Sequence 4: 1 4 1 3 3 3 1 3 2 2 3 4 1 1 4 1 1 3 3 3

2. For each sequence you did not select in Item 1, explain why you ruled out that sequence.

3. The following were the outcomes from 20 spins of a spinner. Design a spinner that you think might have generated this sequence of outcomes. Explain your reasoning in designing the spinner.

3 4 4 4 4 4 2 4 1 4 4 2 4 4 4 3 1 4 4 2

4. Mr. Lund's science class has 30 students. There are 20 students in this class who do not plan to enter a project in the upcoming school science fair.

a. If you were to select a student at random from this class, what is the probability that the selected student is planning to enter the science fair?

b. Is the probability you calculated in part a a theoretical probability or an estimated probability?

c. Could the probability that a student selected at random from this class is a female who plans to enter a project in the science fair be greater than the probability you calculated in part a? Explain why or why not.

d. Could the probability that a student selected at random from this class is a female be greater than the probability you calculated in part a? Explain why or why not.

5. A paper bag contains 100 plastic chips. Each chip has a different number and the chips are numbered from 1 to 100. One chip will be selected at random from the bag.

a. Which of the following events has the greatest probability? Which has the smallest probability?

Event	Description of Event
A	Selected number is an odd number greater than 50
B	Selected number is an odd number less than 20
C	Selected number is even

b. Give an example of an event that would have a probability that is greater than any of the events listed in part a.

c. Give an example of an event that would have a probability that is less than any of the events listed in part a.

6. Give an example of an event and its complement. What do you know about the sum of their probabilities?

7. Give an example of a probability experiment and a probability question that would involve calculating a theoretical probability.

8. A box contains a large number of plastic balls. Some of the balls are red and the rest are green. One ball will be selected at random from the box.

a. Using only the information you have been given, can you conclude that the probability of getting a red ball is $\frac{1}{2}$? Explain why or why not.

b. Based on the description of the probability experiment, can you calculate the theoretical probability that a red ball is selected? Explain why or why not.

c. What would you have to do in order to find an estimated probability of selecting a red ball?

9. If two people each correctly calculate the theoretical probability of an event, will they get the same answer? Explain why or why not.

10. If two people each use a correct method to find an estimated probability of an event, will they always get the same answer? Explain why or why not.

Finding Probabilities
SPINNING SPINNERS AND RANDOM PICKS

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3, 4a-d, 5a-c, 6, 7, 8a-c, 9, 10)	• Clear and accurate understanding of calculating estimated and theoretical probabilities.	• Calculating estimated and theoretical probabilities with few if any errors.	• Difficulty calculating estimated and theoretical probabilities.	• Incorrect or incomplete calculation of estimated and theoretical probabilities.
Problem Solving (Items 4a, 5a)	• Accurately interpreting possible outcomes as probability.	• Interpreting possible outcomes as probability.	• Difficulty interpreting outcomes as probability.	• No clear understanding of interpreting outcomes.
Mathematical Modeling / Representations (Items 1, 3. 5b-c)	• Accurately using theoretical probability to model outcomes of events.	• Using theoretical probability to model outcomes of events.	• Errors in using theoretical probability to model outcomes of events.	• Inaccurate or incomplete use of theoretical probability to model outcomes.
Reasoning and Communication (Items 2, 3, 4c-d, 5b-c, 6, 7, 8a-c, 9, 10)	• Clear and accurate explanation of estimated and theoretical probabilities.	• An adequate explanation of estimated and theoretical probability.	• Difficulty in explaining estimated and theoretical probability.	• An inaccurate explanation of estimated and theoretical probability.

Games and Probability

Rock, Paper, Scissors . . . and Other Games

Lesson 22-1 Rock, Paper, Scissors

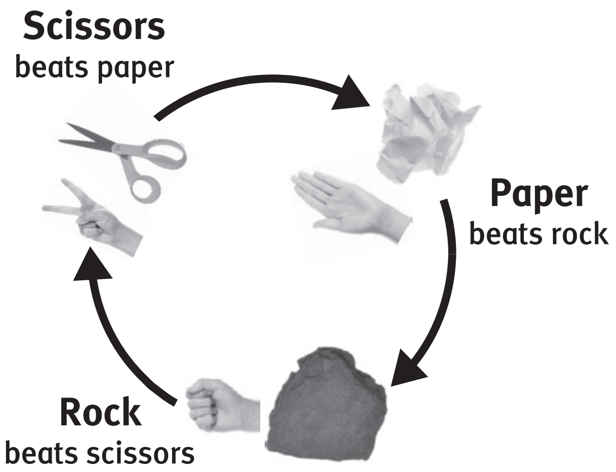
My Notes

Learning Targets:

- Use observed outcomes to estimate probabilities.
- Use tables to represent the possible outcomes of a probability experiment.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

Rock, Paper, Scissors (RPS) is a fun two-person game. Some sources say that more people have played Rock, Paper, Scissors than any other game in the world.



To play RPS, do the following:

- Each player taps a fist in his or her palm twice.
- Then, both players simultaneously extend their hands in the shape of a rock, piece of paper, or pair of scissors. This is called a throw.

The winner of a round is decided based on the following rules:

- Rock beats scissors
- Scissors beats paper
- Paper beats rock

If both players show the same shape, the round results in a tie.

1. With a partner, play 10 rounds of RPS. For each round, record whether you won, you lost, or the round resulted in a tie.

Round	1	2	3	4	5	6	7	8	9	10
Result										

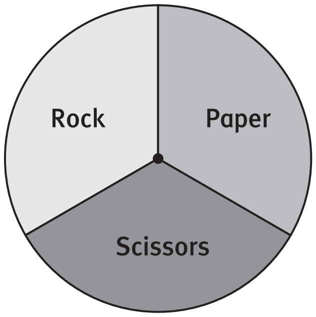
2. Did one player win more often? What fraction of the rounds resulted in a tie?

My Notes

3. **Reason abstractly.** If both players in a RPS game choose between rock, paper, and scissors at random, which do you think would occur most often: Player 1 wins, Player 2 wins, or the round results in a tie? Explain your reasoning.

4. Do you think players really pick at random each time they play RPS or do you think some players tend to favor some moves (rock, paper or scissors) over others?

5. To ensure that a throw is selected at random, use the spinner below to first determine a throw for Player 1. Then spin again to determine a throw for Player 2. (Use a paper clip, just as you did in the previous two activities.) Do this a total of 20 times, filling in the first two columns of the table on the next page.



My Notes

Round	Player 1	Player 2	Winner
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			
16			
17			
18			
19			
20			

6. Complete the last column of the table above by determining the winner for each round. If the round was a tie, enter Tie.

How many times did Player 1 win?

How many times did Player 2 win?

How many rounds were ties?

7. Combine your counts from Item 6 with those of two other students so that you have the outcomes of 60 rounds of play.

What fraction of the time did Player 1 win?

What fraction of the time did Player 2 win?

What fraction of the rounds were ties?

8. Based on the data you have generated from the class, state a hypothesis about the probability of a student choosing rock, paper, or scissors. Do you think the data collected is the same as expected? Why or why not? Find the probability of each.

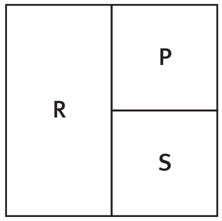
My Notes

MATH TIP

Remember that a fair game does not favor one outcome over the other.

Check Your Understanding

9. RPS is a fair game if each player chooses a throw at random. Suppose a player used the following spinner to choose a throw.



Is this equivalent to selecting a throw at random? Explain why or why not.

10. In a random choice game of RPS, how often do you expect to lose a round of play on average?

Suppose you are playing a game of RPS in which the other player is using the spinner in Item 9 above to select throws. Can you think of a strategy in which you could minimize how often you expect to lose a round? Explain.

LESSON 22-1 PRACTICE

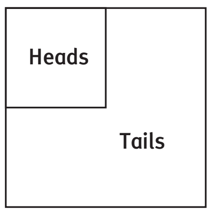
Sam and Jo are playing a coin game. If both coins land on either heads or tails, Sam wins. If one coin lands on heads and the other on tails, Jo wins.

11. The results of fifteen games are shown below. Determine which games were won by Sam and by Jo. (H for heads; T for tails)

Coin 1	H	T	T	T	T	T	T	H	T	T	T	T	T	T	H
Coin 2	H	H	H	H	H	T	H	H	T	T	H	H	H	H	H
Winner															

12. What fraction of the time did Sam win? What fraction of time did Jo win?

The match game is a fair game when played with fair coins. Suppose Sam and Jo use the spinner below to determine whether they get heads or tails.



13. **Make sense of problems.** Make a table to show Sam's and Jo's results using the spinner. What percent of the time should Sam win? What percent of the time should Jo win? Explain.

My Notes

Learning Targets:

- Use tables to represent the possible outcomes of a probability experiment.
- Assign probabilities to outcomes in a sample space.
- Use probabilities assigned to outcomes in a sample space to compute event probabilities.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

1. One possible outcome of a round of RPS is that Player 1 throws a rock and Player 2 throws scissors. This could be written in the form of a (Player 1, Player 2) pair as (R, S).

