# Numerical Relationship 

## Unit Overview

In this unit, you will extend your knowledge of numbers as you investigate patterns, study powers and roots, and exponents and scientific notation. You will apply your knowledge of numbers to practical situations and real-world problems.

## Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

## Academic Vocabulary <br> - refute

## Math Terms

- sequence
- conjecture
- absolute value
- reciprocal
- power
- base
- exponent
- exponential form
- square root
- perfect square
- cubing a number
- index
- cube root
- rational number
- terminating decimal
- repeating decimal
- irrational number
- scientific notation
- standard form


## ESSENTIAL QUESTIONS



Why is it important to understand procedures for working with different kinds of numbers?

How are exponents and scientific notation useful in solving problems?

## EMBEDDED ASSESSMENTS

These assessments, following activities 2,5 , and 8 will give you an opportunity to demonstrate your understanding of numbers and numerical relationships.

Embedded Assessment 1:
Patterns and Quantitative Reasoning

## Embedded Assessment 2:

Representing Rational and Irrational Numbers

Embedded Assessment 3:
Exponents and Scientific
Notation

## Getting Ready

## Write your answers on notebook paper.

Show your work.

1. Find the product and quotient of each pair of numbers.
a. 24.6 and 1.2
b. 1.95 and .25
2. Arrange the following numbers in increasing order.
2.3, 45.1, 18.735, 0.9862, 7
3. Give the next term or figure in the following patterns.
a. $2,8,14$
b. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}$
c.



d.

4. Create a visual representation of each of the following fractions.
a. $\frac{1}{8}$
b. $\frac{1}{4}$
c. $\frac{1}{2}$
d. 1
5. Simplify each of the following rational numbers.
a. $\frac{9}{243}$
b. $\frac{27}{243}$
c. $\frac{81}{243}$
6. For each number, place a check in the box of any set of which the number is a member.

| Number | Natural <br> Numbers | Whole <br> Numbers | Rational <br> Numbers | Integers |
| :---: | :---: | :---: | :---: | :---: |
| 0 |  |  |  |  |
| $\frac{2}{5}$ |  |  |  |  |
| 3.4 |  |  |  |  |
| 13 |  |  |  |  |
| -27 |  |  |  |  |

7. Copy the number line shown and plot the following numbers on your line.

a. 2.5
b. 8
c. $21 \frac{1}{3}$
8. Explain 2 ways you could find the product of $4 \times 4 \times 4 \times 4$.

## Investigating Patterns

## Laws and Orders <br> Lesson 1-1 Analyzing Sequences

## Learning Targets:

- Analyze simple sequences.
- Describe patterns in simple sequences and give the next terms in a sequence.

SUGGESTED LEARNING STRATEGIES: Think-Pair-Share, Look for a Pattern, Visualization, Discussion Groups, Role Play

For much of the late $19^{\text {th }}$ and early $20^{\text {th }}$ centuries, the fictional character Sherlock Holmes was known for his great detective work. In this activity, you will be asked to perform many tasks similar to those Holmes used to solve his cases.

In order to solve mysteries, Holmes used a deductive process that led him to a logical conclusion. First, he would observe a situation and gather as many facts as possible. Next, he would analyze each fact to determine its relevance to the situation. Then he would search for even more clues by considering the smallest of details. Finally, he would use his imagination to link all of the clues together in the most logical manner.

## The Case of the Arabic Symbols

At first glance, the following picture would appear to be a representation of the numbers one through nine. However, the way they are drawn gives a clue to how the symbols for each number were originally created.


1. Make sense of problems. Observe, analyze, and search for clues in the diagram to come up with a guess about why the numbers were first written this way.


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## My Notes

## MATH TERMS

A sequence is an ordered list of numbers or figures. Each number or figure of a sequence is called a term.

## MATH TERMS

A conjecture is a theory or opinion formed without proof.

3. Based on your group's conclusions, explain how this pattern could also be used to describe zero with the symbol 0 .

## The Case of the Multiple Viewpoints

The next case involves investigating the sequence shown below. In order to reconstruct the pattern and solve the mystery, several witnesses have been asked to describe the sequence.


Figure 1


Figure 2


Figure 3
4. The description provided by the first witness, Bob, is given in terms of percents. Bob has determined that Figure 1 represents 25\%, Figure 2 represents $50 \%$ and Figure 3 represents $75 \%$. Is he right? Explain.

ACTIVITY 1
continued
5. The second witness uses fractions to describe the sequence. Provide an example of what this description might be.
6. Construct viable arguments. Witnesses \#3, Julie, and \#4, Greg, provide different accounts of the pattern. They have the following conversation.

Julie: "Analyzing the sequence I noticed that it was increasing." Greg: "I disagree; I believe the sequence is decreasing."
Explain how both of their descriptions could be considered correct.
7. Analyze the descriptions of all four witnesses and draw Figure 4 if the sequence continued.

## Check Your Understanding

Analyze the sequence below to answer Items 8-9.


Figure 1


Figure 2


Figure 3
8. Draw the figure you think is next in the sequence.
9. Write a conjecture for the pattern of the sequence.

## LESSON 1-1 PRACTICE

Analyze the sequence below to answer Items 10-12.

10. Draw and name the figure you think is next in the sequence.
11. Write a description for the figure that you drew in Item 10.
12. Write a conjecture for the pattern of the sequence.

Analyze the sequence below to answer Items 13-14.

13. Draw the figure you think is next in the sequence.
14. Express regularity in repeated reasoning. Draw the eighth figure in the sequence.

## ACTIVITY 1

continuea

## Learning Targets:

- Analyze more complex sequences.
- Describe patterns in sequences and develop methods for predicting any term in a sequence.

SUGGESTED LEARNING STRATEGIES: Quickwrite, Self/Peer Revision, Create Representations, Discussion Groups, Graphic Organizer

Continue to investigate patterns as Sherlock Holmes would with the following case.

## The Case of the Revolving Figure

This next case involves the sequence of figures shown below.


1. Attend to precision. Observe and analyze the patterns in the sequence. Describe the sequence in as much detail as possible.
2. Explain your descriptions with your group members. List any details you may not have considered before. Make sure that you are presenting your details clearly and that you can support them with evidence. Listen to group members' details and determine if they are presenting them clearly and can support their details with evidence.



## My Notes

3. Use the evidence gathered in Items 1 and 2 to draw representations of the fourth and fifth figures in the sequence.
4. Answer the following based on your observations of the patterns in the sequence.
a. Describe the sequence for the number of line segments in each figure.
b. How many line segments would appear in Figure 16?
c. How many line segments would appear in Figure 49?
d. Explain how you could determine the number of line segments in any figure in the sequence.
5. Organize the evidence you gathered about line segments and continue to explore the pattern in the table below.

| Figure | Number of <br> Line <br> Segments | Number of <br> Squares | Sum of Line <br> Segments <br> and Squares |
| :---: | :---: | :---: | :---: |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 16 |  |  |  |
| 49 |  |  |  |

6. Reason quantitatively. Write a conjecture on how you could determine the number of squares and the sum of line segments and squares in any figure in the sequence.

## Check Your Understanding

Analyze the sequence below to answer Items 7-9.

7. Draw the figure you think is next in the sequence.
8. Write a conjecture for the pattern of the sequence.
9. Otis describes the pattern using the number of dots in each figure: 3, $7,11, \ldots$. How many dots would appear in the ninth figure?

## My Notes



## WRITING MATH

One way to describe a number pattern in a sequence is to list several terms in order, followed by ellipses (... ) to indicate that the pattern continues. For example, writing

$$
1,3,5,7, \ldots
$$

implies that the pattern of adding two to each digit continues indefinitely.


## LESSON 1-2 PRACTICE

Analyze the sequence below to answer Items 10-11.

10. Draw what you think are the next two figures in the sequence.
11. Write two different descriptions that could describe the pattern of the sequence.
Analyze the sequence below to answer Items 12-14.

12. Draw what you think are the next two figures in the sequence.
13. Write a conjecture of the pattern of the sequence.
14. Critique the reasoning of others. Felipe predicts that the 10th figure in the sequence will look like this:


Did Felipe draw the $10^{\text {th }}$ figure correctly? Explain your reasoning.

## Learning Targets:

- Understand increasing and decreasing sequences.
- Analyze sequences containing mathematical operations and those based on other patterns.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Think-PairShare, Sharing and Responding, Look for a Pattern, Group Presentation

An increasing sequence is a sequence of numbers where the value of the numbers is increasing, and a decreasing sequence is one where the value of the numbers is decreasing.

1. Express regularity in repeated reasoning. Examine the following sequences and state whether they are increasing or decreasing. Support your answer by describing the pattern.
a. $3,6,12,24, \ldots$
b. $17,14,11,8, \ldots$
2. Provide an example of an increasing and a decreasing sequence. Describe the pattern in the sequence.

## CONNECT TO SCIENCE

One well-known increasing sequence is the Fibonacci numbers. The sequence is represented as:

$$
0,1,1,2,3,5,8, \ldots .
$$

Each number in the sequence is the sum of the two previous numbers. Applications of this sequence occur in the patterns of certain plants as well as in nautilus shells, art, and architecture.


## My Notes

## MATH TERMS

The absolute value of a number is its distance in units from 0 on a number line. Absolute value is always nonnegative. For instance, $|-3|$ and $|3|$ are both 3 since -3 and 3 are the same distance from zero-three units.
3. Complete the table below by investigating each sequence.

| Sequence | Increasing or <br> Decreasing? | Next Term in <br> the Sequence | Description <br> of Pattern |
| :---: | :---: | :---: | :---: |
| $0,5,10,15 \ldots$ |  |  |  |
| $-8,-4,-2,-1, \ldots$ |  |  |  |
| $1.5,2.75,4, \ldots$ |  |  |  |
| $\frac{1}{8}, \frac{1}{4}, \frac{1}{2}, \ldots$ |  |  |  |
| $2, \frac{5}{4}, \frac{1}{2}, \ldots$ |  |  |  |

4. Consider this sequence that uses absolute values of numerical expressions.
$|5-2|,|5-3|,|5-4|,|5-5|,|5-6|,|5-7|$
a. Express the next term in the sequence.
b. Describe the pattern of the sequence.
5. Arrange the numbers below so they form an increasing sequence. Describe the pattern of the sequence.
$|-16| \times|5|$
$|-18-2|$
|3-13|
$|38|+|-2|$
$\left|-\frac{10}{2}\right|$
6. Reason abstractly. Sequences do not always have to include mathematical operations. Look at the sequences below. Give verbal descriptions of what the pattern of the sequence is and give the next three terms.
a. $3.12,3.1212,3.121212, \ldots$
b. $\frac{1}{2}, \frac{1}{22}, \frac{1}{222}, \cdots$
c. $1,12,123, \ldots$
d. Z, Y, X, W, ...
e. J, F, M, A, ...
continued

## Check Your Understanding

Describe the following sequences using a mathematical operation and state the next three terms.
7. $0,8,16,24, \ldots$
8. $27,9,3,1, \ldots$
9. Arrange the numbers below so they form a decreasing sequence.

Describe the pattern of the sequence.
$-2|-4|$
$\frac{|-24|}{6}$
$|-2| \times|-5|$
$40|2 \div 5|$
$|7|-|-9|$

## LESSON 1-3 PRACTICE

Describe the following sequences using either a mathematical operation or a verbal description, indicate if the sequence is increasing or decreasing (if it involves numbers in some way), and state the next three terms.
10. $2,-4,-10,-16, \ldots$
11. $4.5,3.25,2,0.75, \ldots$
12. $\frac{1}{64}, \frac{1}{16}, \frac{1}{4}, 1, \ldots$
13. A, C, E, G, ...
14. Make sense of problems. Darlene wrote a sequence and gave it the following description: "The sequence is increasing by adding 6 to each previous term, and all of the terms are odd." Write five terms from a sequence that could be the one Darlene described.

## ACTIVITY 1 PRACTICE

Write your answers on notebook paper.
Show your work.

## Lesson 1-1

For Items 1-2, describe the pattern of the sequence and draw what you think are the next two terms.
1.


2.

3. Which is the next term in this sequence?

A.
B.

C.

D.

4. Write a conjecture for the pattern of the sequence in Item 3.
5. Draw what you think is the $6^{\text {th }}$ figure in the following sequence:


Figure 1


Figure 2


Figure 3
6. Write a conjecture for the pattern of the sequence in Item 5.

## Lesson 1-2

For Items 7-9, determine two different ways to represent the fourth term in each pattern.
7.

8. $\frac{1}{1}, \frac{1}{4}, \frac{1}{9}, \ldots$
9.

10. Which is the next term in this sequence?

11. Copy and complete the table for the following sequence:


Figure 1


Figure 2


Figure 3


Figure 4

| Figure | Number <br> of squares | Number <br> of line <br> segments | Sum |
| :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 1 |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 10 |  |  |  |
| 25 |  |  |  |

## And The Beat Goes On <br> Lesson 2-1 Adding and Subtracting Fractions

## Learning Targets:

- Represent a real-world context with fractions.
- Simplify expressions involving fractions by adding and subtracting.

> SUGGESTED LEARNING STRATEGIES: Create Representations, Visualization, Graphic Organizer, Think-Pair-Share, Create a Plan

Math can be found in many places in our daily lives. One place you might not realize where math is used is in music.

1. Take a few moments to think about the following questions and discuss them with your group.

What is your favorite song?
Why do you like it?
Why do you prefer one type of music over another?
Believe it or not, the answers you came up with may have more to do with mathematics than you may realize. Consider the following diagrams:

2. The first circle has no division and is represented by the number 1 .
a. Write the fractional equivalents in the sections of the other circles.
b. For each of the circles, justify that the sum of each combination of fractions is 1 .
c. Which circle was not like the others? Explain what made it different and the process you used to determine your answer.

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3. Model with mathematics. Determine two additional ways to divide the circle using combinations of the same fractions that you used in Item 2a. Label each portion with the appropriate fractions, and justify that the sum of each combination of fractions is 1 .


So what does all this have to do with music? As shown in the chart below, certain musical notes correspond to different fractional portions of a measure. In order to write sheet music, a composer must have a working knowledge of fractions.

| Note | Relative Length in <br> Common Time | Beats in <br> Common Time | Fraction of <br> Measure |
| :---: | :--- | :---: | :---: |
| O | Whole note | 4 beats | 1 |
| $\delta$ | Half note | 2 beats | $\frac{1}{2}$ |
| Quarter note | 1 beat | $\frac{1}{4}$ |  |
| $\boldsymbol{O}$ | Eighth note | $\frac{1}{2}$ beat | $\frac{1}{8}$ |
| $\boldsymbol{S}$ | Sixteenth note | $\frac{1}{4}$ beat | $\frac{1}{16}$ |

4. Based on the note chart, how many of each note would it take to fill one measure in common time? Explain your reasoning.

Measure 1
Measure 2


Measure 3
Measure 4

5. For each measure shown above, express each note as the fraction of its respective measure.
a. Measure 1 :
b. Measure 2:
c. Measure 3:
d. Measure 4:
6. Attend to precision. Show that the sum of the fractions in each measure is equal to 1 .
7. Based on your observations in Item 4, explain why you think the note in the fourth measure is called a "whole note."

## My Notes



8. The measures shown above do not contain the required number of beats. Fill out the table below to explore various ways to complete each measure.

| Measure | Fraction of <br> Measure <br> Shown | Fraction of <br> Measure <br> Remaining | Notes to <br> Complete <br> Measure <br> (Example 1) | Notes to <br> Complete <br> Measure <br> (Example 2) |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |

9. Explain the processes you used to determine the fraction of the measures shown and the fraction of the measures remaining.
10. What can you conclude about each of the following expressions? Justify your reasoning.

$$
\begin{aligned}
& \square+\square+Q \\
& \mathrm{O}+\mathrm{O}+\mathrm{O}+\mathrm{O}+\mathrm{Q}+\mathrm{Q}+\mathrm{O}+\mathrm{O}
\end{aligned}
$$

$$
1+1+1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}
$$

11. Write the sum of the expressions in Item 10 as:
a. a mixed number
b. an improper fraction
c. a decimal
d. a percent

## Check Your Understanding

In Items 12-16, simplify the expression.
12. $\frac{1}{3}+\frac{1}{6}$
13. $\frac{2}{5}+\frac{3}{8}$
14. $\frac{5}{12}-\frac{5}{6}$
15. $4 \frac{1}{4}+2 \frac{1}{2}$
16. $6 \frac{3}{5}-2 \frac{2}{3}$

## My Notes





## LESSON 2-1 PRACTICE

17. A trail mix recipe calls for $1 \frac{1}{2}$ cups granola, $\frac{3}{4}$ cup raisins, and $\frac{2}{3}$ cup peanuts. How many cups of trail mix does the recipe yield?