Make a list of all the possible outcomes for a round of RPS. (*Hint:* There are 9 possible outcomes, including (R, S).)

The collection of all the possible outcomes of a probability experiment is the ***sample space***. For the probability experiment that is one round of RPS, the sample space consists of the nine outcomes from Item 1.

MATH TERMS

The **sample space** for a probability experiment is the set of all possible outcomes for that experiment.

Sometimes it helps to have a strategy that will help you find the sample space for a probability experiment.

One strategy is to make a list. For example, for the RPS game, you could begin with one possibility for Player 1—for example, rock. Then you could list rock with each of the different possibilities for Player 2:

(R, R), (R, P), (R, S)

Next, you could list all of the outcomes that have paper for Player 1, and then all the outcomes that have scissors for Player 1.

My Notes

Another possibility is to construct a table like the one below.

		Player 2 Throws		
		R	P	S
Player 1 Throws	R	(R, R)	(R, P)	
	P			
	S			

Each cell in the table above corresponds to one possible outcome.

2. Complete the rest of the table so that it includes all of the possible outcomes.

Recall that an ***event*** is a collection of outcomes from a probability experiment.

MATH TERMS

An **event** is a collection of outcomes of a probability experiment. It is a part of the sample space.

3. For the RPS game, what outcomes are in the event that Player 1 wins?

4. For the RPS game, what outcomes are in the event that Player 2 wins?

5. For the RPS game, what outcomes are in the event that a round results in a tie?

6. Look for and express regularity

a. If players are picking throws at random, would any of the nine possible outcomes be more likely to occur than the others?

b. For the random RPS strategy, what probability would you assign to each outcome in the sample space?

My Notes

7. Calculate the following probabilities for a round of RPS where players are selecting throws at random.

P(Player 1 throws a rock)

P(Player 1 wins)

P(Player 1 throws a rock and wins)

P(The round results in a tie)

8. Suppose two players were selecting throws at random and play 100 rounds of RPS.

a. About how many of them would result in a tie?

b. About how many of them would result in a win for Player 2?

Play the game again, but this time, do not use the spinner to determine the throws. Instead, choose your throws as you would in a real game of RPS.

9. With a partner, play 20 rounds, recording your throw in the table on the next page after each round. Note that for this part of the activity, you only need to record your throw, and you do not need to record your partner's throw.

MATH TIP

To find the probability of an event, you can add the probabilities for each outcome in the event.

If the outcomes in the sample space are equally likely, the probability can also be found by calculating

$$\frac{\left(\text{number of outcomes in the event}\right)}{\left(\text{number of outcomes in the sample space}\right)}$$

My Notes

Round	My Throw (R, P or S)
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

10. Count how many times you threw each of rock, paper and scissors and record your counts here:

Number of rocks:

Number of papers:

Number of scissors:

Add your counts to a class poster that your teacher will provide by adding the appropriate tally marks. (You will be adding a total of 20 tally marks to the poster.)

11. Reason abstractly. Are these counts consistent with what you would expect if you and your classmates were selecting throws at random? Explain why or why not.

MATH TIP

Add tally marks in groups of fives (using four strokes and a fifth stroke across the first four.)

For example:

Class Throws

Rock: 𝍸 𝍸 ||| …

Paper: 𝍸 |||| …

Scissors: 𝍸 𝍸 | …

Make sure to continue the groups of five where the last student left off.

My Notes

Your teacher will also provide two more posters—one for girls and one for boys. Add your counts by adding tally marks to the appropriate poster.

12. Some people believe that boys are more likely than girls to throw a rock when playing RPS.

a. Based on the two posters your class just made, do you think this is the case? Explain why or why not.

b. Determine the theoretical probabilities from the class poster data.

Check Your Understanding

A variation of RPS is played in some countries, which adds a fourth throw called a Well. In this game:

- Well beats Rock
- Well loses to Paper
- Well beats Scissors

13. Construct a table (like the one in Item 9) that shows the different possible outcomes for a RPSW round.

14. a. What outcomes for RPSW result in a win for Player 1?

b. What outcomes result in a win for Player 2?

c. Is the probability of a win the same for Player 1 and for Player 2?

15. RPSW is a fair game if both players select one of the four possible throws at random. Suppose you know that your opponent is selecting throws at random using this spinner:

R	P
W	S

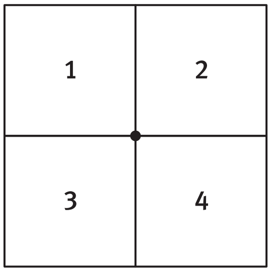
Can you think of a strategy in which you would expect to lose only about $\frac{1}{4}$ of the rounds that you play? Explain why you think this strategy will work.

My Notes

LESSON 22-2 PRACTICE

The following are rules for the High Roller Game, using the spinner below:

- Each player spins, and the player with the greater number wins.
- If both players spin the same number, then it's a tie for that round.



16. Give the sample space for this game, written as ordered pairs (first player, second player).

17. Play the game 20 times, recording the following information:

Round	Player 1's Number	Player 2's Number	Winner

18. Using the table, calculate the following probabilities:
a. P(Player 1 wins)
b. P(tie)
c. P(Player 2 wins)

Instead of spinning a spinner, each player writes down the numbers 1 to 20, representing the rounds of play, and then places five 1's, five 2's, five 3's, and five 4's into the twenty rounds. Next, the players compare numbers to determine the winner of each round. A sample is shown below.

Round	1	2	3	4	5	6	7	8	9	10
Person A	1	2	3	4	1	2	3	4	1	2
Person B	1	1	1	1	1	2	2	2	2	2
Winner										

Round	11	12	13	14	15	16	17	18	19	20
Person A	3	4	1	2	3	4	1	2	3	4
Person B	3	3	3	3	3	4	4	4	4	4
Winner										

19. Determine the winner each time. Then calculate the following:
a. P(Person A wins) **b.** P(Person B wins) **c.** P(tie)

20. **Reason quantitatively.** If you were going to place five 1's, five 2's, five 3's, and five 4's in twenty rounds of the High Roller game, what strategy would you use to maximize your wins?

My Notes

Learning Targets:

- Use observed outcomes to estimate probabilities.
- Use tables and tree diagrams to represent the possible outcomes of a probability experiment.
- Calculate the probabilities of events for a probability experiment with equally likely outcomes.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

A New Game . . .

Now let's think about a different game. Suppose there are two boxes—one big box and one small box. Each box has two drawers.

There is a prize hidden in one of these drawers.

To play the game, you first pick a box, and then you pick either the top drawer or the bottom drawer.

1. Choose a size and then either top or bottom, and then share with your class. Using the class data, estimate the probability that you would win the prize.

2. If you think of this as a probability experiment, there are four possible outcomes. One is (Big, Top), which stands for picking the big box and the top drawer.

List all four possible outcomes for this probability experiment.

(Big, Top)

MATH TIP

You can try a systematic approach to list possible outcomes.

For example, starting with one box and listing the different possibilities is a systematic approach.

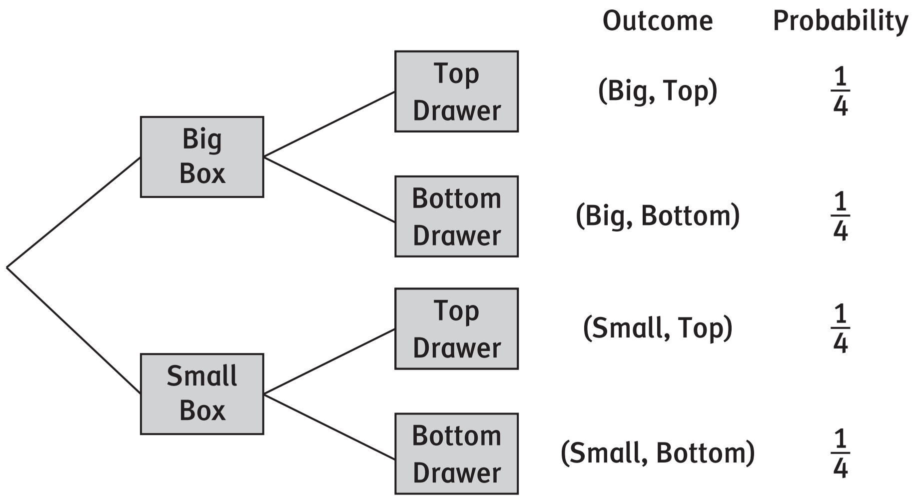
My Notes

3. If you make your selections (box and then drawer) at random, are the four outcomes equally likely? What is the probability of each of these outcomes?