The table below shows rainfall totals for Houston, Texas, during the first six months of the year. Use the table to answer Items 18-20.

| Month | Rainfall (in inches) |
| :---: | :---: |
| January | $3 \frac{2}{3}$ |
| February | $2 \frac{11}{12}$ |
| March | $3 \frac{1}{3}$ |
| April | $3 \frac{1}{2}$ |
| May | $5 \frac{1}{4}$ |
| June | $5 \frac{1}{2}$ |

18. How much rain fell during the two rainiest months?
19. What is the difference in rainfall between the wettest month and the driest month?
20. During which period did more rain fall: January to February or March to April? Explain your reasoning.
21. Reason quantitatively. Write a scenario for the following problem:

$$
5 \frac{1}{8}+2 \frac{4}{5}=7 \frac{37}{40} .
$$

## ACTIVITY 2

## Learning Targets:

- Represent a real-world context with fractions.
- Simplify expressions involving fractions by multiplying and dividing.
- Write the reciprocal of a number.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Create Representations, Visualization, Vocabulary Organizer, Paraphrasing

When multiplying fractions such as $\frac{3}{4} \bullet \frac{1}{2}$, it is helpful to think of the multiplication symbol • as the word "of." So you read, "Find $\frac{3}{4}$ of $\frac{1}{2}$."

1. Reason abstractly. Without completing the problem, do you think the answer will be greater than or less than $\frac{1}{2}$ ? Explain your reasoning.
2. Consider the model below.

a. Explain how this model could be used to represent $\frac{3}{4}$ of $\frac{1}{2}$.
b. Express the shaded portion of the circle as a single fraction.

When discussing ideas in groups, use precise language to present your ideas. Speak using complete sentences and transition words such as for example, because, and therefore. Listen to others as they speak and ask for clarification of terms and phrases they use.

## My Notes

## WRITING MATH

Sometimes a dot is used as a symbol for multiplication. $3 \times 2=6$ and $3 \cdot 2=6$ are both ways to show 3 times 2 equals 6 .

## MATH TIP

After writing explanations to mathematical prompts, share your writing with a peer or your teacher. Have them confirm that your writing demonstrates clear understanding of mathematical concepts.


The problem $\frac{2}{3} \bullet \frac{3}{4}$ could be modeled in the same way. This problem is asking you to find $\frac{2}{3}$ of $\frac{3}{4}$.


Depending on how the problem is completed, several different answers could be given. Using the diagrams above, the answer could be expressed as $\frac{2}{4}$ or $\frac{1}{2}$.
By multiplying the numerators and denominators, the answer could be shown as $\frac{2}{3} \bullet \frac{3}{4}=\frac{2 \cdot 3}{3 \bullet 4}=\frac{6}{12}$.
3. Reason quantitatively. Explain why $\frac{1}{2}, \frac{2}{4}$ and $\frac{6}{12}$ are the same number.

## Example A

Find the product of $\frac{1}{8} \bullet \frac{2}{3}$.
Step 1: Multiply the numerators and multiply the denominators.

$$
\frac{1}{8} \cdot \frac{2}{3}=\frac{1 \cdot 2}{8 \cdot 3}=\frac{2}{24}
$$

Step 2: Simplify to write the product in lowest terms.

$$
\frac{2}{24} \div \frac{2}{2}=\frac{1}{12}
$$

Solution: The product of $\frac{1}{8} \quad \frac{2}{3}$ is $\frac{1}{12}$.

## Try These A

Multiply. Express each product in lowest terms.
a. $\frac{9}{10} \cdot \frac{2}{5}$
b. $\frac{3}{7} \cdot \frac{3}{4}$
c. $\frac{1}{4} \bullet \frac{1}{8}$

## Check Your Understanding

## My Notes

Multiply. Express each product in lowest terms.
4. $\frac{4}{5} \cdot \frac{1}{3}$
5. $\frac{6}{7} \cdot \frac{3}{10}$
6. $\frac{8}{3} \cdot \frac{13}{15}$
7. $1 \frac{2}{3} \cdot \frac{1}{2}$
8. $2 \frac{1}{2} \cdot 3 \frac{2}{3}$

A common strategy used to divide fractions uses the directions "invert and multiply." The problem, however, is that the phrase can often raise more questions than answers. For example:
What does invert mean?
What do I invert?
Why do I invert?
Why do I multiply?
These are all valid questions that deserve equally valid answers. The first two questions are easy to answer. To invert something is to turn it upside down. In mathematics, this is referred to as finding the reciprocal of a number. In other words, inverting the fraction $\frac{2}{3}$ would give the reciprocal $\frac{3}{2}$. When dividing fractions such as in the problem $\frac{2}{5} \div \frac{2}{3}$, the second fraction is inverted to produce $\frac{2}{5} \bullet \frac{3}{2}$.

## MATH TIP

You can gain a greater understanding of a concept by learning about "why" something works as opposed to memorizing "how" something works.

## MATH TERMS

The reciprocal of a number is its multiplicative inverse. The product of a number and its multiplicative inverse is 1 .

9. Make use of structure. Determine the reciprocal of the following:
a. $\frac{4}{3}$
b. $\frac{5}{18}$
c. 15
d. $5 \frac{3}{5}$

As for the last two questions concerning why this works, it's important to understand the concept of inverse operations.
10. Consider the expression $10 \div 2$.
a. Without using the words "divided by," explain what this problem is asking you to do.
b. What number could be multiplied by 10 to reach the same solution?
c. What do you notice about the number 2 and your answers to part b?
d. Use your answer to part b to rewrite the expression using multiplication.
e. Based on your observation, explain what is meant by describing multiplication and division as inverse operations of one another.

ACTIVITY 2
continued

## Example B

Divide $\frac{1}{2} \div \frac{2}{3}$.
Step 1: Find the reciprocal of $\frac{2}{3}$ by inverting the numerator and denominator.

$$
\frac{2}{3} \rightarrow \frac{3}{2}
$$

Step 2: Multiply $\frac{1}{2}$ by the reciprocal of $\frac{2}{3}$, which is $\frac{3}{2}$.

$$
\frac{1}{2} \times \frac{3}{2}=\frac{1 \times 3}{2 \times 2}=\frac{3}{4}
$$

Step 3: Simplify so that the quotient is in lowest terms.
$\frac{3}{4}$ is already in lowest terms, the numerator and denominator cannot be further reduced.
Solution: $\frac{1}{2} \div \frac{2}{3}=\frac{3}{4}$.

## Try These B

Divide. Simplify so the quotient is in lowest terms.
a. $\frac{5}{6} \div \frac{1}{3}$
b. $\frac{1}{2} \div 4$
C. $1 \frac{3}{4} \div \frac{1}{8}$

Background music in movies and television is not chosen at random. In fact, the music is often chosen so that it directly relates to the emotions being portrayed on the screen.
11. The following table lists the average beats per minute of music meant to express various emotions.

| Emotion | Beats per Minute |
| :--- | :---: |
| Joy and Triumph | 120 |
| Mystery and Suspense | 115 |
| Comfort and Peace | 100 |
| Loneliness and Regret | 120 |

a. Describe a scene from a movie or television show that would match each of the four categories in the table.


b. Explain how you could use multiplication or division to determine the number of beats in a three-minute song.

## Check Your Understanding

In Items 12-14, divide and simplify to lowest terms.
12. $\frac{4}{5} \div \frac{3}{7}$
13. $\frac{6}{15} \div \frac{2}{9}$
14. $4 \frac{2}{3} \div 4$

## LESSON 2-2 PRACTICE

15. Write a scenario for this problem: $8 \frac{2}{3} \times 4=34 \frac{2}{3}$.
16. Miranda made some cupcakes for the school bake sale. Of the cupcakes, $\frac{1}{2}$ are chocolate and $\frac{1}{2}$ are strawberry. Of the strawberry cupcakes, $\frac{1}{3}$ are frosted with vanilla frosting, and the rest are frosted with chocolate frosting. What portion of the cupcakes are strawberry with chocolate frosting?
17. Ennis has a length of rope that measures $3 \frac{3}{4}$ yards. He cuts all of the rope into smaller pieces that measure $\frac{3}{4}$ yard each. How many cuts did Ennis make?
18. Draw a picture that proves $\frac{3}{4} \div \frac{1}{8}=6$.
19. Critique the reasoning of others. Brody and Lola each found the product of $4 \frac{2}{3} \times 1 \frac{1}{2}$ in a different way shown below. Who multiplied correctly? Explain your reasoning.

$$
\begin{gathered}
\text { Brody } \\
4 \times 1=4 \\
\frac{2}{3} \times \frac{1}{2}=\frac{2}{6}=\frac{1}{3} \\
=4 \frac{1}{3}
\end{gathered}
$$

## ACTIVITY 2 PRACTICE

Write your answers on notebook paper.
Show your work.

## Lesson 2-1

In Items 1-6, simplify the expression.

1. $\frac{3}{5}+\frac{4}{6}$
2. $\frac{5}{8}-\frac{1}{3}$
3. $2 \frac{3}{4}+7 \frac{3}{5}$
4. $7 \frac{1}{6}-4 \frac{1}{2}$
5. $12 \frac{3}{8}+6 \frac{5}{6}$
6. $8 \frac{9}{10}-3 \frac{1}{4}$
7. Judy has $4 \frac{2}{3}$ yards of fabric and Marie has $5 \frac{1}{2}$ yards. How much fabric do they have altogether?
8. Describe the process you would use to find the solution to the problem $3 \frac{1}{4}-1 \frac{1}{3}$. Express your answer numerically and with a graphical representation.
9. Angelo weighed $5 \frac{1}{4}$ pounds when he was born. His sister, Carmen, weighed $7 \frac{1}{2}$ pounds when she was born. How much heavier was Carmen than Angelo at birth?
10. Copy and complete:

$$
\frac{1}{2}+\frac{1}{3}=\frac{\square}{6}+\frac{2}{6}
$$

11. Gwen went walking three days one week. The first day, she walked $3 \frac{1}{2}$ miles. Each day after that, she walked $\frac{1}{4}$ mile more than the day before. How many miles did Gwen walk this week?
A. $10 \frac{1}{2}$
B. $10 \frac{3}{4}$
C. $11 \frac{1}{4}$
D. $11 \frac{1}{2}$
12. The following week, Gwen walked a total of $15 \frac{7}{8}$ miles. How many more miles did she walk the second week than the first week?
13. Yuri measured the snowfall over a four-day period and calculated a total of $15 \frac{1}{3}$ inches of snowfall. If it snowed $4 \frac{3}{4}$ inches on the first day and $2 \frac{1}{2}$ inches on the second day, how much snow fell on the third and fourth days?

## Lesson 2-2

In Items 14-19, simplify the expression.
14. $\frac{2}{5} \cdot \frac{7}{12}$
15. $3 \frac{1}{2} \cdot 4 \frac{1}{4}$
16. $24 \div 2 \frac{2}{3}$
17. $\frac{2}{3} \cdot \frac{2}{9}$
18. $8 \div 1 \frac{1}{5}$
19. $6 \frac{1}{6} \div 2 \frac{1}{2}$
20. Without performing any calculations, determine which of the following problems will produce the greatest result. Explain your reasoning.
a. $\frac{3}{5} \cdot \frac{4}{7}$
b. $\frac{3}{5} \div \frac{4}{7}$
21. Tony has 36 pages left in the book he is reading. He plans to read $\frac{1}{4}$ of the pages tonight before going to bed. Which of the following expressions would not produce the number of pages Tony is going to read before he goes to bed?
A. $36 \cdot \frac{1}{4}$
B. $36 \div \frac{1}{4}$
C. $36 \div 4$
D. $36 \cdot 0.25$
22. The gas tank in Mr. Yang's car is $\frac{2}{3}$ full. The tank holds a total of 15 gallons of gasoline. How many gallons of gas are in Mr. Yang's car?
A. 10
B. $10 \frac{1}{3}$
C. $10 \frac{2}{3}$
D. 11
23. One-third of the students in Mr. Rose's class have corrected vision. Of those students, $\frac{3}{8}$ wear glasses. What fraction of the class wears glasses?
24. Bananas cost $30 \$$ per pound. How much will $2 \frac{5}{6}$ pounds of bananas cost?
25. A recipe for fried rice yields $4 \frac{1}{2}$ cups, or 6 servings. How big is each serving of fried rice?
26. Owen is having a dinner party and plans to serve the fried rice recipe from Item 25 . He needs to make 10 servings. How many cups of fried rice will Owen need to make?

## MATHEMATICAL PRACTICES Look For and Make Use of Structure

27. Copy and complete this table to examine the similarities and differences between adding, subtracting, multiplying, and dividing fractions.

| Operation | Steps to <br> Evaluate | Example |
| :--- | :---: | :---: |
| Add fractions |  |  |
| Subtract fractions |  |  |
| Multiply fractions |  |  |
| Divide fractions |  |  |

a. How are addition and subtraction of fractions similar?
b. How are multiplication and division of fractions similar?
c. How are the operations different?

## Patterns and Quantitative

## GAME ON

Piper and Lily are in the school math club and they are going to participate in an "Are You Smarter Than An Eighth Grader?" competition. Help Piper and Lily answer the following questions to ensure that they gain the most points possible in this competition.

The first category is finding patterns in sequences.

1. Write what you think are the next two terms of the pattern.
a. $45,34,23, \ldots$
b. $27,18,12, \ldots$
c. $13,103,1003, \ldots$
d. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$

2. For parts a and b in Item 1 , state if the sequences were increasing or decreasing sequences. Describe the patterns using a mathematical operation.

The second category is story problems. Show your work on how to solve the problems.
3. Luke needs to tie his dog Duke to the deck while they are working on the fence. He finds two short leashes with lengths of $6 \frac{1}{2}$ feet and $5 \frac{7}{8}$ feet. If Luke connects the two leashes, how far can Duke travel from the deck?
4. Fletch is making shelves. He has $14 \frac{1}{2}$ feet of wood. He wants each shelf to be $2 \frac{1}{3}$ feet long. How many shelves can Fletch make with the wood he has? How much wood would be left over?
5. Greg is planning a new city park. He has a rectangular piece of land that is $50 \frac{1}{2}$ feet by $25 \frac{2}{3}$ feet. What is the area of Greg's park?

| Scoring Guide | Exemplary | Proficient | Emerging | Incomplete |
| :---: | :---: | :---: | :---: | :---: |
|  | The solution demonstrates these characteristics: |  |  |  |
| Mathematics <br> Knowledge and <br> Thinking <br> (Items 1a-e, 2, 3, 4, 5) | - Clear and accurate understanding of operations with fractions and mixed numbers. <br> - Effective understanding of finding the pattern and missing terms in a sequence. | - Operations with fractions and mixed numbers that are usually correct. <br> - Finding the pattern in a sequence and extending it. | - Partially correct operations with fractions and mixed numbers. <br> - Errors in extending sequences and finding the pattern. | - Incorrect or incomplete computation in operations with fractions and mixed numbers. <br> - Little or no understanding of sequences. |
| Problem Solving (Items 1a-e, 2, 3, 4, 5) | - An appropriate and efficient strategy that results in a correct answer. | - A strategy that may include unnecessary steps but results in a correct answer. | - A strategy that results in some incorrect answers. | - No clear strategy when solving problems. |
| Mathematical <br> Modeling/ <br> Representations <br> (Items 1a-e, 2, 3, 4, 5) | - Writing accurate expressions for operations with fractions and mixed numbers. <br> - Accurately writing an expression to represent a sequence. | - Writing an expression for operations with fractions and mixed numbers. <br> - Writing an expression to represent a sequence. | - Errors in writing expressions for a given problem situation. <br> - Errors in writing an expression to represent a sequence. | - Inaccurately written expressions. <br> - Little or no understanding of writing an expression to represent a sequence. |
| Reasoning and Communication (Item 2) | - Precise and accurate description of a sequence. | - An adequate description of a sequence. | - A misleading or confusing description of a sequence. | - An incomplete or inaccurate description of a sequence. |

## Squares and Cubes <br> Lesson 3-1 Area, Squares, and Square Roots

## Learning Targets:

- Interpret and simplify the square of a number.
- Determine the square root of a perfect square.

> SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Paraphrasing, Create Representations, Look for a Pattern, Note Taking, Critique Reasoning

Dominique Wilkins Middle School is holding its annual school carnival. Each year, classes and clubs build game booths in the school gym. This year, the student council has asked Jonelle's math class for help in deciding what size the booths should be and how they should be arranged on the gym floor. The class will begin this work by reviewing some ideas about area.

## Example A

Find the area of this rectangle. The rectangle has been divided into squares. Assume that the length of each side of a small square is 1 cm .


Step 1: Find the length and the width of the rectangle. The rectangle is 5 cm long and 3 cm wide.

Step 2: To find the area, multiply the length times the width.
Area $=$ length $\times$ width
Area $=5 \mathrm{~cm} \times 3 \mathrm{~cm}=15 \mathrm{~cm}^{2}$
Solution: The area of the rectangle is $15 \mathrm{~cm}^{2}$. Note that the units are centimeters squared and this represents the amount of surface the rectangle covers.

## Try These A

Before deciding on how to arrange the booths, the student council needs to know the area of the gym floor. Several class members went to the gym to measure the floor. They found that the length of the floor is 84 feet and the width of the floor is 50 feet.
a. Find the area of the gym floor. Explain how you found the area and include units in your answer.
b. Explain what the area of the gym floor means.

## My Notes






The class is now going to focus on the area of squares, because this is the shape of the base of many of the game booths.

## Example B

Find the area of the square below. Do you need to know both the length and width of a square to be able to determine its area?


Step 1: The length of one side is given and as this is a square, we know that all four sides have equal lengths.
Step 2: To find the area of the square, multiply the length of a side by itself. In this case, the length is 4 cm .
$4 \mathrm{~cm} \times 4 \mathrm{~cm}=16 \mathrm{~cm}^{2}$
Solution: The area of the square is $16 \mathrm{~cm}^{2}$. You can find the area of a square with only one length given, since all four sides are equal lengths.

## Try These B

This drawing shows the floor space of one of the carnival booths. It is a square with the length of one side labeled with the letter $s$. The $s$ can be given any number value since the booths are going to be different sizes.

a. Express regularity in repeated reasoning. Complete this table by finding the areas of some different sized booths. The length of a side in feet is represented by $s$, as in the drawing above. Include units for area in the last column.

| Length of Side <br> (in feet) | Calculation | Area of the Square |
| :---: | :---: | :---: |
| $s=3$ | $3 \times 3$ | $9 \mathrm{ft}^{2}$ |
| $s=6$ |  |  |
| $s=8$ |  |  |
| $s=11$ |  |  |
| $s=4.2$ |  |  |
| $s=10 \frac{1}{2}$ |  |  |

For each calculation in Try These B, you found the product of a number times itself. The product of a number times itself can be written as a power with a base and an exponent.

## Example C

Write $5 \cdot 5$ as a power with a base and an exponent.
Step 1: Identify the base, which is the number being multiplied. The base is 5 .

Step 2: Identify the exponent, which is the number of times the base is multiplied by itself.
In $5 \cdot 5$, the base 5 is multiplied by itself two times, so the exponent is 2 .

Step 3: The base is written normally and the exponent is written as a superscript: $5^{2}$
Solution: $5 \cdot 5=5^{2}$. Numbers expressed as powers with a base and an exponent are written in exponential form.

## Try These C

Write the following in exponential form.
a. $12 \cdot 12$
b. $\frac{1}{3} \cdot \frac{1}{3}$
c. $3.25 \cdot 3.25$

## Check Your Understanding

1. The table below gives some booth sizes in exponential form. Copy and complete the table.

| Exponential Form | Product Using the Base <br> as a Factor Twice | Standard Form |
| :---: | :---: | :---: |
| $5^{2}$ | $5 \cdot 5$ | 25 |
| $2^{2}$ |  |  |
| $1^{2}$ |  |  |
| $7^{2}$ |  |  |
| $15^{2}$ |  |  |

## My Notes

## MATH TERMS

A power is a number multiplied by itself. A number or expression written with an exponent is in exponential form.

## READING MATH

Read the expression $5^{2}$ as " 5 squared" or the "square of $5 .{ }^{\text {. }} 5^{2}$ means 5 times 5 , or $5 \times 5$.

1. Find the value of the expression $9^{2}$.
2. The number 49 is the square of what number?


## My Notes

## WRITING MATH

The symbol $\sqrt{ }$ is called a radical sign. It is used in expressions to show square roots.

## MATH TERMS

The square root of a number is another number that when multiplied by itself gives the original number. A square root does not have to be an integer.

When an integer is multiplied by itself, the result is a perfect square. In part d of Try These D, the area of each booth's floor is a perfect square, and the side lengths are their square roots.

## MATH TIP

Remember that you solve an equation by isolating the variable. You isolate the variable by performing the same operation on both sides of the equation.

## Example D

The area of the floor of a square booth is $36 \mathrm{ft}^{2}$. What is the length of the side $s$ of this booth?


Step 1: The area is the square of the length of a side. Here, $36 \mathrm{ft}^{2}$ is the square of $s$. To find $s$, find the square root of $36 \mathrm{ft}^{2}$. The symbol for square roots is $\sqrt{ }$.

$$
\sqrt{36}=s
$$

Step 2: To solve $\sqrt{36}=s$, think about which number times itself equals 36 . $\sqrt{36}=6$

Solution: The length of a side of the booth is 6 ft .

## Try These D

Find each square root.
a. $\sqrt{16}$
b. $\sqrt{81}$
c. $\sqrt{100}$
d. The carnival booth sizes are assigned according to the number of members in the club or class. Copy and complete the table.

| Club or Class Size | Area of Square <br> Booth's Floor | Side Length of <br> Booth's Floor |
| :---: | :---: | :---: |
| $1-30$ members | $36 \mathrm{ft}^{2}$ |  |
| $31-60$ members |  | 8 ft |
| $61-90$ members |  | 9 ft |
| $91-120$ members | $121 \mathrm{ft}^{2}$ |  |
| $121-150$ members | $144 \mathrm{ft}^{2}$ |  |

You can use equations to model problems involving sides and areas of squares. The situation in Example D can be modeled with the equation $s^{2}=36$, where $s$ represents the side of the square and 36 is the area in square feet.

## Example E

Solve the equation $s^{2}=36$.
Step 1: To find $s$, you need to find the square root of $s^{2}$.
Step 2: If you take the square root of one side of an equation, you must take the square root of the other side.

$$
\begin{aligned}
& s^{2}=36 \\
& \sqrt{s^{2}}=\sqrt{36} \\
& \sqrt{s^{2}}=s \text { and } \sqrt{36}=6
\end{aligned}
$$

Solution: The solution of the equation $s^{2}=36$ is $s=6$.

## Try These E

Solve each equation.
a. $x^{2}=64$
b. $x^{2}=121$
c. $x^{2}=1.44$

## Check Your Understanding

For Items 2-6, simplify each expression.
2. $11^{2}$
3. $5.5^{2}$
4. $\left(\frac{2}{3}\right)^{2}$
5. $\sqrt{144}$
6. $\sqrt{64}$
7. What would the area of a square booth be if the side length is 9 feet?
8. Solve for $x: x^{2}=16$

## LESSON 3-1 PRACTICE

9. Find the area of this square:

10. Write a rule, in words, for finding the area of a square.
11. Label the diagram using the terms base and exponent.

12. Why do you think the number 25 is called the square of 5 ? Draw a model in the My Notes space as part of your explanation.
13. Reason abstractly. Think about what you have discovered about the area of a square and finding the side length of a square. Write a sentence to explain what the square root of a number means.
14. Solve each equation for $x$.
a. $x^{2}=49$
b. $x^{2}=0.49$
c. $x^{2}=\frac{1}{4}$

## My Notes

9. 

|  |  |  |  |  |  |  | My Notes |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
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## Learning Targets:

- Interpret and simplify the cube of a number.
- Determine the cube root of a perfect cube.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall,
Paraphrasing, Create Representations, Look for a Pattern, Note Taking, Critique Reasoning

The student council is very happy with the work that the class has done on the carnival so far. The class has found the areas of the floors and the side lengths of the booths. One concept remains for the class to review before completing all the needed work. The booths do not just take up floor space; they also have height.


The diagram below represents a cubic foot. Its dimensions are $1 \mathrm{ft} \cdot 1 \mathrm{ft} \cdot 1 \mathrm{ft}$.


Build a solid with the cubes given to you by your teacher with dimensions of 2 units by 2 units by 2 units. This solid is a cube with equal edge lengths of 2 units.

## Example A

Find the volume of the cube you built with edge lengths of 2 units each. Find the volume in exponential form and in cubic units.
Step 1: The volume of a cube is found by multiplying length times width times height. A cube has equal values for length, width, and height.
Volume $=$ length $\times$ width $\times$ height
Step 2: To find the volume of a cube in exponential form, write the volume with a base number and an exponent.
Volume $=2 \times 2 \times 2=2^{3}$
Step 3: To find the volume of a cube in cubic units, multiply the edge lengths and include the cubic units label.
Volume $=2$ units $\times 2$ units $\times 2$ units $=8$ cubic units
Solution: The volume of the cube is $2^{3}$ or 8 cubic units.

## Try These A

a. Make use of structure. Complete this table to show the volume of each cube in exponential form and in cubic feet.

| Length of an <br> Edge of a <br> Cube (in feet) | Calculation for <br> Finding Volume <br> of Cube | Volume of Cube <br> (in exponential <br> form) | Volume of <br> the Cube (in <br> cubic feet) |
| :---: | :---: | :---: | :---: |
| 2 | $2 \times 2 \times 2$ | $2^{3}$ | $8 \mathrm{ft}^{3}$ |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |
| 8 |  |  |  |
| 9 |  |  |  |

The exponent used in each exponential expression of volume in Try These A is the same. This exponent can be used for the volume of a cube. Using this exponent is known as cubing a number.

## Example B

Find the cube of 10 or $10^{3}$.
Cubing a number means to multiply it by itself three times.

$$
10 \cdot 10 \cdot 10=1,000
$$

Solution: The cube of 10 is 1,000 or $10^{3}=1,000$.

## Try These B

Simplify each expression.
a. $3^{3}$
b. 7•7•7
c. $11^{3}$
d. $1.5 \cdot 1.5 \cdot 1.5$

If you know the volume of a cube you can determine the length of the edge of the cube. The operation you use to find the edge length when you know the volume is called finding the cube root. The symbol used for cube roots is $\sqrt[3]{ }$.

## Example C

The volumes of four cubes are given in this table. Find the length of the edge of each cube by finding the cube root of the volume.

| Volume of the Cube <br> (in cubic units) | Length of the Edge <br> (in units) |
| :---: | :---: |
| 1 |  |
| 8 |  |
| 27 |  |
| 64 |  |

## My Notes



## WRITING MATH

The index 3 in the radical sign $\sqrt[3]{ }$ shows a cube root. $\sqrt[3]{8}$ is read "the cube root of 8 ."


## My Notes

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|  |  |  |  |  |  |  |  |
|  | Side Length |  |  |  |  | Volume of Booth |  |
|  | 2 |  |  |  |  |  |  |
|  |  |  |  |  | 27 |  |  |
|  |  |  |  |  | 64 |  |  |
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|  | MATH TIP |  |  |  |  |  |  |
|  | You can solve equations such as $x^{3}=8$ by taking the cube root of both sides. |  |  |  |  |  |  |

Step 1: When the volume of a cube is 1 cubic unit, simplify $\sqrt[3]{1}$ to find the length of the edge. $\sqrt[3]{1}=1$ unit
Step 2: When the volume of a cube is 8 cubic units, simplify $\sqrt[3]{8}$ to find the length of the edge. $\sqrt[3]{8}=2$ units
Step 3: When the volume of a cube is 27 cubic units, simplify $\sqrt[3]{27}$ to find the length of the edge. $\sqrt[3]{27}=3$ units
Step 4: When the volume of a cube is 64 cubic units, simplify $\sqrt[3]{64}$ to find the length of the edge. $\sqrt[3]{64}=4$ units

Solution: Complete the table with the values found in the steps above.

## Try These C

Complete the table to the left. All numbers are in standard form.

## Check Your Understanding

In Items $1-4$, simplify each expression.

1. $5.1^{3}$
2. $\sqrt[3]{1,728}$
3. $\left(\frac{1}{4}\right)^{3}$
4. $20^{3}$
5. Copy and complete the table for the booths listed.

| Volume of the Cubical <br> Booth | Length of each Edge <br> of the Booth |
| :---: | :---: |
| $216 \mathrm{ft}^{3}$ |  |
| $343 \mathrm{ft}^{3}$ |  |
| $729 \mathrm{ft}^{3}$ |  |
| $1000 \mathrm{ft}^{3}$ |  |

## LESSON 3-2 PRACTICE

6. What is the exponent used in cubing a number?
7. Find the volume of the cube shown.

8. Think about what you have discovered about the volume of a cube and finding the edge length of a cube. Write a sentence to explain what the cube root of a number means.
9. A cube has a volume of 216 cubic feet. What is the length of an edge of this cube?
10. Critique the reasoning of others. Aaron says that to find the volume of a cube he uses the formula $l \cdot w \cdot h$. Jonelle says that the formula she uses is $l^{3}$. Whose formula is correct? Justify your answer.
11. Solve each equation for $x$.
a. $x^{3}=64$
b. $x^{3}=216$
c. $x^{3}=1$

## Learning Targets:

- Simplify expressions with powers and roots.
- Follow the order of operations to simplify expressions.

SUGGESTED LEARNING STRATEGIES: Interactive Word Wall, Paraphrasing, Create Representations, Look for a Pattern, Note Taking, Critique Reasoning

Exponents other than 2 and 3 can be used. The exponent tells you the number of times the base is used as a factor.

## Example A

Simplify $2.4^{5}$.
Step 1: $\quad 2.4^{5}$ means 2.4 raised to the power of 5 , which means 2.4 appears 5 times as a factor.
$2.4 \cdot 2.4 \cdot 2.4 \cdot 2.4 \cdot 2.4$
Step 2: Simplify by multiplying. Use a calculator.
$2.4 \cdot 2.4 \cdot 2.4 \cdot 2.4 \cdot 2.4=79.62624$
Solution: $2.4^{5}$ equals 79.62624 .

## Try These A

a. Copy and complete this table.

| Number (in <br> exponential form) | Product Using the Base <br> as a Factor | Number (in <br> standard form) |
| :---: | :---: | :---: |
| $4.5^{3}$ |  |  |
| $2.1^{4}$ |  |  |
| $\left(\frac{4}{5}\right)^{3}$ |  |  |
| $\left(1 \frac{1}{2}\right)^{4}$ |  |  |
| $\left(\frac{2}{3}\right)^{5}$ |  |  |

## My Notes

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## My Notes

## ORDER OF OPERATIONS

1. Parentheses
2. Exponents/Roots (from left to right)
3. Multiplication/Division (from left to right)
4. Addition/Subtraction (from left to right)


## LESSON 3-3 PRACTICE

Evaluate the following expressions.
4. $5^{2} \cdot 2^{4}$
5. $9^{5}$
6. $6 \times(5+3) \div 3-2^{3}$
7. Construct viable arguments. Jose and Juan were given the expression $3+24 \div 2 \times 3$; however, they solved it differently. Who solved it correctly and why?

$$
\begin{array}{cc}
\text { Jose: } 3+24 \div 2 \times 3 & \text { Juan: } 3+24 \div 2 \times 3 \\
3+24 \div 6 & 3+12 \times 3 \\
3+4 & 3+36 \\
7 & 39
\end{array}
$$

## ACTIVITY 3 PRACTICE

Write your answers on notebook paper.
Show your work.

## Lesson 3-1

1. Evaluate each expression.
a. $\sqrt{81}$
b. $3.3^{2}$
c. $\left(\frac{1}{5}\right)^{2}$
2. If the side length of a square is 7.2 inches, what is the area of the square?
3. This figure has an area of $196 \mathrm{in}^{2}{ }^{2}$ and is made up of four small squares. What is the side length of a small square?

4. Which is NOT a way to express $8^{2}$ ?
A. eight multiplied by two
B. eight to the second power
C. eight squared
D. eight times eight
5. Complete the table for a booth with a floor in the shape of a square.

| Side Length <br> (in cm) | Perimeter of <br> Booth (in cm) | Area of Booth <br> (in cm) |
| :---: | :---: | :---: |
| 2 |  |  |
| 3 |  |  |
|  | 16 |  |
|  | 20 |  |
|  |  | 49 |
|  |  | 100 |
| $n$ |  |  |

6. What patterns do you notice in the table you made in Item 5?
7. Which of the following numbers is a perfect square?
A. 32
B. 36
C. 40
D. 44
8. Solve each equation for $x$.
a. $x^{2}=81$
b. $x^{2}=0.16$
c. $x^{2}=\frac{1}{100}$
9. Daisy cut a square out of a sheet of graph paper. The square had an area of 16 square cm . She then trimmed 1 cm from each side of the square. What is the area of the smaller square?

## Lesson 3-2

10. Evaluate each expression.
a. $0.4^{3}$
b. $\sqrt[3]{27}$
c. $\sqrt[3]{0.001}$
11. Write "four cubed" in exponential form.
12. What is the volume of a cube with an edge length of $\frac{2}{3}$ foot?
13. What is the edge length of a cube with a volume of 216 cubic feet?
14. A picture frame cube has a volume of 64 cubic cm . Each of the six faces holds a picture of the same size as the face. What size picture does each face hold?