MATH TERMS

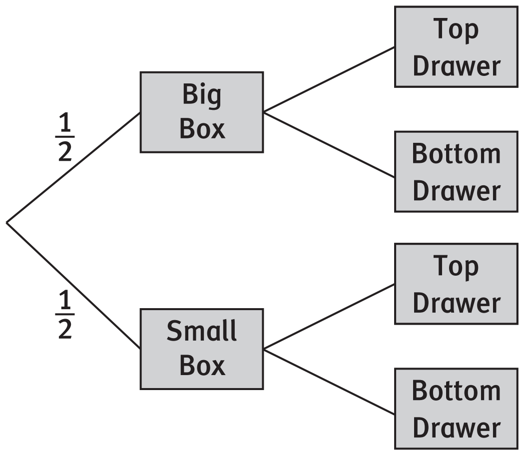
A **tree diagram** is a diagram that represents the total possible outcomes.

When a probability experiment can be carried out in steps, a common way to organize the outcomes is to construct a ***tree diagram***. The tree diagram for the probability experiment just described looks like this:



4. If you select a box at random, what is the probability that you select the big box? What is the probability that you select the small box?

We sometimes write the probabilities on the branches of the tree diagram, as shown below. This indicates that the probability of selecting the big box is $\frac{1}{2}$, and the probability of selecting the small box is $\frac{1}{2}$.

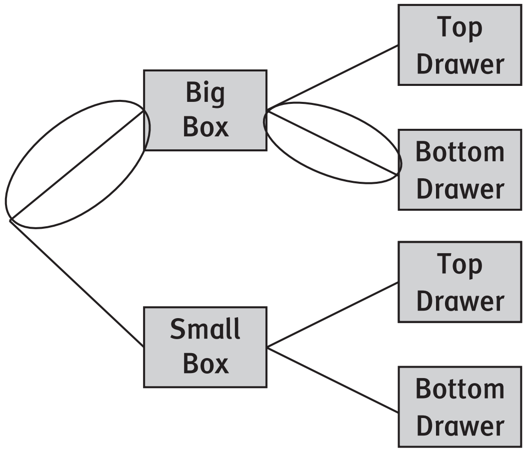


My Notes

5. If you select a drawer at random, what is the probability that you select the top drawer? The bottom drawer?

Label the remaining four branches in the tree diagram on the previous page with the appropriate probabilities.

One way to arrive at a possible outcome is to follow branches of the tree starting at the left. For example, the outcome (Big, Bottom) could be arrived at by following the two circled branches:



6. **Reason quantitatively.** Because the four possible outcomes of the probability experiment are equally likely, once you know how many outcomes there are, you know what probability to assign to each outcome (in this case, $\frac{1}{4}$). What is another way to calculate the probability of an outcome that uses the probabilities on the branches that lead to that outcome?

My Notes

Suppose each box now has three drawers as shown below.

7. Compare the probability from the class data and the probability from Item 6. Explain your comparison.

8. If you pick a box at random and then pick a drawer at random, how many different outcomes are there?

9. Construct a tree diagram that shows the possible outcomes.

10. Put probabilities on the branches of your tree diagram in Item 7. Pick an outcome and multiply the probabilities on the branches that lead to that outcome. What do you notice about the product?

MATH TIP

To find the probability of an outcome in a tree diagram with probabilities on the branches, multiply the probabilities on the branches that lead to that outcome.

My Notes

11. **Express regularity in repeated reasoning.** When there were two boxes and two drawers, there were four possible outcomes. When there were two boxes and three drawers, there were six possible outcomes. How many outcomes would there be if there were four boxes that each had two drawers? How did you find your answer?

12. Use a table or a tree diagram to represent all of the possible outcomes when there are four boxes and each box has two drawers. Was your answer in Item 9 correct?

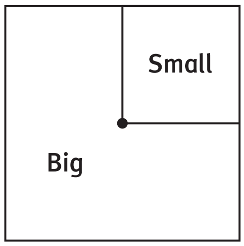
13. How many possible outcomes are there with 20 boxes if each box has four drawers? If you were to pick a box at random and then pick a drawer at random, what is the probability of each of the outcomes?

My Notes

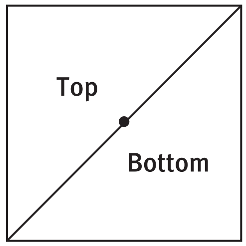
Check Your Understanding

Consider the game in which there are two boxes and each box has two drawers. Instead of picking a box at random and then picking a drawer at random, you decide to use the following two spinners to make your selections.

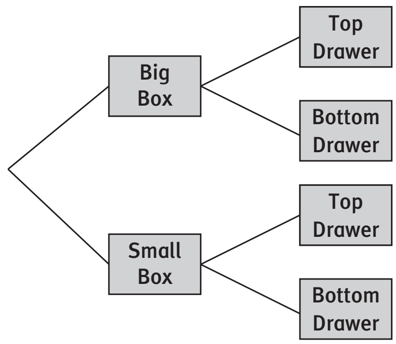
Box Spinner



Drawer Spinner



14. Put the appropriate probabilities on the branches of the tree diagram below.



15. Consider the probability experiment that consists of choosing a box and then choosing a drawer using the spinners above. One possible outcome is (Big, Top).

What is the sample space for this experiment?

LESSON 22-3 PRACTICE

Using the spinner in Check Your Understanding, complete Items 14–18.

16. Make sense of problems. The four outcomes in the sample space are not equally likely. How do you know?

17. If a prize is hidden in the top drawer of the big box, what is the probability that you will win this prize?

18. If a prize is hidden in the top drawer of the small box, what is the probability that you will win this prize?

19. Explain why the probability for a prize in the top drawer of the big box is different from the probability for the prize hidden in the top drawer of the small box.

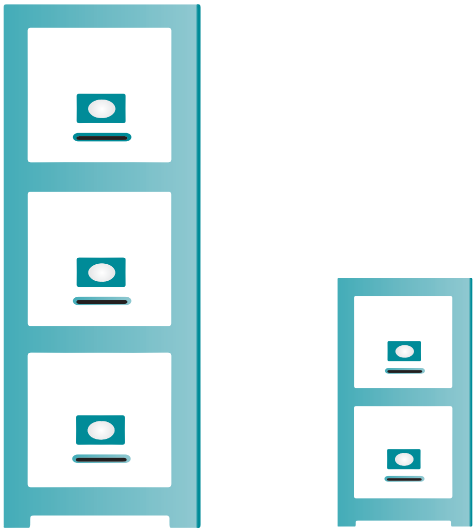
My Notes

Learning Targets:

- Use observed outcomes to estimate probabilities.
- Use tables and tree diagrams to represent the possible outcomes of a probability experiment.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

For the rest of this activity, consider a game with the big boxes and small boxes below. The big box has three drawers (top, middle, and bottom) and the small box has two drawers.



You will pick a box at random and then pick a drawer at random from that box.

1. Draw a tree diagram that shows the possible outcomes.

My Notes

2. Put probabilities on the branches of your tree diagram and multiply across branches to get a probability for each outcome.

3. Reason abstractly. Are all of the outcomes equally likely? Explain.

4. If there is one prize and it is hidden in the top drawer of the small box, what is the probability that you select the drawer with the prize?

5. Suppose that there are two prizes and they are hidden in the top drawer of the big box and in the bottom drawer of the small box. What is the probability that you select a drawer that has a prize? How did you arrive at this probability?

6. Suppose that there are three prizes and they are hidden in the top drawer of the big box, the middle drawer of the big box and the top drawer of the small box. What is the probability that you select a drawer that has a prize? How did you arrive at this probability?

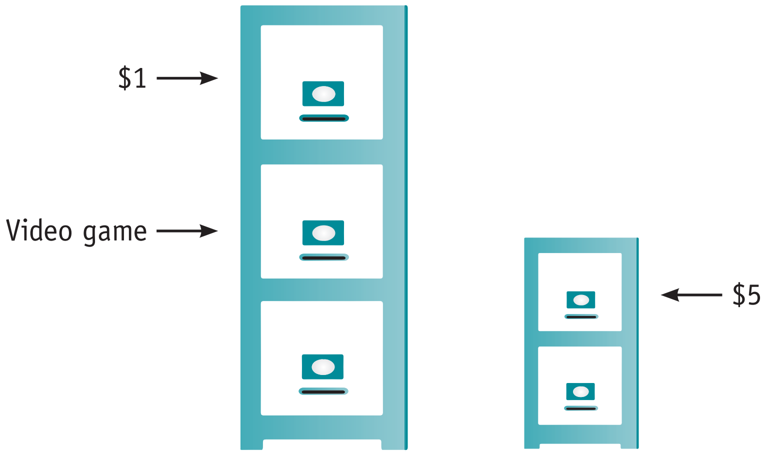
My Notes

Suppose that there are prizes in three different drawers, and you are going to select a box at random and then a drawer at random from that box.

7. **Construct viable arguments.** Is the probability of getting a prize greater if all three prizes are in the big box or if one prize is in the big box and two prizes are in the small box? Support your answer with probabilities.

Check Your Understanding

Suppose that three different prizes are placed in different drawers as shown here.



8. Make a tree diagram for selecting a box at random and then selecting a drawer at random. Write the probabilities along the branches of the tree diagram. Calculate each of the five possible outcomes.