A. $3 \times 3$
B. $4 \times 6$
C. $4 \times 4$
D. $6 \times 6$
15. The dimensions of a cube are $5 \mathrm{~cm} \times 5 \mathrm{~cm} \times 5 \mathrm{~cm}$. What is the volume of this cube?
16. The edge length for a cube is $c$. Which of the following does NOT represent how to find the volume of this cube?
A. $c^{3}$
B. $c \cdot c \cdot c$
C. $3 \times c$
D. $c$ cubed
17. Solve each equation for $x$.
a. $x^{3}=125$
b. $x^{3}=0.008$
c. $x^{3}=\frac{1}{27}$

## Lesson 3-3

18. Write $3 \cdot 3 \cdot 3 \cdot 3$ in exponential form.
19. In science we find that some cells divide to form two cells every hour. If you start with one cell, how many cells will there be after 7 hours?
20. Use $=,>$, or $<$ to complete the following:
a. $1^{9}$ $\qquad$ $1^{4}$
b. $3^{4} \quad 4^{3}$
c. $2^{6}$ $\qquad$ $\sqrt{144}$
21. If you know that $9^{3}=729$, describe how to find $9^{4}$ without having to multiply four 9 s .
22. Which of the following expressions results in the largest amount?
A. $2^{9}$
B. $4.5^{3}$
C. $9^{2}$
D. $8.3^{3}$
23. Evaluate the following expressions.
a. $(16-10)^{2} \div 4$
b. $7^{2}+(5-3)^{3}$
c. $\sqrt[3]{125} \times 2^{3}$
d. $\left(\frac{1}{2}\right)^{4}$
e. $1.8^{5}$
f. $\sqrt{(50+50)}$
24. Write 9 raised to the power of 6 in exponential form.
25. What is the first step in simplifying the expression $(8+2)^{3} \div \sqrt{(20-4)}-50$ ?
A. add $8+2$
B. cube 2
C. subtract 50
D. cube 8
26. Complete the table.

| $3^{1}$ |  |
| :---: | :---: |
|  | 9 |
| $3^{3}$ | 27 |
| $3^{4}$ |  |
|  | 243 |
| $3^{6}$ |  |
|  | 2,187 |

## MATHEMATICAL PRACTICES

## Reason Abstractly and Quantitatively

27. Write a letter to the student council that describes the relationship between the side of a booth and its area and volume. Use examples to illustrate your relationship.

## Rational Numbers

Know When to Fold 'Em
Lesson 4-1 Modeling Fractions

## Learning Targets:

- Model fractions graphically.
- Convert between fractions, decimals, and percents.

SUGGESTED LEARNING STRATEGIES: Manipulatives, Discussion
Group, Graphic Organizer, Sharing and Responding
A popular urban myth is that it is impossible to fold a piece of paper in half more than seven times.

1. Remove a sheet of paper from your notebook and try it for yourself. You may fold the paper in any direction you wish as long as you fold the paper in half each time. When you're done experimenting, share your results with your group members.
2. Complete the following table based on the first six folds you made in your paper.

| Folds | Number <br> of Regions <br> on Paper | Each Region's <br> Fraction of the <br> Original Paper | Sketch of <br> Unfolded Paper <br> Showing Folds |
| :---: | :---: | :---: | :---: |
| 0 |  |  |  |
| 1 |  |  |  |
| 2 |  |  |  |
| 3 |  |  |  |
| 4 |  |  |  |
| 5 |  |  |  |
| 6 |  |  |  |

## My Notes

## CONNECT TO AP

In AP and higher-level math classes you will learn about asymptotes, which are lines on a graph that a function gets closer and closer to, but never actually touches.

b. The region's fractions are different than the previous fractions. Why?
5. Use another piece of paper to show how you could find other fractional values between 0 and 1 . Discuss your results with your group.

The fractions you created in the previous tables can also be represented in different forms, including decimals, percents and graphical representations.
6. When given a fraction, decimal, or percent, it can be converted into either of the other two. Use the graphic organizer below to discuss the process of converting between the different representations. Describe how to interpret the graphic organizer.


## My Notes


## Example A

Convert 0.75 to a fraction.
Step 1: Determine the power of 10 for the fraction's denominator. Count the number of digits to the right of the decimal.
For the number 0.75 , there are 2 digits to the right of the decimal so the exponent is 2 .
Step 2: Write the whole number 75 for the numerator, and 10 with an exponent of 2 for the denominator.

$$
\frac{75}{10^{2}}=\frac{75}{100}
$$

Step 3: Simplify the fraction: $\frac{75}{100}=\frac{3}{4}$
Solution: $0.75=\frac{3}{4}$

## Try These A

Convert the following decimals into fractions. Simplify each fraction.
a. 0.59
b. 0.4
c. 0.235
7. Janice says that you can remove any zeros after the decimal point before converting to a fraction, unless they are between nonzero digits.
a. Give an example of a decimal that supports Janice's claim. How do these zeros relate to simplifying the fraction?
b. Give an example of a decimal that refutes Janice's claim. How would you explain to Janice when her statement doesn't make sense?

## Check Your Understanding

8. Copy and complete the table. Round decimals to the nearest hundredth.

| Fraction | Decimal Form | Percent |
| :---: | :---: | :---: |
| $\frac{1}{10}$ |  |  |
|  | 0.2 |  |
| $\frac{1}{3}$ |  | $25 \%$ |
| $\frac{1}{2}$ | 0.6 | $33 \frac{1}{3} \%$ |
| $\frac{2}{3}$ | 0.75 |  |
|  |  | $66 \frac{2}{3} \%$ |
| $\frac{9}{10}$ |  | $80 \%$ |

## LESSON 4-1 PRACTICE

9. Write the fraction and decimal equivalents for each percent.
a. $60 \%$
b. $80 \%$
10. Show the graphical representation of $\frac{2}{3}$ using a pie chart.
11. Convert 0.14 to a fraction and percent.
12. Model with mathematics. Describe how you could use paper folding to illustrate the fraction $\frac{1}{9}$.

|  |  |  |  |  |  |  | My Notes |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |

## MATH TERMS

A rational number is any number that can be written as the ratio of two integers, $\frac{a}{b}$, where the divisor, $b$, is not zero.

## Learning Targets:

- Define and recognize rational numbers.
- Represent repeating decimals using bar notation.
- Convert a repeating decimal to a fraction.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Group Presentations, Think-Pair-Share, Debriefing

Rational numbers may be represented as fractions or as decimals.

1. If rational numbers need to be able to be expressed as fractions, how can decimals also be rational numbers?
2. Convert the following fractions into decimals.
a. $\frac{7}{20}=$
b. $\frac{6}{25}=$

Some decimals are terminating decimals. Terminating decimals have a finite or limited number of digits following the decimal point. It is possible to express these numbers as the ratio of two integers, or as a fraction.
3. Write the decimals below as a ratio of two integers. Express each answer as a fraction.
a. 0.65
b. 0.004
4. Describe how any terminating decimal can be written as a fraction.

Some decimals are repeating decimals. Repeating decimals have one or more digits following the decimal point that repeat endlessly. For instance, 0.777 . . . is a repeating decimal as well as $2.5111 \ldots$. Repeating decimals are also rational numbers.
5. Rewrite each of the following rational numbers as a decimal.
a. $\frac{1}{3}=$
b. $\frac{4}{9}=$
c. $\frac{5}{6}=$

Repeating decimals can also be converted into fractions.

## Example A

Convert 0.51111 . . . to a fraction.
Step 1: Let $x=$ the repeating decimal.

$$
x=0.51111 \ldots
$$

Step 2: Determine the repeating digit or digits.
The repeating digit is 1 .
Step 3: Multiply both sides of the original equation by the least power of 10 so that the digits that repeat align after the decimal point.

$$
\begin{aligned}
x & =0.5111 \ldots \\
10 x & =5.1111 \ldots
\end{aligned}
$$

Step 4: Subtract one equation from the other. The repeating digits will not be in the difference.

$$
\begin{aligned}
10 x & =5.1111 \ldots \\
-x & =0.5111 \ldots \\
\hline 9 x & =4.6
\end{aligned}
$$

Step 5: Solve the resulting equation.

$$
\begin{aligned}
\frac{9 x}{9} & =\frac{4.6}{9} \\
x & =\frac{46}{90}=\frac{23}{45}
\end{aligned}
$$

## Try These A

Convert the following repeating decimals into fractions.
a. $0.444444 \ldots$
b. $0.121212 \ldots$
c. 2.505050 . .

## My Notes

## READING MATH

Bar notation can be used to represent the repeating digits of a repeating decimal.
$0.777 \ldots=0 . \overline{7}$
$2.5111 \ldots=2.5 \overline{1}$


## Check Your Understanding

6. Convert the following to decimals.
a. $\frac{6}{15}$
b. $\frac{7}{9}$
c. $\frac{23}{99}$
d. $\frac{5}{9}$
7. Is the decimal $0.12345 \ldots$ a repeating decimal? Explain.

## LESSON 4-2 PRACTICE

8. How are terminating decimals different than repeating decimals?
9. Write $\frac{7}{8}$ as a decimal. What type of decimal is it?
10. Convert the decimals to fractions.
a. 0.50
b. 0.60
c. 0.25
11. Write $0.2222222 \ldots$ as a fraction.
12. Attend to precision. Write $1.024242424 \ldots$ as a fraction.
13. Hannah knows that the repeating decimal $0.0545454 \ldots$ is equal to $\frac{3}{55}$.
a. What decimal can be added to $0.0545454 \ldots$ to get $0.1545454 \ldots$ ?
b. Write your answer from part a as a fraction.
c. Add two fractions to determine the fraction equal to $0.1545454 \ldots$.

## Learning Targets:

- Compare rational numbers in different forms.
- Represent repeating decimals using bar notation.
- Utilize various forms of rational numbers.

SUGGESTED LEARNING STRATEGIES: Activating Prior Knowledge, Group Presentations, Think / Pair / Share, Sharing and Responding

1. What is the difference between the decimals $0 . \overline{7}$ and $0.00 \overline{7}$ ?

To convert a repeating decimal to a fraction, it is often necessary to multiply by a greater power of 10 .

## Example A

Convert 1.0525252 . . . to its fractional form.
Step 1: Let $x=$ the repeating decimal.

$$
x=1.0525252 \ldots
$$

Step 2: Determine the repeating digit or digits.
The repeating digits are 52 .
Step 3: Multiply both sides of the original equation by the least power of 10 so that the digits that repeat align after the decimal point.

$$
\begin{aligned}
x & =1.052525252 \ldots \\
100 x & =105.252525252 \ldots
\end{aligned}
$$

Step 4: Subtract one equation from the other. The repeating digits will not be in the difference.

$$
\begin{aligned}
& 100 x=105.252525252 \ldots \\
& -x=1.052525252 \ldots \\
& \hline 99 x=104.2
\end{aligned}
$$

Step 5: Solve the resulting equation.

$$
\begin{aligned}
\frac{99 x}{99} & =\frac{104.2}{99} \\
x & =\frac{1,042}{990}=\frac{521}{495}
\end{aligned}
$$

Solution: $1.0525252 \ldots=\frac{521}{495}$



## Try These A

Convert each repeating decimal to a fraction.
a. $0.0555555 \ldots$
b. $3.00121212 \ldots$
c. $0.023333 \ldots$
2. Rational numbers can be written as fractions, decimals, or percents. How does the ability to convert between these forms help to compare the values of rational numbers?
3. For each type of rational number, explain how to compare two numbers. a. fractions
b. decimals
c. percents

## Check Your Understanding

4. Convert $1 . \overline{30}$ to a fraction.
5. Compare the number from Item 4 to the following rational numbers. Explain your reasoning.
a. $\frac{4}{3}$
b. 1.30

## Example B

List the rational numbers $2 . \overline{4}, \frac{49}{20}$, and $240 \%$ in increasing order.
Step 1: Convert the fraction and percent to a decimal.

$$
\begin{aligned}
\frac{49}{20} & =2 \frac{9}{20}=2.45 \\
240 \% & =2.4
\end{aligned}
$$

Step 2: Align the values on the decimal point and compare the digits from the right.

$$
\begin{aligned}
2 . \overline{4} & =2.444 \ldots \\
\frac{49}{20} & =2.450 \\
240 \% & =2.400
\end{aligned}
$$

Step 3: List the rational numbers in increasing order.

$$
240 \%, 2 . \overline{4}, \frac{49}{20}
$$

## Try These B

List the rational numbers in increasing order.
a. $25 \%, \frac{1}{5}, 0 . \overline{2}$
b. $\frac{1}{3}, 0.33,33.5 \%$

## My Notes

6. Fill the blank in with a $>,<$ or $=$ sign.
a. $40 \%$ $\qquad$ $\frac{4}{10}$
b. $0 . \overline{3}$ $\qquad$ $0.0 \overline{3}$
c. $\frac{2}{9}$ $\qquad$ 29\%
7. Critique the reasoning of others. Samuel enters the fraction $\frac{5}{17}$ on his calculator. The display shows 2941176471 , which are the most digits that can be displayed. He concludes that the fraction is a terminating decimal. Is Samuel correct?
8. Write the rational numbers from Example B on the previous page as fractions. Use the fractions to list the numbers in increasing order.

## LESSON 4-3 PRACTICE

9. Which number is $\frac{3}{10}$ greater than?
a. $\frac{4}{10}$
b. $28 \%$
c. $\frac{3}{5}$
10. What is a rational number? Give three examples in different forms.
11. Convert $1.063636363 \ldots$ to a fraction.
12. Convert $8.6 \overline{3}$ to a fraction.
13. How can you compare fractions with the same denominators? How can you compare fractions with the same numerators?
14. Reason quantitatively. List the rational numbers in increasing order. $0.555, \frac{5}{9}, 50.9 \%$

## ACTIVITY 4 PRACTICE

Write your answers on notebook paper.
Show your work.

## Lesson 4-1

1. George ate the portion of pizza represented by the letter A in the pizza pie shown below. What fraction is this portion equivalent to?

A. $\frac{1}{4}$
B. $\frac{1}{2}$
C. $\frac{3}{4}$
D. 4
2. How much pizza would be left after George eats his portion? Write as a fraction.
3. Convert the fractions to decimals and percents.
a. $\frac{3}{5}$
b. $\frac{2}{25}$
4. Convert the decimals to fractions and percents.
a. 0.8
b. 0.32
5. Convert the percents to decimals and fractions.
a. $20 \%$
b. $72 \%$
6. Convert the following to decimals.
a. $\frac{3}{9}$
b. $\frac{123}{999}$
C. $\frac{45}{99}$
7. Which of the following is NOT equivalent to $\frac{60}{80}$ ?
A. $\frac{3}{4}$
B. 0.6
C. $\frac{6}{8}$
D. 0.75

## Lesson 4-2

8. Which of the following does NOT describe a rational number?
A. a decimal that terminates
B. a decimal that does not terminate and does not repeat
C. a decimal that repeats
D. a fraction
9. Convert each of the following repeating decimals to a fraction.
a. $0 . \overline{6}$
b. $0 . \overline{12}$
10. Write three examples of each type of rational number.
a. fractions that are repeating decimals
b. fractions that are terminating decimals
11. Is 0.232323 a repeating decimal? Why or why not?
12. Are all whole numbers also rational numbers? Justify your answer.
13. Consider the two repeating decimals shown below.

$$
\begin{aligned}
& 0.31313131 \ldots \\
& 0.02020202 \ldots
\end{aligned}
$$

a. By simply looking at the decimal representations, what is the sum of these two rational numbers? Write as a decimal and as a fraction.
b. Verify your answer by finding fractions that represent each decimal. Add the fractions.
c. Write the difference of the two numbers as a repeating decimal.
14. Which number represents a repeating decimal?
$\frac{1}{6}, 25 \%, 0.5020202$
15. Which repeating decimal is equivalent to the fraction $\frac{1}{9}$ ?

## Lesson 4-3

16. Arrange the following rational numbers from least to greatest.
a. $\frac{1}{6}, 16 \%, 0 . \overline{16}$
b. $\frac{6}{11}, 0 . \overline{5}, 5.9$
17. Fill the blank in with a$\rangle,\langle$ or $=$ sign.
a. $45 \%$ $\qquad$
b. $0 . \overline{38}$ $\qquad$ $0.0 \overline{32}$
c. $\frac{21}{9}$ $2 . \overline{33}$
18. Which number is the largest of the group? $70 \%, \frac{1}{2}, 15 \%$
19. Convert $1.0353535 \ldots$ to a fraction.
20. Convert $0.022222 \ldots$ to a fraction.
21. List the fractions below in increasing order.

Describe the method you used. $\frac{1}{2}, \frac{5}{6}, \frac{7}{10}, \frac{2}{3}$
22. Write a convincing argument that the repeating decimal $0.999 \ldots$ is equal to 1 .
23. Tanya says that 1 out of every 3 students at her school plays a sport. William says the actual number is $33 \%$ of the students. Who thinks a greater number of students plays a sport than the other does?
24. Write a fraction that is between the rational numbers 0.3 and $\frac{1}{3}$. Explain what you did.

## MATHEMATICAL PRACTICES Reason Abstractly and Quantitatively

25. Rational numbers are used every day. Write three instances in which you use each of the representations of a rational number-fraction, decimal, and percent-in your daily activities. Does a certain representation appear to be used most often?

## Rational and Irrational Numbers

## Where Am I?

## Lesson 5-1 Estimating Irrational Numbers

## Learning Targets:

- Differentiate between rational and irrational numbers.
- Approximate an irrational number in terms of a rational number.

SUGGESTED LEARNING STRATEGIES: Vocabulary Organizer, Create Representations, Look for a Pattern, Critique Reasoning, Simplify the Problem

Many early mathematicians believed that all numbers were rational; that is, they could be written as a quotient of two integers. However, as early as the seventh century $\operatorname{BCE}$, mathematicians from India became aware of numbers that could not be expressed as the quotient of two integers. Eventually, it became accepted that the square roots of most real numbers could not be expressed rationally. These numbers were considered irrational.

1. Some examples of irrational numbers are $\sqrt{2}$ or $\pi$. Look at the screen shots in the My Notes sections of $\sqrt{2}$ and $\pi$.
a. Based on what you know about $\pi$, is the screen shot the entire value of $\pi$ ?
b. Based on your answer to part a, is the screen shot the entire value of $\sqrt{2}$ ?
c. What is the difference between the decimal forms of rational numbers and the decimal forms of irrational numbers?

## My Notes

## TECHNOLOGY TIP

While some decimals continue on forever, calculator screens will round the decimal when there is no space left on the screen.

## V(2)

1.414213562
$\pi$
3.141592654

## MATH TERMS

Irrational numbers are real numbers that cannot be written as a ratio or fraction.

Rational numbers are real numbers that can be written as a ratio or fraction.


## My Notes

|  |  |  |  |  |  |  |  | My Notes |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |

2. Use this number line and the method described in parts a-f below to determine the approximate square root of 18 .

a. Which perfect square is less than 18 but closest to 18 ? Mark this integer on the number line.
b. Which perfect square is greater than 18 but closest to 18 ? Mark this integer on the number line.
c. What is the square root of the first integer you marked? Write the square root above the integer on the number line.
d. What is the square root of the second integer you marked? Write the square root above the integer on the number line.
e. Put an $X$ on 18. The $X$ should be between the two perfect squares. Is the $X$ closer to the smaller or the larger perfect square?
f. Attend to precision. Above the $X$ you put on 18, write a decimal number to the nearest tenth that you think is the square root of 18 .
g. Check your estimate by squaring your approximation to see how close you are.
h. Using the number line below, fill in the two perfect squares that you determined were below and above 18 on the top of the number line. Write the whole numbers that represent those perfect squares on the bottom of the number line. Put your estimation of $\sqrt{18}$ on the number line.

3. Using the method you used in Item 2, estimate the value of the number.

## My Notes

a. $\sqrt{42}$

b. $\sqrt{98}$

4. Use appropriate tools strategically. Use a calculator to determine the values of the square roots you estimated using a number line in Items 2 and 3 . Round the answer to the nearest tenth. How close were your estimates?
a. $\sqrt{18}$
b. $\sqrt{42}$
c. $\sqrt{98}$
5. Using the method you used in Items 2 and 3, estimate the value of the cube root. $\sqrt[3]{20}$

6. Explain why $\sqrt[3]{64}$ is rational and $\sqrt[3]{12}$ is not.

## Check Your Understanding

Name the irrational number in each set.
7. $\left\{\frac{17}{2}, \sqrt{8}, \sqrt[3]{8}\right\}$
8. $\left\{-0.578 \overline{9}, 6^{3}, \sqrt{65}\right\}$
9. Estimate the following square roots to the tenths place without using a calculator.
a. $\sqrt{10}$
b. $\sqrt{28}$
c. $\sqrt{73}$
d. Place the above square roots on their approximate location on the number line.


## LESSON 5-1 PRACTICE

10. Name the irrational number in each set:
a. $\left\{12.89, \sqrt{12}, \frac{12}{144}\right\}$
b. $\{\sqrt[3]{16}, \quad \sqrt{16}$,
$2.5 \overline{6}\}$
11. Estimate the following square and cube roots to the tenths place without using a calculator.
a. $\sqrt{92}$
b. $\sqrt{139}$
c. $\sqrt{68}$
d. $\sqrt{19}$
e. $\sqrt[3]{24}$
f. $\sqrt[3]{56}$
12. Place the approximate value of $\sqrt{26}$ on the number line below.

13. Place the approximate value of $\sqrt{74}$ on the number line below.

14. In previous math courses, you have studied the sets of whole numbers and integers.
a. Create a Venn diagram showing the relationship of the set of whole numbers and the set of integers.
b. How would you add the set of rational numbers to the Venn diagram you made in part a?
c. Would the set of irrational numbers have any overlapping regions with the Venn diagram you made in part a?
15. Critique the reasoning of others. Dale says that every number can be represented as a decimal. Is Dale correct? Explain.

## Learning Targets:

- Approximate an irrational number in terms of a rational number.
- Compare and order irrational and rational numbers.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Think Aloud, Construct an Argument, Simplify the Problem

Approximate an irrational number in terms of a rational number to understand its value.

1. Using a calculator, $\sqrt{3}$ is shown to be 1.7320508 ...
a. Explain where $\sqrt{3}$ would be in relation to the estimates of 1.7 and 1.8 .
b. Explain the connection between these estimates and the actual value with rational and irrational numbers.

## Example A

Find examples of a rational number and an irrational number between 4.8 and 4.9.

Step 1: A rational number can be expressed as a ratio of integers or a decimal that terminates or repeats.
4.859 is between 4.8 and 4.9
4.859 terminates and can be expressed as the ratio: $\frac{4,859}{1,000}$.

Step 2: An irrational number cannot be expressed as a ratio of integers and, when expressed as a decimal, does not terminate or repeat. $4.8598979486 \ldots$ is between 4.8 and 4.9
4.8598979486 . . . does not terminate and does not repeat.

Solution: A rational number between 4.8 and 4.9 is 4.859 . An irrational number between 4.8 and 4.9 is $4.8598979486 \ldots$

## My Notes

|  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| , |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |
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## MATH TIP

There are infinitely many examples of rational and irrational numbers between any two numbers.


## Try These A

Give an example of a rational and an irrational number that is between the following numbers.
a. 2 and 2.1
b. 5.3 and 5.4
c. 10.6 and 10.7
2. Explain why it is always possible to find another number (rational or irrational) between any two numbers.

## Example B

Order the following numbers from least to greatest.

$$
\begin{array}{llll}
\sqrt{71}, & 9.3, & \sqrt{84}, & 8.1
\end{array}
$$

Step 1: Approximate values for the irrational numbers in terms of rational numbers.
$\sqrt{71}$ is between $\sqrt{64}=8$ and $\sqrt{81}=9$.
$\sqrt{71} \approx 8.4$
$\sqrt{84}$ is between $\sqrt{81}=9$ and $\sqrt{100}=10$.
$\sqrt{84} \approx 9.2$
Step 2: Using the approximated values for the irrational numbers, order the numbers from least to greatest.

$$
\text { 8.1, 8.4, 9.2, } 9.3
$$

Solution: The numbers in order from least to greatest are

$$
\begin{array}{llll}
8.1, & \sqrt{71}, & \sqrt{84}, \quad 9.3 .
\end{array}
$$

## Try These B

Order the following numbers from least to greatest.
a. $\sqrt{16}, \sqrt{14}, 4.1,3.6$
b. $\sqrt{150}, 12,11.8, \sqrt{135}$
c. $\pi, \sqrt{6}, 3.4, \sqrt{14}$
3. Complete the comparisons using $>$ or $<$. Explain your reasoning.
a. $\sqrt{10} \bigcirc 2.5$
b. $7 \bigcirc \sqrt{50}$
c. $\sqrt[3]{8} \bigcirc \sqrt[3]{9}$
d. $\sqrt{110} \bigcirc \sqrt{66}$
4. Reason quantitatively. In Item 3d above, explain how you could have completed the comparison without using rational number approximations for $\sqrt{110}$ and $\sqrt{66}$.

## Check Your Understanding

5. Give an example of a rational and an irrational number that is between the following numbers:
a. 12 and 12.1
b. 3.4 and 3.5
6. Complete the comparison with $>$ or $<$ :

$$
11.2 \bigcirc \sqrt{120}
$$

7. A right trapezoid has sides that measure 2 units, 6 units, $\sqrt{13}$ units, and 3 units. Order the lengths of the sides from least to greatest.

## My Notes

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## LESSON 5-2 PRACTICE

8. Give an example of a rational and an irrational number between the following numbers:
a. 8 and 8.1
b. 9.6 and 9.7
c. 10.2 and 10.3

Order the numbers from greatest to least.
9. $\sqrt[3]{18}, 2.4, \pi, 2.7$
10. $5.6, \sqrt{25}, \sqrt[3]{140}, 6.2$
11. $\sqrt[3]{343}, 7.4, \sqrt{52}, 8.2$
12. The following numbers were placed in order from greatest to least. Complete the list with irrational numbers.
6.8, $\qquad$ , 6.2, $\qquad$ , 5.1
13. Construct viable arguments. Describe the vastness of the sets of rational and irrational numbers. Why is it always possible to find a rational or irrational number between any two numbers?

## ACTIVITY 5 PRACTICE

Write your answers on notebook paper.
Show your work.

## Lesson 5-1

Name the irrational numbers in each set.

1. $\{8.4, \sqrt{12}, \pi, \sqrt{64}, \sqrt{65}\}$
2. $\left\{\frac{5}{6}, 2.1, \sqrt{45}, \sqrt{78}, \frac{2}{3}\right\}$
3. $\left\{\sqrt{25}, \sqrt{36}, \sqrt{17}, \frac{1}{8}, 3\right\}$
4. $\left\{1.24562 \ldots, 5.9, \sqrt{82}, \sqrt{20}, \frac{9}{16}\right\}$

Determine a reasonable estimate to the tenths place.
5. $\sqrt{18}$
6. $\sqrt{130}$
7. $\sqrt{2}$
8. $\sqrt{86}$
9. $\sqrt[3]{117}$
10. $\sqrt[3]{24}$
11. $\sqrt[3]{41}$
12. $\sqrt[3]{56}$
13. Which of the following square roots would not be between 5 and 6?
A. $\sqrt{27}$
B. $\sqrt{32}$
C. $\sqrt{37}$
D. $\sqrt{29}$
14. Which of the following square roots would not be between 8 and 9 ?
A. $\sqrt{74}$
B. $\sqrt{62}$
C. $\sqrt{80}$
D. $\sqrt{77}$
15. Describe the relationship between irrational numbers and rational numbers.
16. Cody is asked to find a cube root that is found between 4.2 and 4.3. He provides the number, $\sqrt[3]{75}$. Is he correct? Explain.
17. Ling marked the approximate value of $\sqrt{32}$ on the number line as shown. Did Ling mark the value correctly? Explain your reasoning.

18. Which is not an irrational number?
A. $\sqrt{15}$
B. $\sqrt{16}$
C. $\sqrt{17}$
D. $\sqrt{18}$

## Lesson 5-2

Determine a rational and an irrational number between each of the following pairs of numbers.
19. 12.1 and 12.2
20. 4.5 and 4.6
21. 9 and 9.1
22. 14.2 and 14.3
23. 15 and 15.1

Order the following from least to greatest.
24. $\sqrt{27}, 5.5, \sqrt{24}, 5$
25. $\sqrt[3]{19}, \pi, 2.65, \sqrt{4}$
26. $6.8, \sqrt{49}, 7.1,6.8556546 \ldots$
27. $\sqrt[3]{67}, \sqrt{16}, 4.2,4.15$
28. $12 \frac{1}{3}, \sqrt{144}, 12.99,3^{2}$
29. $\sqrt[3]{88}, \sqrt[3]{8}, \sqrt[3]{108}, \sqrt[3]{18}$

Compare using $>,<$, or $=$.
30. $\sqrt[3]{729} \bigcirc \sqrt[3]{729}$
31. $13.225 \bigcirc \sqrt{170}$
32. $\sqrt[3]{5} \bigcirc \sqrt{5}$
33. $\sqrt{72} \bigcirc 8$
34. Roxie thinks that the irrational number $6.00289467 \ldots$ is less than the rational number 6. Do you agree with Roxie? Explain your reasoning.
35. Which of the following irrational numbers is the greatest?
A. $\frac{\pi}{2}$
B. $2 \pi$
C. $\pi^{2}$
D. $\frac{3 \pi}{2}$
36. Which of the following is an irrational number between 8.8 and 8.9?
A. $8.089703487 \ldots$
B. $8.888888888 \ldots$
C. $8.912984065 \ldots$
D. $8.871985703 \ldots$

## MATHEMATICAL PRACTICES

## Look for and Make Use of Structure

37. Consider the following numbers:

$$
\sqrt[3]{8}, \sqrt[3]{9}, \sqrt[3]{27}, \sqrt[3]{42}, \sqrt[3]{110}, \sqrt[3]{125}
$$

a. Which of these are rational?
b. Which of these are irrational?
c. Write a conjecture explaining why some of these cube roots are rational and why some are irrational.
d. Support your conjecture with a different example of a rational cube root and an irrational cube root.

# Representing Rational and Irrational Numbers 

Natural disasters can happen anywhere in the world. Examples of natural disasters include tornados, earthquakes, hurricanes, and tsunamis. Two of the most well-known natural disasters are Hurricane Katrina (2005), which hit New Orleans, Louisiana, and Japan's tsunami (2011).

1. After Hurricane Katrina, $\frac{8}{10}$ of the city of New Orleans was flooded.

Represent this number in the following ways:
a. decimal
b. visual representation
c. percent
2. Only $58 \%$ of the people in the coastal areas of Japan took the warning system seriously that a tsunami was coming and evacuated the area.
Represent this number in the following ways:
a. decimal
b. fraction

The area of land that is affected by a natural disaster can vary greatly. While the destruction areas of these disasters were not perfect squares, thinking about the areas as squares can give you a good visualization of how much area was affected.
3. The total square miles affected by the natural disasters are given below. Find the side length of the area affected if it was a square.
a. Hurricane Katrina: 90,000 square miles
b. Japan tsunami: 216 square miles
4. Given that a storm has a destruction area in the shape of a square, give the total area affected if the side lengths of the square were:
a. 315.2 miles
b. $30 \frac{1}{2}$ kilometers

When natural disasters occur, organizations such as the Red Cross help by sending in crates of supplies to those who are affected. The crates contain first aid, food, drinks, and other supplies.
5. Explain how, given an edge length of a cubical crate, the volume of the crate could be determined. Provide an example with your explanation.
6. Given the following volumes of the cubical crates, determine the edge length. Explain how you found the edges.
a. $8 \mathrm{ft}^{3}$
b. $27 \mathrm{ft}^{3}$

| Scoring Guide | Exemplary | Proficient | Emerging | Incomplete |
| :---: | :---: | :---: | :---: | :---: |
|  | The solution demonstrates these characteristics: |  |  |  |
| Mathematics Knowledge and Thinking (Items 1a-c, 2a-b, 3a-b, $4 a-b, 5,6 a-b)$ | - Clear and accurate understanding of converting between fractions, decimals, and percent. <br> - Effective understanding of squares and square roots; cubes and cube roots. | - Converting between fractions, decimals, and percent. <br> - Understanding of squares and square roots; cubes and cube roots. | - Errors in converting between fractions, decimals, and percent. <br> - Some errors in working with squares and square roots; cubes and cube roots. | - Incorrect or incomplete converting between fractions, decimals, and percent. <br> - Little or no understanding of squares and square roots; cubes and cube roots. |
| Problem Solving (Items 3a-b, 4a-b, 6a-b) | - An appropriate and efficient strategy that results in a correct answer. | - A strategy that may include unnecessary steps but results in a correct answer. | - A strategy that results in some incorrect answers. | - No clear strategy when solving problems. |
| Mathematical Modeling/ Representations (Items 1a-c, 2a-b, 3a-b, $4 a-b, 5,6 a-b)$ | - Clear and accurate understanding of representing a rational number as a fraction, decimal, or percent. <br> - Clearly and accurately relating a volume to a cube, an area to a square, and the root to a side length. | - Representing a rational number as a fraction, decimal, or percent. <br> - Relating a volume to a cube, an area to a square, and the root to a side length. | - Errors in representing a rational number as a fraction, decimal, or percent. <br> - Errors in relating volume to a cube, area to a square, and the root to a side length. | - Inaccurately representing a rational number as a fraction, decimal, or percent. <br> - Little or no understanding of relating volume to a cube, area to a square, and the root to a side length. |
| Reasoning and Communication (Items 5, 6a-b) | - Precise explanation of the difference between rational and irrational numbers. <br> - Clear and precise explanation of the relationship between volume and edge length of a cube. | - Adequate explanation of the difference between rational and irrational numbers. <br> - Adequate explanation of the relationship between volume and edge length of a cube. | - A misleading or confusing explanation of the difference between rational and irrational numbers. <br> - A confusing explanation of the relationship between volume and edge length of a cube. | - An incomplete or inaccurate explanation of the difference between rational and irrational numbers. <br> - An incomplete or inaccurate explanation of the relationship between volume and edge length of a cube. |

## Properties of Exponents

## That's a Lot of Cats <br> Lesson 6-1 Multiplying and Dividing with Exponents

## Learning Targets:

- Understand and apply properties of integer exponents.
- Simplify multiplication expressions with integer exponents.
- Simplify division expressions with integer exponents.

SUGGESTED LEARNING STRATEGIES: Marking the Text,
Paraphrasing, Look for a Pattern, Critique Reasoning, Work Backward
As I was going to St. Ives,
I met a man with seven wives.
Every wife had seven sacks,
And every sack had seven cats.
Every cat had seven kittens.
Kittens, cats, sacks, wives,
How many were going to St. Ives?
In addition to being an $18^{\text {th }}$ century translation of what the Guinness Book of World Records claims is the oldest mathematical riddle in history, this riddle can be used to explain how exponents work.