9. If you select a box at random and then a drawer at random, what are the following probabilities?
   a. P(win video game) =
   b. P(do not win a prize) =
   c. P(win a prize) =
   d. P(win \$5) =
   e. P(win money) =
   f. P(win a prize that is not money) =
   g. P(win a prize that is worth more than \$1) =
   h. P(win the video game or \$5) =

MATH TIP

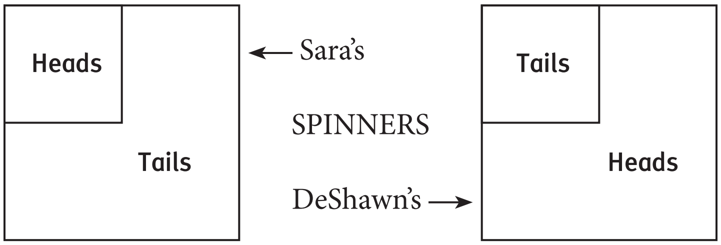
Since a tree diagram shows all of the outcomes, you can use a tree diagram to help you determine probabilities.

For example, you can use the tree diagram you drew in Item 8 to help you determine the probabilities in Item 9.

My Notes

LESSON 22-4 PRACTICE

Sara and DeShawn are playing the Match Game with two spinners.



Both players spin at the same time.

10. **Model with mathematics.** Draw a tree diagram with probabilities to summarize the outcomes of the Match Game.

11. Write the sample space of the two spinners as ordered pairs (Sara's result; DeShawn's result).

12. What is the probability that DeShawn's spinner will result in tails?

13. What is the probability that both spinners will show heads?

14. If you were advising Sara, should she play to win if the results of both spinners are the same (match) or if they are different? Explain.

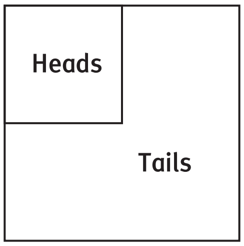
ACTIVITY 22 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 22-1

Use the following situation to answer Items 1–3.

Tia and Li are playing a coin game using the spinner below. If both spins land on either heads or tails, Tia wins. If one spin lands on heads and the other on tails, Li wins.



1. Complete the table for ten games. Determine which games were won by Tia and Li.

Coin 1										
Coin 2										
Winner										

2. What fraction of the time did Tia win?

3. What fraction did Li win?

Lesson 22-2

4. Calculate the following probabilities for a round of the Rock, Paper, Scissors game where players are selecting throws at random.
a. P(Player 2 throws paper)
b. P(Player 2 wins)
c. P(Player 2 throws paper and wins)

Lesson 22-3

Use the following situation to answer Items 5–9.

Three bags each contain four balls. Bag 1 has three red balls and one blue ball. Bag 2 has four red balls. Bag 3 has two red balls and two blue balls.

A probability experiment consists of selecting a bag at random and then selecting a ball at random from that bag, and noting its color.

5. One possible outcome for this probability experiment is (Bag 1, Red).
Draw a tree diagram that shows all of the possible outcomes for this probability experiment.

6. How many different outcomes are there?

7. Put appropriate probabilities on the branches of the tree diagram. Then for each outcome, multiply across the branches to obtain the probability of the outcome.

8. Are the outcomes equally likely? If not, give an example of two outcomes that do not have the same probability.

9. Calculate the following probabilities.
a. P(ball is from Bag 1 and is red)
b. P(ball is from Bag 2 and is blue)
c. P(ball is from Bag 1)
d. P(ball is red)
e. P(ball is blue)

Lesson 22-4

Use the information below to answer Items 10 and 11.

A game has three boxes. Box 1 has one drawer, Box 2 has two drawers, and Box 3 has three drawers.

You will pick a box at random and then pick a drawer from that box at random.

10. A prize is hidden in the top drawer of Box 3. What is the probability that you select this drawer?

11. A prize is hidden in the only drawer of Box 1. What is the probability that you select this drawer?

12. Make up a box game like the ones in this activity that has two boxes and eight possible outcomes. How many drawers are in each box?

13. Consider a box game with two boxes. Box 1 has one drawer and Box 2 has 5 drawers. A player will select a box at random and then pick a drawer from that box at random. You have two prizes to place in the drawers. Where should you place them to create the greatest probability that someone who plays the game will win a prize? Explain your reasoning.

14. Suppose you perform a probability experiment in which you first toss a fair coin and then roll a fair number cube with the faces labeled with the numbers 1 through 6.

a. Draw a tree diagram to show all of the possible outcomes for this experiment. How many outcomes are in the sample space?

b. Calculate the probability that the coin will show heads and the number cube will show a three.

c. Calculate the probability that the coin will show tails and the number cube will show an odd number.

d. Are all the outcomes equally likely? Explain.

15. Consider a two-player game called Otto-Mamu based on the tree diagram you created with the following rules:

- An Otto occurs whenever (heads and an odd number) or (tails and an even number) results.
- A Mamu occurs for all other throws.

a. Is the Otto-Mamu game a fair game? Explain your reasoning.

b. If it is not a fair game, what strategy should a player choose to gain the winning advantage?

MATHEMATICAL PRACTICES
Construct Viable Arguments and Critique the Reasoning of Others

16. When is knowing how many outcomes are in the sample space enough to tell you what probability should be assigned to each outcome?

Probability

Estimating Probabilities Using Simulation
Lesson 23-1 What is Simulation?

My Notes

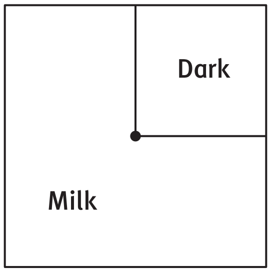
Learning Targets:

- Use artificial processes to simulate outcomes.
- Assign random digits to outcomes.
- Carry out a simulation using random digits.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

In Activity 20, you learned that it is possible to estimate probabilities by observing many outcomes of a probability experiment. In Activity 21, you learned that it is sometimes possible to generate outcomes by using an artificial process (like spinning a spinner) in place of actually carrying out the probability experiment.

As an example, the spinner below was used as a substitute for selecting a candy at random from a bag of 40 candies in which 30 were milk chocolate and 10 were dark chocolate.



In this activity, you will see how ***random digits*** can be used as a substitute for spinners to observe the outcome of a probability experiment.

The Random Digit Table on page 317 shows 30 rows of random digits. Each row has 25 random digits organized in groups of five. Carefully tear out that page so that you can use it as you answer the items in this activity.

We will start with a well known probability experiment—tossing a fair coin. It is easier to develop some important ideas in a simple setting. Then we will move on to more complex experiments.

1. What are the two possible outcomes that make up the sample space when you toss a coin?

MATH TERMS

Random digits are digits (0, 1, 2, . . . , 8, 9) arranged in a random order.

Because the order is random, it is not possible to predict what the next digit will be from knowing the digits that come before it.

You can think of a list of random digits as having been generated by spinning a spinner with ten equal sections labeled 0 through 9.

My Notes

2. Toss a penny 10 times and record the outcomes in the table below. Then estimate the probability of getting heads using the fraction of the 10 tosses that resulted in heads.

Toss	1	2	3	4	5	6	7	8	9	10
Outcome										

Estimated probability of tossing heads:

3. What is the theoretical probability of getting heads when a fair coin is tossed? How does the estimated probability from Item 2 compare to the theoretical probability?

4. Combine your toss results with those of four other students so that you know how many heads were observed in a total of 50 tosses. Use these 50 tosses to estimate the probability of heads. Is this estimated probability closer to the theoretical probability of heads than your estimate from Item 2?

ACADEMIC VOCABULARY

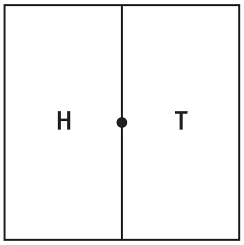
A ***simulation*** is when an artificial process for generating an outcome is substituted for a probability experiment.

It is important that the artificial process generates outcomes that are like outcomes from the actual experiment.

The estimated probability in Item 4 was based on actually carrying out the probability experiment of tossing a coin many times. Instead of actually tossing a coin, we could substitute an artificial process that behaves in a similar way and use the artificial process to generate outcomes. This is called carrying out a ***simulation.***

My Notes

5. A simple artificial process that could be substituted for tossing a fair coin is spinning a spinner like the one shown below. Explain why using this spinner to generate outcomes is like tossing a fair coin.



Even though spinning this spinner could be substituted for tossing a fair coin, it is not an improvement over actually tossing the coin. In fact, it may even take longer to spin the spinner and observe an outcome than it would take to toss a coin! Instead, look at how random digits can be used to simulate the process.

Example A

Random integers can be chosen using a table of values or by using a calculator.
Suppose we use a random digit to represent the outcome of a coin toss with even digits (0, 2, 4, 6, and 8) representing heads and odd digits (1, 3, 5, 7, and 9) representing tails.

A sequence of 10 random digits written in row 1 of the table below could be interpreted as ten tosses.

Digit	7	1	1	3	6	4	6	8	8	9
Outcome	T	T	T	T	H	H	H	H	H	T

Try These A

a. Explain why using even digits to represent heads and odd digits to represent tails produces outcomes that are like tossing a fair coin.

b. Should every group of ten random digits produce five heads and five tails? Explain.