Use this table to determine the number of kittens in the riddle-in expanded form, exponential form, and standard form. Expanded form is expressing the number in terms of multiplication, exponential form is expressing the number with a base and exponent, and standard form is the product.

|  | Number Written in. . . |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Expanded <br> Form | Exponential <br> Form | Standard <br> Form | Base |
| Wives | 7 | $7^{1}$ | 7 | 7 |
| Sacks | $7 \bullet 7$ | $7^{2}$ | 49 | 7 |
| Cats | $7 \bullet 7 \bullet 7$ | $7^{3}$ | 343 | 7 |
| Kittens |  |  |  |  |

1. Describe any patterns you observe in the table.
2. Use the patterns you observe to complete the last row of the table.

## My Notes

## CONNECT TO HISTORY

Problem 79 on the Rhind Mathematical Papyrus (c. 1650 BCE) contains the algorithm that is said to be the basis for the mathematics in this riddle.

3. Make sense of problems. Suppose each kitten had seven stripes. Write an expression to determine the total number of stripes. Write your expression in the following forms.
a. Expanded form:
b. Exponential form:
c. Standard form:

When you multiply two exponential expressions with the same base, add the exponents.

## Example A

Simplify $7 \cdot 7 \cdot 7 \bullet 7 \bullet 7 \bullet 7 \bullet 7$ in various ways. Then compare your results.
Step 1: Use the associative property of multiplication to rewrite the expanded form $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$ as

$$
(7 \bullet 7 \cdot 7)(7 \bullet 7 \bullet 7 \bullet 7) \text { or }(7 \bullet 7)(7 \bullet 7 \bullet 7 \bullet 7 \bullet 7)
$$

Step 2: Rewrite each expression in exponential form.

$$
\begin{aligned}
& (7 \bullet 7 \cdot 7)(7 \cdot 7 \cdot 7 \cdot 7)=7^{3} \cdot 7^{4} \\
& (7 \cdot 7)(7 \cdot 7 \cdot 7 \cdot 7 \cdot 7)=7^{2} \cdot 7^{5}
\end{aligned}
$$

Step 3: Simplify each power. Notice that the exponents are being added as the bases are multiplied.

$$
\begin{aligned}
& 7^{3} \cdot 7^{4}=7^{3+4}=7^{7}=823,543 \\
& 7^{2} \cdot 7^{5}=7^{2+5}=7^{7}=823,543
\end{aligned}
$$

Solution: Each of these expanded forms simplifies to the same product, 823,543 , and same exponential form, $7^{7}$.

## Try These A

Consider the product $(x \bullet x)(x \bullet x \bullet x)$.
a. Rewrite the product using exponents.
b. Simplify the expression, and write the answer in exponential form.

## Check Your Understanding

Simplify the expressions.
4. $3^{9} \cdot 3^{3}$
5. $a^{7} \cdot a^{4}$

## Example B

Simplify $\frac{7^{4}}{7^{3}}$ ．
Step 1：Write $\frac{7^{4}}{7^{3}}$ in expanded form．

$$
\frac{7 \cdot 7 \cdot 7 \cdot 7}{7 \cdot 7 \cdot 7}
$$

Step 2：Simplify by dividing common factors．

$$
\frac{\text { 又 } \cdot \text { 又 } \cdot \text { 又 } \cdot 7}{\text { 又 } \cdot 又 \cdot \text { 又 }}=7
$$

Solution：$\frac{7^{4}}{7^{3}}=7$

## Try These B

Consider the expression $\frac{x^{8}}{x^{5}}$ ．
a．Rewrite the expression with the numerator and denominator in
b．Simplify the expression you wrote in part a．
c．Describe how you could simplify the original expression without writing it in expanded form．

6．What is the exponent of the solution to Example B？

7．How does the exponent of the solution to Example B relate to the exponents of the original expression？

8．Reason quantitatively．What pattern do you notice about the exponents when dividing two powers with the same base？

## My Notes


#### Abstract

expanded form．


## Check Your Understanding

Simplify the expressions.
9. $\frac{12^{15}}{12^{10}}$
10. $\frac{r^{9}}{r}$

## LESSON 6-1 PRACTICE

11. Re-read the riddle at the beginning of the lesson. This riddle has been the subject of great debate over the years as the riddle is said to have multiple answers. Determine which of the following could be considered a reasonable answer to the riddle. Justify your reasoning.
A. 1
B. 30
C. 2,403
D. 2,802
12. Simplify each expression:
a. $t^{2} \cdot t^{5}$
b. $\frac{8^{8}}{8^{6}}$
13. Write a rule for multiplying terms with exponents that have the same base in your own words.
14. Write a rule for dividing terms with exponents that have the same base in your own words.
15. Critique the reasoning of others. Sebastian and Georgia are examining the expression $\frac{4^{7}}{4^{5}}$. Sebastian says the answer is $1^{2}$. Georgia knows the answer is $4^{2}$. Help Georgia explain to Sebastian what he did incorrectly.

## Learning Targets:

- Understand and apply properties of integer exponents.
- Simplify expressions with negative exponents.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm,
Paraphrasing, Look for a Pattern, Visualization

## Example A

Consider the expression $\frac{4^{3}}{4^{8}}$. Simplify the expression by dividing and by writing the numerator and denominator in expanded form. Compare the results.

Step 1: Divide by subtracting the exponents.

$$
\frac{4^{3}}{4^{8}}=4^{3-8}=4^{-5}
$$

Step 2: Write the numerator and denominator in expanded form and simplify.

$$
\frac{4^{3}}{4^{8}}=\frac{A \cdot A \cdot A}{A \cdot A \cdot A \cdot 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}=\frac{1}{4^{5}}
$$

Step 3: Compare the two results.

$$
4^{-5}=\frac{1}{4^{5}}
$$

Solution: The two strategies used to divide $4^{3}$ by $4^{8}$ yielded the same answer in two different forms, $4^{-5}$ and $\frac{1}{4^{5}}$.

## Try These A

Consider the expression $\frac{x^{3}}{x^{5}}$.
a. Rewrite the numerator and denominator in expanded form and simplify.
b. Simplify the expression by dividing (subtract the exponents).
c. Compare your results for parts $a$ and $b$.

## MATH TERMS

The reciprocal of a number is its multiplicative inverse. For example, the reciprocal of 6 is $\frac{1}{6}$.

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When an exponent has a negative value, it actually means to divide 1 by the base that number of times. An expression with a negative exponent can be written as an equivalent expression with a positive exponent by writing the reciprocal.

1. Simplify $\frac{x^{4}}{x^{2}}$.
2. Simplify $\frac{x^{2}}{x^{4}}$.
3. Reason quantitatively. Compare and contrast your answers to Items 1 and 2.

## Check Your Understanding

Simplify the following expressions. Compare the answers.
4. $\frac{2^{9}}{2^{5}}$ and $\frac{2^{5}}{2^{9}}$
5. $\frac{a^{6}}{a^{3}}$ and $\frac{a^{3}}{a^{6}}$

## Example B

Write $6^{-2}$ as an equivalent expression without a negative exponent.
Write the reciprocal of $6^{2}$ to make an equivalent expression with a positive exponent.
Solution: $6^{-2}=\frac{1}{6^{2}}$.

## Try These B

Rewrite these expressions without a negative exponent.
a. $a^{-4}$
b. $8^{-3}$
c. $\frac{1}{3^{-2}}$

## Example C

Simplify the expression $\frac{r^{8}}{r^{15}}$. Your final answer should not contain a negative exponent.
Step 1: Divide $r^{8}$ by $r^{15}$ by subtracting the exponents.

$$
r^{8-15}=r^{-7}
$$

Step 2: Rewrite $r^{-7}$ without a negative exponent by writing the reciprocal.

$$
r^{-7}=\frac{1}{r^{7}}
$$

Solution: $\frac{r^{8}}{r^{15}}$ equals $\frac{1}{r^{7}}$.

## Try These C

Attend to precision. Simplify these expressions. Your final answer should not contain a negative exponent.
a. $\frac{y^{4}}{y^{8}}$
b. $\frac{10^{12}}{10^{15}}$
c. $\frac{4^{-2} 5^{7}}{4^{3} 5^{-2}}$

## Check Your Understanding

Simplify the follow expressions.
6. $x^{-7}$
7. $\frac{5^{6}}{5^{9}}$
8. $\frac{x^{5} y^{-2}}{x^{-4} y^{4}}$

## LESSON 6-2 PRACTICE

Simplify the expressions in Items 9-12. Write final answers in exponential form without negative exponents.
9. $9^{-10}$
10. $\frac{k^{-5}}{k^{2}}$
11. $3^{3} \cdot 3^{-6}$
12. $\frac{2^{4} \cdot 10^{-6}}{2^{2} \cdot 10^{-2}}$
13. Reason abstractly. Write in your own words how to write an expression with a negative exponent as an equivalent expression with a positive exponent.

## My Notes




## Learning Targets:

- Understand and apply properties of integer exponents.
- Simplify expressions with zero as the exponent.
- Simplify expressions with exponents raised to a power.

SUGGESTED LEARNING STRATEGIES: Paraphrasing, Look for a
Pattern, Graphic Organizer, Construct an Argument
Although the riddle As I was going to St. Ives has only one narrator, the number 1 can also be written as a base of 7 . To see how this works, you can examine several ways to express the number 1.

## Example A

Simplify $\frac{7}{7}$ using exponents and explain the result.
Step 1: Rewrite the fraction by expressing the numerator and denominator in exponential form.

$$
\frac{7}{7}=\frac{7^{1}}{7^{1}}
$$

Step 2: Simplify the expression by dividing (by subtracting the exponents).

$$
\frac{7^{1}}{7^{1}}=7^{1-1}=7^{0}
$$

Solution: Since $\frac{7}{7}$ is equal to 1 and also equal to $7^{0}$, it follows that 7 (or any number) raised to the power of 0 is equal to 1 .

## Try These A

Express regularity in repeated reasoning. Simplify these expressions.
a. $y^{0}$
b. $9^{0}$
c. $125^{0}$

## Check Your Understanding

1. $83,567^{0}$
2. $\frac{b^{4}}{b^{4}}$

## ACTIVITY 6

continuea

Recall from the riddle that the man had 7 wives, each wife had 7 sacks, each sack had 7 cats, and each cat had 7 kittens. Suppose that each kitten had 7 stripes. Now assume each stripe on each kitten contains seven spots. The situation is becoming more complicated, and the need for using exponents has grown . . . exponentially. The number of spots can be written as a power raised to another power.
When an exponential expression is raised to a power, multiply the exponents to simplify.

## Example B

Show that $\left(7^{3}\right)^{2}=7^{3 \times 2}$
Step 1: $\quad\left(7^{3}\right)^{2}=\left(7^{3}\right) \cdot\left(7^{3}\right)$
Step 2: $\quad 7^{3} \cdot 7^{3}=(7 \cdot 7 \cdot 7) \cdot(7 \cdot 7 \cdot 7)=7^{6}=7^{3 \times 2}$
Solution: $\left(7^{3}\right)^{2}=7^{3 \times 2}$

## Try These B

Simplify these expressions by multiplying the exponents. Write your answer in exponential form.
a. $\left(6^{3}\right)^{4}$
b. $\left(n^{7}\right)^{5}$
c. $\left(12^{6}\right)^{3}$

## My Notes

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## MATH TIP

Create an organized summary of the properties used to simplify and evaluate expressions with exponents.


## LESSON 6-3 PRACTICE

5. In your own words, write the outcome of raising any base to the power of zero.

Simplify each expression in Items 6-8. Write your answer in exponential form.
6. $9^{0} \cdot\left(6^{12}\right)^{2}$
7. $\left(17^{9}\right)^{3}$
8. $\left(w^{4}\right)^{5}$
9. Construct viable arguments. Explain how the product $10,324^{0} \cdot 8,576^{\circ}$ can be done using mental math.

## ACTIVITY 6 PRACTICE

Write your answers on notebook paper.
Show your work.

## Lesson 6-1

Simplify. Express your answer in exponential form.

1. $3^{5} \cdot 3^{4}$
2. $5^{6} \cdot 5^{2}$
3. $x^{16} \cdot x^{1}$
4. $\frac{4^{3}}{4^{2}}$
5. $\frac{10^{8}}{10^{6}}$
6. $\frac{x^{11}}{x^{4}}$
7. Which of the following is $12^{6} \cdot 12^{4}$ simplified in exponential form?
A. $12^{2}$
B. $12^{10}$
C. $12^{12}$
D. $12^{24}$
8. Which of the following is $\frac{a^{8}}{a^{3}}$ simplified in
exponential form?
A. $a^{-5}$
B. $a^{5}$
C. $a^{11}$
D. $a^{24}$
9. Kwon multiplied $5^{5} \cdot 5^{4}$ and found the product to be $5^{1}$. Do you agree with Kwon? Explain your reasoning.

## Lesson 6-2

Simplify. Express your answer in exponential form.
10. $\frac{2^{4}}{2^{7}}$
11. $\frac{r^{7}}{r^{12}}$
12. $3^{-2}$
13. $z^{-5}$
14. $8^{-8} \cdot 8^{6}$
15. $\frac{x^{8} y^{2}}{x^{2} y^{8}}$
16. $\frac{5^{-3}}{5^{2}}$
17. Which of the following is equivalent to $4^{-10}$ ?
A. $\frac{1}{10}$
B. $\frac{4}{10}$
C. $\frac{1}{4^{-10}}$
D. $\frac{1}{4^{10}}$
18. Which of the following is equivalent to $\frac{c^{-3} d^{5}}{c^{2} d^{-1}}$ ?
A. $c^{1} d^{4}$
B. $c^{5} d^{6}$
C. $\frac{c^{5}}{d^{6}}$
D. $\frac{d^{6}}{c^{5}}$

## MATHEMATICAL PRACTICES Look For and Make Use of Structure

28. In the table below, summarize the rules for exponents you discovered in this activity.

| Situation | Verbal <br> Description | Numeric <br> Example |
| :--- | :--- | :--- |
| Multiplying <br> powers with <br> the same <br> base |  |  |
| Dividing <br> powers with <br> the same <br> base |  |  |
| Raising a <br> term to an <br> exponent of <br> zero |  |  |
| Raising a <br> power to <br> another <br> exponent |  |  |

A. The exponents are multiplied.
B. The exponents are subtracted.
C. The exponents are divided.
D. The exponents are added.

## Lesson 6-3

Simplify. Express your answer in exponential form.
19. $5^{0}$
20. $1,734^{0}$
21. $\left(b^{3}\right)^{9}$
22. $\left(4^{4}\right)^{5}$
23. $\left(6^{3}\right)^{4}$
24. $\left(99^{9}\right)^{0}$
25. Which of the following is $\left(4^{x}\right)^{y}$ simplified in exponential form?
A. $4^{x y}$
B. $4^{x-y}$
C. $4^{x+y}$
D. $4^{\frac{x}{y}}$
26. Victor was asked to simplify this expression in exponential form: $8^{0}+12^{0}$. He says the answer is 2. Do you agree with Victor? Explain your reasoning.
27. When raising a power to another power, how are the exponents simplified?

## Scientific Notation

## A Traveler's Tale <br> Lesson 7-1 Scientific Notation vs. Standard Form

## Learning Targets:

- Express numbers in scientific notation.
- Convert numbers in scientific notation to standard form.
- Use scientific notation to write estimates of quantities.

Suggested Learning Strategies: Interactive Word Wall, Graphic Organizer, Marking the Text, Look for a Pattern, Work Backward

The story Gulliver's Travels describes the adventures of Lemuel Gulliver, a ship's doctor, who becomes stranded in many strange places. In Lilliput, Gulliver finds that he is a giant compared to the people and the world around him. During another voyage Gulliver is stranded in another land,
Brobdingnag, where he is as small to the inhabitants as the Lilliputians were to him.

The story never says how tall Gulliver is, but it does tell how the heights of the Lilliputian people and the people from Brobdingnag compare to Gulliver's height. The many descriptions of size in this tale provide ways to explore the magnitude, or size, of numbers. Powers of 10 will be used to express these very large and very small numbers. For this activity assume that Gulliver is 5 feet tall.

With a partner or in your group, discuss the story of Gulliver. By asking questions and making notes, confirm that you understand who Gulliver and the Lilliputians are, as well as what you know about their heights.

## Example A

A person from Brobdingnag is 10 times as tall as Gulliver. Determine the height of the person.
Step 1: Gulliver is 5 feet tall, and the person from Brobdingnag is 10 times as tall, so multiply to find the height of the person from Brobdingnag:

$$
5 \times 10=50 \text { feet tall }
$$

Step 2: Write an expression using Gulliver's height and a power of 10 to represent the height of 50 feet:

$$
5 \times 10^{1}=50
$$

Solution: A person from Brobdingnag is 50 feet tall, or in terms of Gulliver's height, the person from Brobdingnag is $5 \times 10^{1}$ feet tall.

## Try These A

a. If Adadahy is 10 times as tall as a person from Brobdingnag, how tall is she?
b. Write an expression using Gulliver's height and a power of 10 to represent the height of the person in part a.

## CONNECT TO LITERATURE

Gulliver's Travels was a book written in 1726 by Jonathan Swift. The book has been a popular read since the day it was published and has never been out of print. The book is about Lemuel Gulliver and his voyages.



## MATH TERMS

Scientific notation is a way to write a number as a product of the number, $a$, and $10^{n}$, when $1 \leq a<10$ and $n$ is an integer.

$$
7 \times 10^{8}
$$

Standard form is a way to write a number using a digit for each place.

700,000,000

## MATH TIP

Integers consist of all natural numbers, their opposites, and 0 : $\{\ldots,-3,-2,-1,0,1,2,3, \ldots\}$.


Notice the expressions you have written show the product of a factor and a power of 10 with an exponent that is a positive integer.

## Example B

Find the value of the expression $6 \times 10^{4}$. This expression is the product of a factor and a power of 10 with an exponent that is a positive integer:
Step 1: Simplify $10^{4}$.

$$
10^{4}=10 \cdot 10 \cdot 10 \cdot 10=10,000
$$

Step 2: Multiply by 6 .

$$
6 \times 10,000=60,000
$$

Solution: $6 \times 10^{4}=60,000$

## Try These B

Find the value of these expressions.
a. $15 \times 10^{3}$
b. $2 \times 10^{6}$
c. $43.2 \times 10^{3}$
d. Describe any patterns you noticed in evaluating these expressions.

After Gulliver's boat capsizes in a violent storm, he swims ashore to Lilliput and falls asleep. When he wakes, Gulliver finds he has been tied to the ground and can only look up into the bright sun. The sun has a diameter of $1.39 \times 10^{9} \mathrm{~m}$ and a mass of $2.0 \times 10^{20} \mathrm{~kg}$.

The measurements, $1.39 \times 10^{9} \mathrm{~m}$ and $2.0 \times 10^{20} \mathrm{~kg}$, are written in scientific notation. A number written in scientific notation is the product of a factor, $a$, and a power of 10 with an exponent that is an integer, $n$. It is expressed in the form $a \times 10^{n}$, where $1 \leq a<10$ and $n$ is an integer. A number is written in standard form when it is expressed in digits only. Scientific notation is especially helpful when working with numbers with very large and very small magnitudes.

## Example C

Rewrite the diameter of the sun, $1.39 \times 10^{9} \mathrm{~m}$, in standard form.
Step 1: Simplify $10^{9}$.

$$
10^{9}=1,000,000,000
$$

Step 2: Multiply by 1.39 .
$1.39 \times 1,000,000,000=1,390,000,000$
Solution: The diameter of the sun, $1.39 \times 10^{9} \mathrm{~m}$, in standard form is $1,390,000,000 \mathrm{~m}$.

## Try These C

a．What is the relationship between the exponent of the power of 10 and the number of places the decimal moves？
b．Rewrite the mass of the sun， $2.0 \times 10^{20} \mathrm{~kg}$ ，in standard form．

Rewrite these expressions in standard form：
c． $4.3 \times 10^{7}$
d． $7 \times 10^{12}$
e． $3.2 \times 10^{5}$
f． $9 \times 10^{24}$

## Example D

Convert $25,000,000,000$ from standard form to scientific notation．
Step 1：Identify the location of the decimal point in $25,000,000,000$ ．
25,000,000,000.
$\uparrow$ decimal point
Step 2：Move the decimal point to the left until you have a number that is greater than or equal to 1 and less than 10．Count the number of places you moved the decimal point．
$25,000,000,000$
勾勾余勾余
Step 3：Rewrite the number in scientific notation： $2.5 \times 10^{10}$
Solution： $25,000,000,000=2.5 \times 10^{10}$

## Try These D

Convert each number from standard form to scientific notation．Work backwards from your answer to check your work．
$\begin{array}{lll}\text { a．} 6,000 & \text { b．} 436,000,000 & \text { c．} 16,000\end{array}$
$\begin{array}{lll}\text { a．} 6,000 & \text { b．} 436,000,000 & \text { c．} 16,000\end{array}$
$\begin{array}{lll}\text { a．} 6,000 & \text { b．} 436,000,000 & \text { c．} 16,000\end{array}$

Large numbers can also be written using words. For example, you can write $9,000,000,000$ or 9 billion or $9 \times 10^{9}$.

This table shows names for some very large numbers.

| Standard Form | Power <br> of 10 | Name |
| :---: | :---: | :---: |
| 1,000 | $10^{3}$ | Thousand |
| $1,000,000$ | $10^{6}$ | Million |
| $1,000,000,000$ | $10^{9}$ | Billion |
| $1,000,000,000,000$ | $10^{12}$ | Trillion |
| $1,000,000,000,000,000$ | $10^{15}$ | Quadrillion |
| $1,000,000,000,000,000,000$ | $10^{18}$ | Quintillion |
| $1,000,000,000,000,000,000,000$ | $10^{21}$ | Sextillion |
| $1,000,000,000,000,000,000,000,000$ | $10^{24}$ | Septillion |
| $1,000,000,000,000,000,000,000,000,000$ | $10^{27}$ | Octillion |
| $1,000,000,000,000,000,000,000,000,000,000$ | $10^{30}$ | Nonillion |
| $1,000,000,000,000,000,000,000,000,000,000,000$ | $10^{33}$ | Decillion |
| There is not enough space to write this number. | $10^{100}$ | Googol |

Make use of structure. Complete this table showing standard form, scientific notation, and name of some large numbers.

|  | Standard Form | Scientific Notation | Name |
| :--- | :---: | :---: | :---: |
| 1.,400,000,000,000,000 | $7.4 \times 10^{15}$ | 7.4 quadrillion |  |
| 2. |  | $3 \times 10^{3}$ |  |
| 3. | $1,200,000,000$ |  |  |
| 4. |  | 5 trillion |  |
|  | $9,000,000$ |  |  |
|  |  |  |  |

You can use scientific notation to write estimates of large numbers that have many non-zero digits. Estimates consist of a single digit times a power of 10 .

## Example E

According to the U.S. Census, the population of California in the year 2010 was $37,253,956$. Write an estimate of the population using scientific notation.

Step 1: The greatest place value is ten million, so round the population to the nearest ten million.
$37,253,956 \approx 40,000,000$

Step 2: Write the population to the nearest ten million in scientific notation.
$40,000,000=4 \times 10^{7}$
Solution: An estimate of the population of California in 2010 is $4 \times 10^{7}$.

## Try These E

Write an estimate of each number using scientific notation.
a. 284,116
b. $5,218,996$

## Check Your Understanding

Convert each number from scientific notation to standard form.
5. $5.2 \times 10^{4}$
6. $4.23 \times 10^{6}$
7. $2 \times 10^{3}$
8. $1.03 \times 10^{4}$

Convert each number from standard form to scientific notation.
9. 20,000
10. $1,340,000$

## LESSON 7-1 PRACTICE

11. Gulliver is so much bigger than the Lilliputians that he consumes more food than 1,000 Lilliputians do. Write this number in scientific notation.
For Items 12-16, identify whether the expression is written in scientific notation. For those not written in scientific notation, explain why and rewrite in scientific notation.
12. $6 \times 10^{4}$
13. $15 \times 10^{3}$
14. $2 \times 10^{6}$
15. $3.2 \times 10^{5}$
16. $43.2 \times 10^{3}$
17. The kingdom of Lilliput is said to have an area of 24 million square miles. Write this amount using scientific notation.
18. Construct viable arguments. Explain why someone would want to write $52,000,000,000,000,000$ in scientific notation instead of standard form.
19. According to the U.S. Census of 2010, the population of persons aged 65 years and older was $40,267,984$. Write an estimate of this population using scientific notation.

## My Notes

## 




## My Notes



## Learning Targets:

- Express numbers in scientific notation.
- Convert numbers in scientific notation to standard form.
- Compare and order numbers in scientific notation.
- Use scientific notation to write estimates of quantities.

Suggested Learning Strategies: Summarizing, Close Reading,
Visualization, Construct an Argument, Work Backward
Suppose that Gulliver is 5 feet tall.

1. Reason quantitatively. Describe how to write Gulliver's height in scientific notation.
a. Is 5 an appropriate factor to use? Why or why not?
b. What is the exponent for the power of 10 ? Justify your response.
2. Convert each number from standard form to scientific notation.
a. 2
b. 8.3
3. Convert each number from scientific notation to standard form.
a. $9 \times 10^{0}$
b. $6.12 \times 10^{0}$

The Lilliputians are 10 times as short as Gulliver. In other words, they are $\frac{1}{10}$ of Gulliver's height.
4. Express the height of a Lilliputian in scientific notation.
a. Explain what factor is appropriate for the value of $a$ in the form $a \times 10^{n}$.
b. Determine what power of 10 is appropriate for the value of $n$ in the form $a \times 10^{n}$.
c. Use the values you found in parts $a$ and $b$ to write the height of $a$ Lilliputian in scientific notation.
d. What do you notice about a fractional number in standard form when it is written in scientific notation?

ACTIVITY 7
continued
5. Convert each number from standard form to scientific notation.
a. 0.125
b. 0.00006
c. 7
d. 0.000000000025

## Example A

The heights measured in feet, expressed in scientific notation, of Gulliver, a person from Brobdingnag, and a Lilliputian are shown:

$$
5 \times 10^{0}, 5 \times 10^{1}, 5 \times 10^{-1}
$$

Order these numbers from least to greatest.
Step 1: Use the values of the exponents to help determine the order.

$$
5 \times 10^{-1} \text { is the least, then } 5 \times 10^{0} \text {, then } 5 \times 10^{1}
$$

Step 2: Write the numbers in standard form to check the order.

$$
\begin{array}{ll}
5 \times 10^{-1}=0.5 & \text { least } \\
5 \times 10^{0}=5 & \downarrow \\
5 \times 10^{1}=50 & \text { greatest }
\end{array}
$$

Solution: From least to greatest, the heights are $5 \times 10^{-1}, 5 \times 10^{0}$, and $5 \times 10^{1}$ feet.

## Try These A

Order these numbers from least to greatest.
a. $8 \times 10^{0}, 9 \times 10^{-2}, 2 \times 10^{3}$
b. $4.14 \times 10^{2}, 1.4 \times 10^{-4}, 4.1 \times 10^{-3}$

Order these numbers from greatest to least.
c. $0.007,5 \times 10^{1}, 7 \times 10^{-5}$
d. $1 \times 10^{0}, 0.87,7.8 \times 10^{1}$

## My Notes








|  |  |  |  |  |  |  | My Notes |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  |  |  |  |  |  |  |  |  |  |

You can use scientific notation to write estimates of small numbers. Estimates consist of a single digit times a power of 10 .

## Example B

A micrometer is a small metric measure for length. Using her computer, Jenny found that a micrometer equals 0.000039370078740157 inch. Use scientific notation to estimate the inch equivalent of a micrometer.
Step 1: The greatest place value is hundred thousandths, so round to the nearest hundred thousandth. $0.000039370078740157 \approx 0.00004$
Step 2: Write the measure to the nearest hundred thousandth in scientific notation. $0.00004=4 \times 10^{-5}$
Solution: An estimate of the length of a micrometer is $4 \times 10^{-5}$ inch.

## Try These B

Write an estimate for each number using scientific notation.
a. 0.00018
b. 0.00619023

## Check Your Understanding

Convert each number from scientific notation to standard form.
6. $5.2 \times 10^{-4}$
$7.4 .23 \times 10^{-6}$
8. $2.5 \times 10^{0}$

Convert each number from standard form to scientific notation.
9. 0.0002
10. 0.000000513
11.6 .9
12. Order these numbers from least to greatest.
$0.0051,8 \times 10^{-4}, 1.89 \times 10^{-3}, 0.00079$

## LESSON 7-2 PRACTICE

13. In terms of Gulliver's height, use scientific notation to express the height of an insect if the insect is 10 times shorter than a person from Lilliput.
14. Write a rule for converting a number from scientific notation to standard form that will always work when the scientific notation of a number includes $10^{\circ}$.
15. Lena converted $1.2 \times 10^{-7}$ to the standard form $12,000,000$. Do you agree with Lena? Explain your reasoning.
16. Explain why someone would want to write the number 0.000000000064 in scientific notation instead of standard form.
17. Make sense of problems. The fictional land of Brobdingnag had an area of $1.8 \times 10^{7}$ square miles. In its army were 32,000 cavalry and $2.07 \times 10^{5}$ soldiers. Order these numbers from greatest to least.
18. Write an estimate for each number using scientific notation.
a. 0.000079013
b. 0.0022978

## ACTIVITY 7 PRACTICE

Write your answers on notebook paper.
Show your work.

## Lesson 7-1

Write the following in scientific notation.

1. $25,000,000,000$
2. 60,000
3. $713,000,000,000,000,000$
4. 99

Write the following in standard form.
5. $7 \times 10^{2}$
6. $8.92 \times 10^{8}$
7. $4 \times 10^{20}$
8. $6.07 \times 10^{6}$
9. Is $10.2 \times 10^{4}$ written in scientific notation? Explain.
10. Copy and complete.

| Standard <br> Form | Scientific <br> Notation | Name |
| :---: | :---: | :---: |
| $2,300,000,000$ |  |  |
|  | $3.4 \times 10^{3}$ |  |
|  |  | 9 million |

11. Which of the following shows $9,200,000,000,000,000$ in scientific notation?
A. $9.2 \times 10^{9}$
B. $9.2 \times 10^{12}$
C. $9.2 \times 10^{15}$
D. $9.2 \times 10^{18}$
12. Which of the following is in correct scientific notation form?
A. $0.8 \times 10^{2}$
B. $8 \times 10^{2}$
C. $80 \times 10^{2}$
D. $8 \times 10^{\frac{1}{2}}$
13. The sun is 93 million miles away from Earth. Write this number in scientific notation.
14. A fast food restaurant claims to have served 245 billion hamburgers. Write this number in standard form.
15. The Milky Way galaxy is estimated to be 13.2 billion years old. Write this number in scientific notation.
16. According to the U.S. Census of 2010, the population of Florida was $18,801,310$. Write an estimate of this population using scientific notation.

## Lesson 7-2

17. The following table shows the attendance for a year at four major-league baseball stadiums. Order the attendance from greatest to least.

| Yankees | Mariners | Red Sox | Dodgers |
| :---: | :---: | :---: | :---: |
| $3.8 \times 10^{6}$ | $2,000,000$ | 2.5 million | $3.2 \times 10^{6}$ |

18. Copy and complete.

| Standard <br> Form | Scientific <br> Notation |
| :---: | :---: |
| 0.00009 |  |
|  | $1.7 \times 10^{-3}$ |
|  | $6.99 \times 10^{-7}$ |
| 0.00086 |  |

19. Wire 1 has a diameter of $9 \times 10^{-2}$ inches. Wire 2's diameter is $2.4 \times 10^{-3}$ inches and Wire 3 is 0.0023 inches in diameter. Order the wire diameters from smallest to largest.
20. Describe how scientific notation aids the discussion of very small or very large numbers. Provide an example.

## MATHEMATICAL PRACTICES Reason Abstractly and Quantitatively

34. Think about the value of each expression.

$$
\begin{array}{llll}
4.3 \times 10^{3} & 3.8 \times 10^{-5} & 2.4 \times 10^{12} & 3.0 \times 10^{0} \\
2.2 \times 10^{-2} & 7.8 \times 10^{-4} & 7.1 \times 10^{0} & 9.8 \times 10^{5} \\
6.4 \times 10^{-3} & 3.8 \times 10^{-14} & 6.4 \times 10^{8} & 4.8 \times 10^{0}
\end{array}
$$

a. Copy the table and place each expression in the appropriate column.

| Between <br> 0 and 1 | From <br> 1 to 10 | 10 and <br> greater |
| :---: | :---: | :---: |
|  |  |  |
|  |  |  |
|  |  |  |

b. Explain what you notice about the exponents in the scientific notation form of the numbers you sorted.

Numbers between 0 and 1
Numbers from 1 to 10
Numbers 10 and greater
c. Which expression has the greatest value?
d. Which expression has the least value?
31. Which of the following is 4 micrometers in standard form?
A. 0.04
B. 0.0004
C. 0.000004
D. 0.00000004
32. Cooper's best high jump was measured as 1.1 m . Write this number in scientific notation.
33. An insect known as a fairy wasp can be as little as 0.007874 inch long. Write an estimate of this length using scientific notation.

# Operations with Scientific Notation 

## How Big is That Planet?

Lesson 8-1 Multiplying and Dividing in Scientific Notation

## Learning Targets:

- Multiply numbers expressed in scientific notation.
- Divide numbers expressed in scientific notation.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Predict and Confirm, Critique Reasoning, Look for a Pattern, Create a Plan

Our solar system includes eight planets. The one that is closest to the sun is Mercury, followed by Venus, Earth, Mars, Jupiter, Saturn, Uranus and Neptune. In the previous activity, you learned that scientific notation is commonly used to represent very large or very small numbers. Scientific notation can be used to write the numbers representing the distance of the planets from the sun and the mass of the planets.

1. Earth is $93,000,000$ miles away from the sun and has an approximate mass of $6,000,000,000,000,000,000,000,000$ kilograms.
a. Write Earth's distance from the sun in scientific notation.
b. Write Earth's mass in scientific notation.
2. Earth moves at an approximate average speed of $107,000 \mathrm{~km} / \mathrm{h}$. It takes Earth approximately 8,800 hours to orbit the sun.
a. Using the numbers written in standard form, find the total approximate distance Earth travels when orbiting the sun.
b. Express the average speed of Earth in scientific notation.
c. Express the total amount of hours it takes Earth to orbit the sun in scientific notation.
d. Write your answer to part a in scientific notation.