TECHNOLOGY TIP

For this example, use a graphing calculator to simulate random number generation. Choose the probability simulation and enter the starting number, the ending number, and how many numbers you want.

My Notes

6. Can we use random digits to represent the outcome of a fair coin toss if we use 0, 1 and 2 to represent heads and 3, 4, 5, 6, 7, 8, and 9 to represent tails? Explain why or why not.

7. Make sense of problems. What assignment of digits different than the one already given would produce results similar to tossing a fair coin? Explain your reasoning.

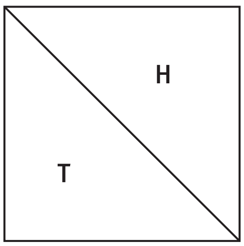
8. Use the digits you chose in Item 7 to determine heads and tails for the digits in the chart. Did you find the same number of heads as tails? Explain how this could occur.

Digit	7	1	1	3	6	4	6	8	8	9
Outcome										

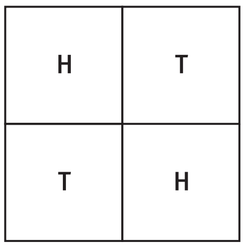
Check Your Understanding

9. Which of the following spinners could be substituted for tossing a fair coin?

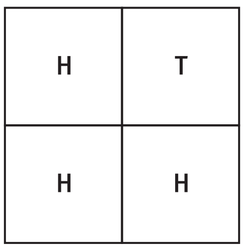
Spinner 1



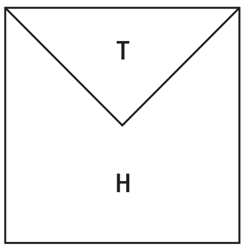
Spinner 2



Spinner 3



Spinner 4



My Notes

10. Suppose you wanted to simulate tossing a fair coin by selecting a chip from a bag containing blue chips and red chips. A blue chip represents heads and a red chip represents tails. How many of each color should you put in the bag to make a total of 30 chips?

11. Suppose you plan to use a random digit to represent a toss of a fair coin. Consider the following assignment of digits:

- 1, 2 and 3 represent heads
- 4, 5 and 6 represent tails
- Ignore 0, 7, 8 and 9. If one of these digits occurs, just skip it and move on to the next digit.

Will this assignment of digits produce outcomes (heads or tails) that are like tossing a fair coin? Explain your reasoning.

LESSON 23-1 PRACTICE

Suppose you simulate rolling a fair number cube using the Random Digit Table. Let 1, 2, 3, 4, 5, and 6 represent the numbers on the number cube and ignore the digits 0, 7, 8, and 9.

12. Fill in the chart with numbers that are rolled on the number cube according to the assignment of digits in the instructions above.

Digit	7	1	1	3	6	4	6	8	8	9
Outcome										

Digit	9	2	2	2	7	4	9	2	1	0
Outcome										

13. On a fair number cube, what is the probability of rolling a 5?

14. **Reason quantitatively.** Give the estimated probabilities for rolling each number on a number cube as simulated in Item 12.

$P(1) =$ $P(2) =$ $P(3) =$

$P(4) =$ $P(5) =$ $P(6) =$

15. Do the answers for $P(5)$ in Items 13 and 14 agree? Explain.

16. Write out a different assignment of digits to represent rolling a fair number cube. Then fill in the chart below with the results. Calculate $P(5)$ for this simulation. Are you surprised by the result? Explain.

Digit	7	1	1	3	6	4	6	8	8	9
Outcome										

Digit	9	2	2	2	7	4	9	2	1	0
Outcome										

My Notes

Learning Targets:

- Design and carry out a simulation.
- Use a simulation to estimate a probability.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

You can use a group of five random digits to represent five tosses of a fair coin.

Your teacher will assign you a row in the Random Digit Table on page 317.

1. Copy your row from the Random Digit Table into the table below in groups of five digits. Then, translate each group into outcomes (such as HHTHT). Use even digits to represent heads and odd digits to represent tails.

Random Digits					
Translated Outcomes					

2. For each group of five digits, determine if the event *E* occurred, where *E* = event that two or more heads occur in five tosses. For example, if the five tosses were TTHTH, there are two heads, and the event *E* occurred.

Digit Group	**1**	**2**	**3**	**4**	**5**
Did *E* Occur? (Yes or No)					

Add your results to a class poster that your teacher will provide by adding five tally marks. For example, if the event *E* occurred for two of your groups and did not occur for three of your groups, add two tally marks for *yes* and three tally marks for *no*.

My Notes

3. Use the results from the entire class to estimate the probability of getting two or more heads in five tosses of a fair coin. This is an estimated probability based on a simulation.

4. **Construct viable arguments.** Explain how you would use simulation to estimate the probability of getting four or more heads when a fair coin is tossed six times.

MATH TIP

When you are asked to explain a simulation, you do not have to carry out the simulation—just describe the steps as you would carry them out.

Discuss your answer to Item 4 with a partner. Then answer Items 5–7 based on the simulation described in that item.

5. How many digits do you need to represent an outcome for the probability experiment?

6. Did you use even digits to represent heads and odd digits to represent tails? Why is this an appropriate way to assign digits to heads and tails?

7. Suppose you used the simulation steps you described in Item 4 to simulate 50 outcomes. If 19 of these simulated outcomes included four or more heads, what is the estimated probability of getting four or more heads in six tosses of a fair coin?

My Notes

Now think about a probability experiment that consists of selecting a candy at random from a bag that contains 40% milk chocolate candies and 60% dark chocolate candies.

8. If you are interested in observing whether the selected candy is milk chocolate or dark chocolate, which of the following would be an appropriate way to use random digits to represent this probability experiment? Explain your choice.

Choice 1: Use a random digit to represent a selected candy, with even digits representing milk chocolate and odd digits representing dark chocolate.

Choice 2: Use a random digit to represent a selected candy, with 0, 1, 2 and 3 representing milk chocolate and 4, 5, 6, 7, 8 and 9 representing dark chocolate.

9. Explain how you would use random digits to represent selecting a candy at random from a bag that contained 70% milk chocolate candies and 30% dark chocolate candies.

10. **Reason abstractly and quantitatively.** In Activity 21, you saw a probability experiment for selecting a candy at random from a bag that contained 75% milk chocolate candies and 25% dark chocolate candies. Explain why it is not possible to use a single random digit to represent selecting a candy at random from this bag.

11. Can you think of a way to use a *pair* of random digits (such as 27, 34, and so on) to represent selecting a candy at random from a bag that contains 75% milk chocolate candies and 25% dark chocolate candies? Explain how you would do this.

My Notes

Check Your Understanding

12. A probability experiment consists of selecting a chip at random from a paper bag that contains two red chips and eight green chips. A random digit will be used to represent the selected chip. How would you assign digits to the two possible outcomes?

Outcome	Digits Assigned
Red	
Green	

13. A probability experiment consists of selecting a chip at random from a paper bag that contains 45 red chips and 55 green chips. For each of the following, explain whether the process described would be a correct way to use random digits to simulate this experiment.

a. Use one random digit to represent a selection, with 0, 1, 2, and 3 representing a red chip and 4, 5, 6, 7, 8 and 9 representing a green chip.

b. Use one random digit to represent a selection, with even digits representing a red chip and odd digits representing a green chip.

c. Use two random digits to represent a selection, with even numbers (like 34) representing a red chip and odd numbers (like 35) representing a green chip.

d. Use two random digits to represent a selection, with 00, 01, 02, . . ., 44 representing selecting a red chip and 45, 46, . . ., 99 representing selecting a green chip.

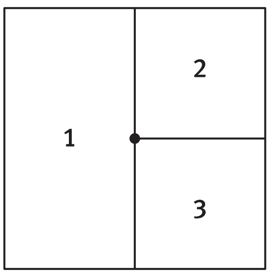
LESSON 23-2 PRACTICE

14. **Make sense of problems.** The groupings of four numbers represent the results of four rolls of a standard cube numbered 1 through 6. Event *S* represents two numbers that are the same. Based on the simulation, what is the estimated probability for Event *S* to occur?

2235 6225 2526 3523 2211 6214 4463
3314 1366 4345

5525 6316 4264 3165 3516 4153 3366
3262 3664 1526

15. To use random digits to simulate the results from a spinner, what assignment of digits could be made to accurately model the spinner below?



16. A bag contains 10% blue marbles, 40% red marbles and 50% green marbles. The probability experiment involves drawing one marble from the bag, returning the marble to the bag, and then shaking the bag before selecting the next marble. What assignment of digits can accurately simulate probability in this experiment?

17. Groups of four random digits (0 to 9) are listed below. In a two-player game, one player wins when the sum of the four digits is greater than 18, and the other player wins when the sum of the digits is less than 18. Is this a fair game? Explain your reasoning.

5963 2434 6122 6196 3352 6920
5957 2725 5967 9478

7839 0335 3132 5163 7703 1942
7347 3572 7472 2338

18. A bag contains 100 chips with 20 blue chips and 80 red chips. Describe an assignment of random digits that can be used to simulate drawing one chip from the bag.

My Notes

Learning Targets:

- Design and carry out the simulation of a compound event.
- Use a simulation to estimate the probability of a compound event.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

Now we would like to introduce you to Sophie—a young dog who loves to play Frisbee. Sophie will be helping with the rest of this activity.

Even though Sophie loves to catch a Frisbee, she isn't very good at actually catching the Frisbee! In the long run, she catches the Frisbee only 10% of the time.

1. If you throw the Frisbee 20 times, about how many of the throws do you think Sophie will catch?

2. You can use a random digit to represent a Frisbee throw. Describe how you would assign the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 to the two possible outcomes—Success (Sophie catches the Frisbee) and Failure (Sophie doesn't catch the Frisbee).

3. Use your answer to Item 2 to translate the following sequence of 20 random digits into an outcome for the probability experiment that consists of throwing the Frisbee for Sophie 20 times.

8 5 9 4 5 9 9 6 1 5 4 5 8 4 6 1 2 7 0 6

4. How many of the 20 throws in your answer to Item 3 were successes?

5. What is the probability that Sophia will be successful based on the results of the simulation in item 3?

So far, we only have one simulated outcome. To estimate the probability of the event F, we would need many more simulated outcomes.

6. If available, use technology to complete this question, or follow the steps below:
- Toss a paper clip onto the page of random digits on page 317.
- The digit on the page that is nearest to the center of the larger loop of the paper clip is the first random number. Write the digit in row 1, Group 1, in the table below.
- Repeat until you have picked and recorded 20 random numbers for row 1, Group 1.
- Continue the activity until you have picked 20 random numbers for groups 2–5.
- Translate each group of 20 random digits into an outcome by indicating which throws were successes and which throws were failures. You will have five groups, each of which has 20 random digits and the results of 20 frisbee throws.

Group 1	
Results	
Group 2	
Results	
Group 3	
Results	
Group 4	
Results	
Group 5	
Results	

My Notes

CONNECT TO LANGUAGE ARTS

A ***compound sentence*** consists of two or more independent clauses. In a similar way, a compound event consists of two or more simple events.

TECHNOLOGY TIP

The random number generator on a graphing calculator can be "seeded," which means that every calculator would generate different numbers. For example, you might use your birth date as the starting point (such as January 3, or 0103).

My Notes

7. For each of the five groups in Item 6, determine if the event *T* (two or more catches) occurred or not. Write yes or no.

Group 1: ______________

Group 2: ______________

Group 3: ______________

Group 4: ______________

Group 5: ______________

8. For each of the five outcomes in Item 7, determine if a new event *F* (four or more catches) occurred or not.

Add your results to a class poster that your teacher will provide by adding five tally marks. For example, if the event *S* occurred for two of your groups of 20 throws and did not occur for three of your groups, add two tally marks for *yes* and three tally marks for *no*.

9. Use the results from the entire class to estimate the probability that Sophie catches four or more of the 20 Frisbee throws.

Check Your Understanding

10. Suppose that in the long run Sophie could catch a Frisbee 30% of the time instead of 10% of the time. How would you assign digits to the outcomes *success* and *failure* in this situation?

11. Below is a sequence of 10 random digits. Use your assignment of digits from Item 10 to translate this into an outcome for the probability experiment of throwing the Frisbee for Sophie 10 times.

7 6 2 8 4 6 5 0 7 8

12. For the simulated outcome in Item11, how many times out of 10 throws did Sophie catch the Frisbee?

My Notes

LESSON 23-3 PRACTICE

13. Ameda is an 80% free-throw shooter.

a. In preparing to run a simulation, which digits should be assigned to Ameda for making a free throw?

b. Predict how many times Ameda will make at least eight free throws out of 10 in 100 free throws.

c. Using Row 12 of the Random Digits Chart, select 10 digits at a time for 10 events. In how many of these ten simulated events will Ameda make at least eight free throws?

d. What percent of the time did the simulation indicate that Ameda will make at least eight free throws? Was your prediction in part b correct? Explain.

14. Sidney hits the mark 60% of the time when shooting at targets with his bow and arrow.

a. When creating a simulation of his shooting accuracy, which digits should be assigned to Sidney for hitting the target?

b. Predict what percent of the time Sidney will miss the target two times or fewer in five shots, if ten groups of five shots each are taken.

c. Using Row 2 of the Random Digits Chart, select five digits at a time to simulate ten events of five shots each. How many times does the event *missing two or fewer targets* occur?

d. What percent of the time will Sidney miss the target two times or fewer according to the simulation? Was your prediction in part b correct? Explain.

15. Reason abstractly. What can be done to increase the accuracy of the results for the events in Items 13 and 14?

16. A machine manufactures good light bulbs with 98% accuracy. How should random digits be assigned to indicate whether the light bulb is good or bad?

My Notes

Learning Targets:

- Design and carry out the simulation of a compound event.
- Use a simulation to estimate the probability of a compound event.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Predict and Confirm, Create Representations, Look for a Pattern, Summarizing, Paraphrasing, Interactive Word Wall

Kirby is a dog that is more talented than Sophie when it comes to catching a Frisbee. Kirby catches the Frisbee 70% of the time.

1. How would you change the assignment of random digits for the outcomes *success* (catches a Frisbee) and *failure* (does not catch a Frisbee) if Kirby is playing instead of Sophie? Be specific about the digits to represent success and the digits to represent failure.

2. Use your assignment of random digits from Item 1 and the 100 random digits shown below to simulate 50 outcomes for the probability experiment that involves throwing the Frisbee for Kirby twice. Write *S* or *F* above each digit.

4 0	3 5	2 3	5 9	7 2	1 8	0 7	2 0	6 4	4 3
1 4	1 8	6 2	4 6	9 2	2 1	0 3	0 4	9 8	0 8
8 2	4 4	3 7	6 4	8 8	1 1	8 6	2 8	2 2	5 9
4 1	2 4	6 8	7 9	3 7	6 9	6 8	1 5	9 9	4 5
1 1	3 6	1 3	9 4	6 1	6 2	2 7	9 7	9 4	8 8

CONNECT TO COMPUTER SCIENCE

Computer simulations are often used for experiments with a continuum of possible outcomes.

My Notes

Use the 50 simulated outcomes to estimate the following probabilities.

3. *P*(Kirby catches the Frisbee on both throws)

4. *P*(Kirby catches the Frisbee on exactly one of the two throws)

5. *P*(Kirby misses on both throws)

6. *P*(Kirby does not miss on both throws)

7. What do you notice about the sum of the probabilities from Items 5 and 6? Explain why the sum has this value.

Consider throwing the Frisbee to Kirby until he catches it. We want to find the number of throws needed. For example, one possible outcome is that Kirby catches the first throw and then we stop. Another possibility is that Kirby misses the first throw and then catches the second throw, which means that two throws are needed.

We can carry out a simulation of this probability experiment using the following steps:

- Use a random digit to represent each throw.
- The digits 0, 1, 2, 3, 4, 5 and 6 represent a throw that Kirby catches.
- The digits 7, 8 and 9 represent a throw that Kirby misses.
- Use a sequence of random digits to represent an outcome. Start with one digit. If it is a digit that represents a catch, stop and record an outcome of one throw. If the digit is 7, 8 or 9, Kirby did not make the catch and so we “throw” again by looking at the next random digit. We continue until Kirby makes a catch and record the number of throws needed.

For example, using the list of random digits below, we could mark off outcomes using vertical bars as shown.

7 3 | 1 | 6 | 9 4 | 8 6 |

The first outcome used two throws. The second outcome used one throw. The third outcome used one throw. The fourth and fifth outcomes each used two throws.

My Notes

Take a minute to make sure you understand how the sequence of random digits was divided up into outcomes and that the number of throws associated with each outcome is correct.

This gives five simulated outcomes. We have added these outcomes to the tally count shown here.

Number of Throws	Tally Count
1	||
2	|||
3	
4	
5	
6	
7	
8	

TECHNOLOGY TIP

To generate random numbers, enter the following:

1234
math
prob
rand
enter

8. If available, use a graphing calculator to generate 45 more outcomes. Otherwise, mark the list of random digits below to show 45 more outcomes. Then add the appropriate tally marks to the tally count above. You should have a total of 50 tally marks when you are done.

4 0 3 5 2 3 5 9 7 2 1 8 0 7 2 0 6 4 4 3
1 4 1 8 6 2 4 6 9 2 2 1 0 3 0 4 9 8 0 8
8 2 4 4 3 7 6 4 8 8 1 1 8 6 2 8 2 2 5 0

Use the 50 simulated outcomes to estimate the following probabilities.

9. *P*(more than 2 throws are needed)

10. *P*(Only 1 or 2 throws are needed)

11. *P*(Kirby makes a catch on the first throw)

12. **Reason quantitatively.** Is the estimated probability in Item 11 close to 0.70? Explain why this makes sense.

My Notes

13. Explain why it makes sense that the estimated probability in Item 9 is small.

14. **Make sense of problems.** How would you modify the simulation steps (listed after Item 7) if the probability experiment consisted of throwing the Frisbee for Kirby until he makes his second successful catch?

Check Your Understanding

Tracker is a great Frisbee catcher. He makes a catch 90% of the time! Representing a Frisbee throw with a random digit, you decide to use 1, 2, 3, 4, 5, 6, 7, 8 and 9 to represent a success (a catch) and 0 to represent a failure (a miss).

15. Use the random digits below to estimate the probability that Tracker will catch four out of four Frisbee throws.

7710	6867	2696	3995
6243	2793	1007	6597
8536	7831	9432	4241
5157	9197	8914	7169
3587	1083	5976	2246

Number of simulated outcomes:

Estimated probability:

16. Remember Kirby? In the long run, Kirby catches 70% of Frisbee throws. If you were to estimate the probability that Kirby would catch four out of four throws, do you think this probability would be much greater than, about the same as, or much less than the probability you computed for Tracker in Item 9? Explain your reasoning.

17. Use the random digits below with 0, 1, 2, 3, 4, 5 and 6 representing a catch to estimate the probability that Kirby can catch four out of four Frisbee throws.

7710	6867	2696	3995
6243	2793	1007	6597
8536	7831	9432	4241
5157	9197	8914	7169
3587	1083	5976	2246

Number of simulated outcomes:

Estimated probability:

18. How does the estimated probability for Kirby in Item 17 compare to the estimated probability for Tracker in Item 15? Was your answer in Item 16 correct?

My Notes

LESSON 23-4 PRACTICE

Write your answers on notebook paper. Show your work.

Use the information below to answer Items 19–22.

A six-sided number cube contains the numbers 1 through 6. You decide to simulate rolling this number cube using a random digit to represent one roll. The digits 0, 7, 8 and 9 are ignored and the other digits represent the corresponding numbers on the number cube. For example, the sequence of random digits 5, 9, 7, 6, and 4 correspond to three rolls with outcomes 5, 6 and 4 (the 9 and 7 are ignored).

Use the following sequence of random digits to carry out a simulation and then answer Items 19–21.

1	8	3	8	4	8	0	5	0	4
0	0	7	6	6	8	1	1	9	5
8	9	9	4	9	2	5	9	0	4
8	1	4	2	9	7	4	3	2	7
4	4	8	3	1	1	8	5	8	5
1	0	1	8	7	0	3	1	9	9

19. How many rolls of the number cube are represented?

20. How many of the simulated rolls resulted in an even number?

21. Based on this simulation, what is the estimated probability that a number greater than 4 will be rolled?

22. Suppose you plan to roll two number cubes and are interested in the sum of the two numbers rolled. You can simulate an outcome using two random digits—each digit represents an outcome for one of the cubes.

- Using the same assignment of digits described above (ignoring 0, 7, 8 and 9), use the following sequence of random digits to obtain 17 simulated outcomes.
- Then use the simulated outcomes to estimate the probability that the sum will be less than or equal to 5.

1	6	7	1	5	1	2	2	6	4
5	4	5	7	3	6	7	9	0	8
0	1	8	8	1	6	6	8	8	0
3	9	0	3	6	8	0	9	0	7
9	6	6	0	2	8	6	7	2	4
4	1	4	2	4	1	8	4	5	7

My Notes

Use the information below to answer Items 23–26.

A probability experiment consists of selecting a chip at random from a bag that contains 60% green chips and 40% yellow chips.

23. **Construct viable arguments.** Explain how you would use a random digit to represent an outcome for this probability experiment.

24. Suppose the probability experiment is performed three times. Use your assignment of digits from Item 23 to translate the three digits below into outcomes for three repetitions of this experiment.

7 2 6

25. Were all three selections in Item 24 green chips?

26. Use the random digits below to estimate the probability of getting three green chips three times, when a chip is selected at random from a bag that has 60% green chips.

5	9	0	4	3	8	3	9	1
4	3	2	7	9	4	5	1	9
8	5	8	5	3	1	9	5	0

Number of simulated outcomes:

Estimated probability:

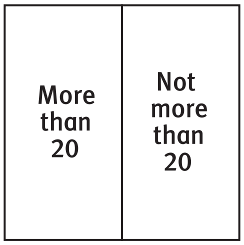
ACTIVITY 23 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 23-1

1. Thirty percent of the employees at a large company live more than 20 miles away from where they work. One employee of this company will be selected at random and asked if he or she lives more than 20 miles from work. For each of the following artificial processes, state whether or not it could be used to simulate an outcome.

Process 1: Spin the spinner below to generate an outcome.



Process 2: Use a random digit to represent an outcome, with even digits representing a person who lives more than 20 miles away and odd digits representing a person who does not live more than 20 miles away.

Process 3: Use a random digit to represent an outcome, with 0, 1, 2, 3, 4, 5 and 6 representing a person who lives more than 20 miles away and 7, 8 and 9 representing a person who does not live more than 20 miles away.

Process 4: Use a random digit to represent an outcome, with 7, 8 and 9 representing a person who lives more than 20 miles away and 0, 1, 2, 3, 4, 5 and 6 representing a person who does not live more than 20 miles away.

2. For each process described in Item 1 that you said could not be used, explain why you said this.

Lesson 23-2

3. Suppose that 60% of the teachers in a large school district plan to travel over the winter break. A probability experiment consists of selecting a teacher from this district at random and determining if that teacher plans to travel over winter break. If you were going to simulate this probability experiment using a random digit table, how would you assign digits to the two outcomes *plans to travel* and *does not plan to travel*?

4. Suppose that 30% of the adult residents of a large city donate money to a charity during the month of December. You wonder how likely it is that more than half of a group of 10 randomly selected residents would have donated money to a charity during the previous December. You decide to carry out a simulation to estimate this probability. Using a random digit to represent one randomly selected resident, you decide to use 1, 2 and 3 to represent a person who donated and 0, 4, 5, 6, 7, 8 and 9 to represent a person who did not donate. A group of 10 random digits will represent the 10 randomly selected residents.

a. One group of 10 random digits is shown below. Use *D* to stand for *donated* and *N* to stand for *did not donate* to translate these digits into a simulated outcome.

1 2 8 5 1 8 0 2 6 8

b. How many of the 10 in this simulated outcome donated? Did more than half donate?

Random Digit Table

Row 1	7 1 1 3 6	4 6 8 8 9	1 9 0 5 4	1 6 7 1 5	1 2 2 6 4
Row 2	6 8 5 7 5	8 8 4 4 9	0 5 6 1 7	5 4 5 7 3	6 7 9 0 8
Row 3	9 3 0 9 8	7 0 5 8 7	5 4 2 3 2	0 1 8 8 1	6 6 8 8 0
Row 4	9 6 7 7 9	6 9 3 1 3	1 9 1 6 8	3 9 0 3 6	8 0 9 0 7
Row 5	1 7 5 4 0	5 9 9 8 2	1 9 7 8 3	9 6 6 0 2	8 6 7 2 4
Row 6	5 5 0 9 4	2 0 6 7 3	6 8 3 3 6	4 1 4 2 4	1 8 4 5 7
Row 7	1 4 5 3 6	1 1 4 1 4	3 9 6 1 1	6 3 0 4 4	6 4 8 8 5
Row 8	1 8 3 8 4	8 0 5 0 4	9 3 8 7 7	7 2 6 3 8	5 4 6 5 2
Row 9	0 0 7 6 6	8 1 1 9 5	6 8 4 1 4	1 3 2 4 2	1 7 3 5 9
Row 10	8 9 9 4 9	2 5 9 0 4	3 8 3 9 1	1 8 0 8 8	2 0 2 9 5
Row 11	8 1 4 2 9	7 4 3 2 7	9 4 5 1 9	9 7 0 7 4	5 7 2 4 3
Row 12	4 4 8 3 1	1 8 5 8 5	3 1 9 5 0	4 4 8 7 9	1 2 9 6 7
Row 13	1 0 1 8 7	0 3 1 9 9	5 5 2 7 5	4 5 3 6 2	6 8 8 8 2
Row 14	2 1 2 5 1	8 1 4 3 0	1 0 8 8 3	2 5 5 3 8	3 5 4 9 1
Row 15	6 4 3 1 0	6 4 3 2 2	7 9 9 7 9	4 1 3 7 6	6 3 2 3 7
Row 16	0 1 0 5 3	0 0 2 1 3	1 1 1 9 8	1 8 4 0 7	7 8 9 7 0
Row 17	8 8 3 6 4	8 5 9 4 5	9 9 6 1 5	4 5 8 4 6	1 2 7 0 6
Row 18	1 0 7 7 7	9 5 8 3 5	8 7 3 5 3	6 4 6 4 9	1 3 2 6 9
Row 19	4 6 4 7 6	7 8 5 7 9	3 2 1 0 3	1 2 8 5 1	8 0 2 6 8
Row 20	9 4 0 3 5	9 4 9 9 6	5 5 9 2 2	9 5 1 1 5	1 2 1 4 7
Row 21	8 8 4 2 7	5 1 9 1 1	1 4 0 9 8	3 1 6 4 0	6 3 0 0 3
Row 22	7 2 6 3 1	3 2 4 2 3	2 2 6 9 9	9 2 1 4 5	8 3 6 7 6
Row 23	7 6 3 8 4	6 5 0 7 8	7 4 1 0 9	3 6 9 5 3	6 1 0 9 6
Row 24	9 9 3 2 4	3 0 4 8 8	0 4 7 1 4	5 3 9 0 4	2 4 9 1 8
Row 25	9 7 9 9 2	6 4 8 4 0	6 0 6 1 5	6 9 9 2 2	3 7 4 0 3
Row 26	7 7 1 0 6	6 8 6 7 2	2 6 9 6 8	3 9 9 5 2	8 8 2 3 8
Row 27	6 2 4 3 2	2 7 9 3 5	1 0 0 7 7	6 5 9 7 2	2 8 8 7 1
Row 28	8 5 3 6 4	7 8 3 1 2	9 4 3 2 7	4 2 4 1 2	8 4 1 9 7
Row 29	5 1 5 7 8	9 1 9 7 5	8 9 1 4 3	7 1 6 9 4	3 6 4 5 5
Row 30	3 5 8 7 0	1 0 8 3 3	5 9 7 6 4	2 2 4 6 0	8 7 1 3 4

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c. Below are 20 groups of 10 random digits. Translate each into a simulated outcome and indicate how many donated for each group of 10.

7960323437	2765708474
0097926919	5778989670
9568551023	5423785394
1554531421	9458264639
3147891746	0648225841
0488453882	3187471549
0654288233	7436697726
0877147658	0869477704
3612939234	4296869289
4819503479	6594533331

d. What is the estimated probability for more than half of the group to donate?

Lesson 23-3

5. Explain how you might select a person at random from the students in your math class.

6. A brown paper bag contains 60 plastic chips. Each chip has a different number, and the chips are numbered from 1 to 60. Suppose that a chip is selected at random from this bag.
- **a.** What is the probability that the selected number is divisible by 5?
- **b.** Give an example of an event that has probability $\frac{1}{2}$.
- **c.** Give an example of an event that has probability $\frac{1}{3}$.

7. A box contains many colored balls. Some of the balls are yellow. Describe what you would do in order to calculate an *estimated probability* of selecting a yellow ball if a ball is selected at random from this box.

Lesson 23-4

8. Recall the probability experiment of selecting a student at random from Ms. Bailey's seventh grade class. Some events are defined below.

Event	Description
M	Selected student is male
F	Selected student is female
R	Selected student chose read minds as super power
S	Selected student sent more than 60 texts

Use the survey data on page 271 to find the following probabilities.
- **a.** $P(M) =$
- **b.** $P(R) =$
- **c.** $P(R') =$
- **d.** $P(S') =$
- **e.** $P(M \text{ and } R) =$
- **f.** $P(F \text{ and } R) =$
- **g.** $P(F \text{ and } S) =$
- **h.** $P(M \text{ and } S) =$

9. State the events represented by each complement.
- **a.** R'
- **b.** S'

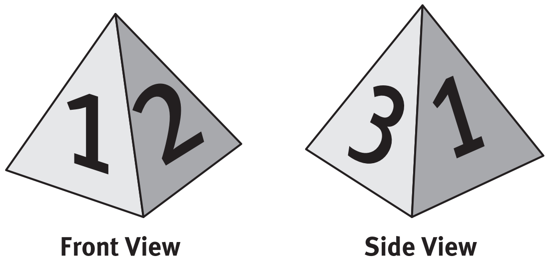
MATHEMATICAL PRACTICES
Use Appropriate Tools Strategically

10. A probability experiment consists of tossing five fair coins. You want to estimate the probability that at least four of the coins land heads up. What would be the advantage of using simulation to estimate this probability rather than estimating the probability by observing many outcomes of the actual experiment?

Write your answers on notebook paper or grid paper. Show your work.

Use the information below to answer Items 1–4.

A probability experiment consists of flipping a fair coin and rolling a four-sided number pyramid that has the numbers 1, 2, 3 and 4 written on its sides. The outcome of a roll of a four-sided number pyramid is the number on the face that lands on the bottom. For example, for the four-sided number pyramid shown below (in two views, so that you can see the three sides that are not on the bottom), the outcome of the throw would be a 4.



One outcome for this probability experiment is (H, 1) which represents a head on the coin and a 1 on the number pyramid.

1. List all of the possible outcomes for this probability experiment. How many different possible outcomes are there?
2. Construct a table with two rows and four columns that shows the different possible outcomes.
3. Construct a tree diagram that shows the possible outcomes for this probability experiment.
4. Explain why the table and the tree diagram are both ways of representing the sample space for this experiment.

Use this information below to answer Items 5–9.

A middle school has four teachers who teach seventh-grade math. Ms. Roy and Mr. Daly each teach two sections of seventh-grade math, and Ms. Kotz and Mr. Teague each teach three sections of seventh-grade math. A probability experiment consists of selecting one of the four teachers at random and then selecting one of that teacher's sections of seventh-grade math at random. One possible outcome for this experiment is (Roy, First Section).

5. Construct a tree diagram that shows all of the outcomes for this probability experiment.
6. How many different outcomes are in the sample space of this probability experiment?
7. Add appropriate probabilities to the branches of your tree diagram.
8. Multiply across the appropriate branches to obtain a probability for each outcome.

9. Are the outcomes equally likely? If not, give an example of two outcomes that do not have the same probability.

10. A probability experiment consists of selecting a chip at random from a box that contains 40 chips. There are 30 red chips in the box and 10 green chips in the box. Allan plans to carry out a simulation of this probability experiment. He will use a random digit to represent a selection and if the digit is a 1, 2 or 3, the simulated outcome is red. If the random number is not 1, 2 or 3, the simulated outcome will be green. Is Allan's plan a good one? Explain why or why not.

Use the information below to answer Items 11–14.

Suppose that 30% of the students at Laguna Middle School participate in school-sponsored after-school activities. A student will be selected at random and asked whether or not he or she participates in after-school activities.

11. If you want to simulate the probability experiment of selecting a Laguna Middle School student at random and you are just interested in whether the selected student participates in after-school activities or not, which of the following would be a correct plan?

Plan 1: Use a random digit to represent a selection. If the selected digit is 1, 2 or 3, the simulated outcome will be a student who participates in after-school activities, and if the digit is 0, 4, 5, 6, 7, 8 or 9, the simulated outcome will be a student who does not participate in after-school activities.

Plan 2: Use a random digit to represent a selection. If the selected digit is even, the simulated outcome will be a student who participates in after-school activities, and if the digit is odd, the simulated outcome will be a student who does not participate in after-school activities.

12. Using the correct plan from Item 11, translate the random digits below into five simulated outcomes for this probability experiment. How many of the five simulated outcomes were students who participated in after-school activities?

4	1	3	7	6

13. Now consider the probability experiment of selecting a student at random five times and counting how many of the five students selected participate in after-school activities. Translate the random digits below into 10 simulated outcomes. (In other words, look at 10 sets of five selections and count the number of students who participate in after-school activities for each set of five. Use the correct plan from Item 11.)

Use the simulated outcomes to estimate the probability that three or more students out of five randomly selected participate in after-school activities.

9	6	7	7	9	6	9	3	1	3	1	9	1	6	8	3	9	0	3	6
1	7	5	4	0	5	9	9	8	2	1	9	7	8	3	9	6	6	0	2
5	5	0	9	4	2	0	6	7	3										

14. Now consider a probability experiment that consists of selecting students at random until you get a student who participates in after-school activities. Using the random digits below and the correct assignment of digits from Item 11, how many students would be selected to complete the *first* simulated outcome?

8	0	2	6	8	1	2	7	0	6

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14)	• Clear and accurate understanding of listing outcomes and probabilities of experiments, and of conducting simulations.	• Adequate understanding of listing outcomes and probabilities of experiments, and of conducting simulations.	• Difficulty with listing outcomes and probabilities of experiments, and with conducting simulations.	• Incorrect or incomplete understanding of outcomes and probabilities of experiments and of simulations.
Problem Solving (Items 7, 8, 12, 13, 14)	• Accurately and precisely interpreting an experiment or simulation.	• Interpreting an experiment or simulation.	• Difficulty interpreting an experiment or simulation.	• No understanding of interpreting an experiment or simulation.
Mathematical Modeling / Representations (Items 2, 3, 4, 5, 11, 12, 13, 14)	• Accurately using tables and trees to represent outcomes, and simulations to model experiments.	• Adequate use of tables and trees to represent outcomes, and simulations to model experiments.	• Difficulty using tables and trees to represent outcomes, and simulations to model experiments.	• Inaccurate or incomplete use of tables and trees to represent outcomes, and simulations to model experiments.
Reasoning and Communication (Items 4, 9, 10)	• Clear and accurate explanation of sample spaces and simulations.	• Adequate explanation of sample spaces and simulations.	• Difficulty explaining sample spaces and simulations.	• An inaccurate explanation of sample spaces and simulations.