## My Notes

## CONNECT TO ASTRONOMY

Astronomers used to consider Pluto a planet, making a total of nine planets in our solar system. In 2006, the International Astronomical Union changed the definition of a planet. The definition excluded Pluto as a planet, in part because it was too small.


## My Notes

## MATH TIP

The associative and commutative properties of multiplication state that the product of several factors will be the same, no matter how the factors are grouped or ordered. Discuss with your peers how these properties apply to multiplying and dividing numbers in scientific notation.


Simplify each expression. Write the answer in scientific notation.
b. $\left(9 \times 10^{5}\right)\left(3 \times 10^{4}\right)$
c. $\left(1.6 \cdot 10^{8}\right)\left(3 \cdot 10^{4}\right)$
d. $\left(4 \cdot 10^{12}\right)\left(6 \cdot 10^{5}\right)$
3. Use appropriate tools strategically. Compare the products of each expression from Try These A using a calculator.
a. Write each output as shown on your calculator.

$$
\begin{aligned}
& \left(9 \times 10^{5}\right)\left(3 \times 10^{4}\right) \\
& \left(1.6 \cdot 10^{8}\right)\left(3 \cdot 10^{4}\right) \\
& \left(4 \cdot 10^{12}\right)\left(6 \cdot 10^{5}\right)
\end{aligned}
$$

b. Explain what the outputs on your calculator mean.
4. Jupiter is the largest planet with a mass of about $1,900,800,000,000,000,000,000,000,000 \mathrm{~kg}$ and Mercury is the smallest planet with a mass of about $330,000,000,000,000,000,000,000 \mathrm{~kg}$.
a. Using the numbers that are written in standard form, determine how many times as large Jupiter is than Mercury.
b. Write Jupiter's mass in scientific notation.
c. Write Mercury's mass in scientific notation.
d. Write your answer to part a in scientific notation.
e. Reason abstractly. Using the numbers written in scientific notation, how could you determine how many times as large Jupiter is than Mercury without changing their masses to standard form?
5. Simplify each expression using the process you stated in Item 4 e . Write the answer in scientific notation.
a. $\frac{16.4 \times 10^{9}}{4.1 \times 10^{5}}$
b. $\frac{2 \times 10^{8}}{8 \times 10^{2}}$
c. $\frac{5.5 \times 10^{2}}{1.1 \times 10^{4}}$

## My Notes

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## MATH TIP

The law of exponents for dividing powers with the same base states that the exponent of the divisor is subtracted from the exponent of the dividend.


## Check Your Understanding

Simplify each expression. Write answers in scientific notation.
6. $\frac{6 \times 10^{5}}{3 \times 10^{4}}$
7. $\left(2 \times 10^{-5}\right)\left(3 \times 10^{4}\right)$
8. $\left(4.2 \times 10^{8}\right)\left(3 \times 10^{-5}\right)$
9. $\frac{3.2 \times 10^{5}}{4 \times 10^{8}}$

## LESSON 8-1 PRACTICE

Simplify each expression and write answers in scientific notation for Items 10-13.
10. $\left(7.5 \times 10^{6}\right)\left(2.3 \times 10^{4}\right)$
11. $\frac{\left(1.2 \times 10^{4}\right)}{\left(2.5 \times 10^{6}\right)}$
12. $\left(4 \times 10^{15}\right)\left(6 \times 10^{-7}\right)$
13. $\left(2.75 \times 10^{-2}\right)\left(8 \times 10^{-6}\right)$

The table below shows the approximate mass of some planets. Use this table for Items 14-15.

| Planet | Approximate Mass (in kg) |
| :--- | :---: |
| Mercury | $3.3 \times 10^{23}$ |
| Earth | $6 \times 10^{24}$ |
| Jupiter | $1.9008 \times 10^{27}$ |

14. How many times bigger is Jupiter than Earth?
15. How many times bigger is Earth than Mercury?
16. Make sense of problems. The speed of light is $3 \times 10^{8} \mathrm{~m} / \mathrm{s}$ and the distance from Earth to the sun is $1.5 \times 10^{11} \mathrm{~m}$. How many seconds does it take for sunlight to reach Earth? Write your answer in scientific notation.
17. The average distance from Earth to the moon is $3.84 \times 10^{10} \mathrm{~cm}$.
a. Choose a more appropriate unit of length to use.
b. Find the approximate distance from Earth to the moon using the unit you chose in part a. Write your answer in scientific notation.

## Learning Targets:

- Add numbers expressed in scientific notation.
- Subtract numbers expressed in scientific notation.

SUGGESTED LEARNING STRATEGIES: Group Presentation, Critique Reasoning, Look for a Pattern, Graphic Organizer, Summarizing

1. Consider the expression $4.2 \times 10^{3}+2.9 \times 10^{3}$.
a. Change the numbers in the expression to standard form and add the numbers together.
b. Write your answer to part a in scientific notation.

## Example A

Add $4.2 \times 10^{3}+2.9 \times 10^{3}$, keeping the numbers in scientific notation.
Step 1: To add or subtract numbers in scientific notation, the exponents must be the same. If they are not the same, rewrite the terms so that the exponents are the same.
$4.2 \times 10^{3}+2.9 \times 10^{3} \rightarrow$ the exponents are the same
Step 2: Add the digits. Write the sum in scientific notation.
$4.2+2.9=7.1$
$7.1 \times 10^{3}$
Solution: $4.2 \times 10^{3}+2.9 \times 10^{3}=7.1 \times 10^{3}$

## Try These A

Add or subtract.
a. $2.3 \times 10^{9}+5.6 \times 10^{9}$
b. $9.1 \times 10^{-2}-2.5 \times 10^{-2}$
c. $8.4 \times 10^{-5}+7.2 \times 10^{-5}$


Mercury is the closest planet to the sun, and Neptune is the farthest planet from the sun. Neptune is $2.8 \times 10^{9}$ miles from the sun and Mercury is $2.6 \times 10^{7}$ miles from the sun.
2. Critique the reasoning of others. Colten and Drake wanted to find the distance between the two planets. They each decided to solve the problem different ways. Compare and contrast the two methods they used to find the distance between the two planets.

| Colten | Drake |
| :---: | :---: |
| $2.8 \times 10^{9}-2.6 \times 10^{7}$ | $2.8 \times 10^{9}-2.6 \times 10^{7}$ |
| $2,800,000,000-26,000,000$ | $280 \times 10^{7}-2.6 \times 10^{7}$ |
| $2,774,000,000$ | $277.4 \times 10^{7}$ |
| $2.774 \times 10^{9}$ | $2.774 \times 10^{9}$ |

3. Earth, the third closest planet to the sun, is $9.3 \times 10^{7}$ miles from the sun. Find the distance from Neptune to Earth using either of the methods shown in Item 2.

## Check Your Understanding

4. $\left(2.5 \times 10^{6}\right)+\left(4 \times 10^{6}\right)$
5. $\left(3.4 \times 10^{-3}\right)+\left(8.1 \times 10^{-3}\right)$
6. $\left(6.23 \times 10^{5}\right)-\left(2.1 \times 10^{5}\right)$
7. $\left(7.2 \times 10^{-2}\right)-\left(2.1 \times 10^{-2}\right)$

## LESSON 8-2 PRACTICE

9. $\left(6.7 \times 10^{10}\right)+\left(4.1 \times 10^{10}\right)$
10. $\left(5.5 \times 10^{9}\right)+\left(2.6 \times 10^{8}\right)$
11. $\left(9.9 \times 10^{-4}\right)-\left(3.26 \times 10^{-5}\right)$
12. Earth is $9.3 \times 10^{7}$ miles from the sun. Mars is $1.4 \times 10^{8}$ miles from the sun. What is the distance from Earth to Mars?
13. Reason abstractly. When adding or subtracting numbers in scientific notation, why do you think the exponents need to be the same?

## ACTIVITY 8 PRACTICE

Write your answers on notebook paper.
Show your work.

## Lesson 8-1

Simplify each expression. Write the answers in scientific notation.

1. $\left(2.2 \times 10^{5}\right)\left(4 \times 10^{7}\right)$
2. $35,000 \cdot 9,000,000,000$
3. $\left(8.1 \times 10^{12}\right)(5.3 \times 10)$
4. $\left(6.5 \times 10^{-13}\right)\left(2 \times 10^{-4}\right)$
5. $\frac{2.7 \times 10^{4}}{1.2 \times 10^{9}}$
6. $\frac{4.2 \times 10^{10}}{3 \times 10^{3}}$
7. If Saturn has a mass of about $569,600,000,000,000,000,000,000,000 \mathrm{~kg}$ and Mars has a mass of $640,000,000,000,000,000,000,000 \mathrm{~kg}$, how many times as big is Saturn than Mars? Write your answer in scientific notation.
8. When multiplying two numbers in scientific notation, which of the following statements is true?
A. Add the exponents.
B. Subtract the exponents.
C. The exponents must be the same.
D. The exponents must be different.
9. The diameter of the sun is approximately $1.4 \times 10^{9} \mathrm{~km}$ and the diameter of Earth is approximately $1.28 \times 10^{4} \mathrm{~km}$. About how many times Earth's diameter is the sun's diameter?
10. Brigitte simplified the expression $\left(7.4 \times 10^{6}\right) \div$ $\left(5 \times 10^{-2}\right)$ as $1.48 \times 10^{8}$. Do you agree with her answer? Explain your reasoning.
11. Neptune is $4.9 \times 10^{9} \mathrm{~km}$ from the sun and light travels at a speed of $3 \times 10^{5} \mathrm{~km} / \mathrm{s}$. In seconds, how long does it take for sunlight to reach Neptune? Write your answer in scientific notation.
12. There are $3.6 \times 10^{3}$ seconds in an hour. How many hours does it take for sunlight to reach Neptune?
13. The mass of the moon is approximately $7.3 \times 10^{22} \mathrm{~kg}$, and the mass of Earth is approximately $6 \times 10^{24} \mathrm{~kg}$. How many times the mass of the moon is the mass of Earth?
14. The mass of the sun is $1.9891 \times 10^{30} \mathrm{~kg}$, and the mass of the largest planet, Jupiter, is $1.9008 \times 10^{27} \mathrm{~kg}$. How many times the mass of Jupiter is the mass of the sun? Write your answer in scientific notation.
15. Over the past three years, Gerry has grown at the rate of $6 \times 10^{-2}$ meter per year. Estimate Gerry's growth using a more appropriate unit.

## Lesson 8-2

16. $3.4 \times 10^{5}+9.1 \times 10^{5}$
17. $7.5 \times 10^{-3}-2.1 \times 10^{-3}$
18. Which is the answer to $2.3 \times 10^{5}+5.1 \times 10^{4}$ ?
A. $28.1 \times 10^{4}$
B. $28.1 \times 10^{5}$
C. $2.81 \times 10^{4}$
D. $2.81 \times 10^{5}$
19. Which is the answer to $9.1 \times 10^{-2}-5.4 \times 10^{-4}$ ?
A. $9.046 \times 10^{-2}$
B. $9.046 \times 10^{-3}$
C. $9.046 \times 10^{-4}$
D. $9.046 \times 10^{-5}$
20. Earth is $9.3 \times 10^{7}$ miles from the sun and Mercury is $2.6 \times 10^{7}$ miles from the sun. Find the distance between Earth and Mercury.
21. When subtracting two numbers in scientific notation, which of the following statements is true?
A. Add the exponents.
B. Subtract the exponents.
C. The exponents must be the same.
D. The exponents must be different.

The table below shows the distance from the sun traveled by two space probes. Use this table for Items 22-23.

| Object | Distance From the <br> Sun (in miles) |
| :---: | :---: |
| Voyager 1 | 11.3 billion |
| Voyager 2 | 9.3 billion |

22. The space probes Voyager 1 and Voyager 2 were launched in 1977. How much farther has Voyager 1 traveled than Voyager 2? Express your answer in scientific notation.
23. How many miles total have been traveled by Voyager 1 and Voyager 2? Express your answer in scientific notation.
24. $\left(2.5 \times 10^{8}\right)+\left(3.8 \times 10^{7}\right)$
25. $\left(6.7 \times 10^{4}\right)-\left(6.1 \times 10^{2}\right)$
26. $\left(3.2 \times 10^{3}\right)+\left(5.4 \times 10^{3}\right)$
27. $\left(1.5 \times 10^{27}\right)-\left(1.4 \times 10^{26}\right)$
28. Rupert added $1.72 \times 10^{4}$ to $8 \times 10^{6}$ and got the sum $9.72 \times 10^{10}$. Do you agree with his answer? Explain your reasoning.
29. Venus is the second planet from the sun. Earth is the third planet from the sun. The distance from Venus to the sun is $6.72 \times 10^{7}$ miles and the distance from Earth to the sun is $9.3 \times 10^{7}$ miles. What is the distance between Earth and Venus?

## MATHEMATICAL PRACTICES Look For and Make Use of Structure

30. Use a Venn diagram to compare and contrast the processes of adding and subtracting numbers in scientific notation or multiplying and dividing numbers in scientific notation.

While checking her e-mail, Luisa stumbles across a cryptic message from someone named 5up3r H4xx0r. In the message, 5up3r H4xx0r claims to have developed a computer virus and is set to release it on the Internet. Once the virus has infected two computers, the potential exists for it to spread exponentially, because each infected computer has a chance to pass it along to the next computer it connects with.
The only way for the virus to be stopped, says the hacker, is if Luisa correctly answers each of the following questions.

1. The pattern of the spread of the virus will be $1,2,4,8, \ldots$ Identify the next three numbers in this pattern.
2. Express the first seven numbers in the pattern as a power of 2 .
3. Describe how the $18^{\text {th }}$ term in the pattern could be determined.
4. Determine which base could be used to write the numbers below in exponential form. Rewrite each product using exponential forms of the base you determined.
a. $32 \cdot 128$
b. $4 \cdot 256$
c. $16 \cdot 64$
5. Simplify each of the products in Item 4. Leave your answer in exponential form.
6. Describe how to simplify each of the following expressions. Simplify each expression and leave your answer in exponential form.
a. $\left(2^{13}\right)^{4}$
b. $\frac{2^{12}}{2^{3}}$
7. Replace the variables with numbers in the expressions so that the expression would result in the answer of $2^{6}$.
a. $\frac{a^{x}}{a^{y}}$
b. $a^{x} \cdot a^{y}$
c. $\left(a^{x}\right)^{y}$
d. $\frac{1}{a^{x}}$
8. Write each number in scientific notation.
a. $20,000,000$
b. 2,400
9. Simplify each expression. Leave your answer in scientific notation.
a. $\left(2 \cdot 10^{5}\right)\left(2 \cdot 10^{3}\right)$
b. $\frac{8 \cdot 10^{4}}{4 \cdot 10^{2}}$
10. Write a reply to 5 up3r H4xx0r about your success in foiling the virus plan. Include in your reply a description of the problems that were difficult for you, and the ones that you were able to complete easily. Also include a final statement that summarizes your overall success.

| Scoring Guide | Exemplary | Proficient | Emerging | Incomplete |
| :---: | :---: | :---: | :---: | :---: |
|  | The solution demonstrates these characteristics: |  |  |  |
| Mathematics <br> Knowledge and <br> Thinking <br> (Items 1, 2, 3, 4a-c, 5, <br> $6 a-b, 7 a-c, 8 a-b, 9)$ | - Clear and accurate understanding of multiplying and dividing with exponents and scientific notation. <br> - Clear and accurate understanding of writing numbers in exponential form and in scientific notation. | - Multiplying and dividing with exponents and scientific notation. <br> - Writing numbers in exponential form and in scientific notation. | - Errors in multiplying and dividing with exponents and scientific notation. <br> - Errors in writing numbers in exponential form and in scientific notation. | - Incorrect or incomplete multiplication and division with exponents and scientific notation. <br> - Little or no understanding of writing numbers in exponential form and in scientific notation. |
| Problem Solving (Items 2, 3, 4, 7a-b, 8a-b) | - An appropriate and efficient strategy that results in a correct answer. | - A strategy that may include unnecessary steps but results in a correct answer. | - A strategy that results in some incorrect answers. | - No clear strategy when solving problems. |
| Mathematical Modeling/ Representations (Items 1, 2, 4a-c, 8) | - Clear and accurate understanding of representing a number in exponential form and in scientific notation. | - Representing a rational number in exponential form and in scientific notation. | - Errors in representing a number in exponential form or in scientific notation. | - Inaccurately representing a number in exponential form and in scientific notation. |
| Reasoning and Communication (Items 3, 6, 10) | - Precise and accurate explanation of how to multiply and divide with exponents. <br> - Clear and precise explanation of the level of difficulty experienced with the problems. | - Adequate explanation of how to multiply and divide with exponents. <br> - Adequate explanation of the level of difficulty experienced with the problems. | - A misleading or confusing explanation of how to multiply and divide with exponents. <br> - A confusing description of the level of difficulty experienced with the problems. | - An incomplete or inaccurate explanation of how to multiply and divide with exponents. <br> - An incomplete description of the level of difficulty experienced with the problems. |

