Functions

4

Unit Overview

In this unit you will study relations and functions. You will evaluate functions and represent them graphically, algebraically, and verbally. You you will compare and contrast linear and non-linear patterns and write expressions to represent these patterns

Key Terms

As you study this unit, add these and other terms to your math notebook. Include in your notes your prior knowledge of each word, as well as your experiences in using the word in different mathematical examples. If needed, ask for help in pronouncing new words and add information on pronunciation to your math notebook. It is important that you learn new terms and use them correctly in your class discussions and in your problem solutions.

Academic Vocabulary

- contraption

Math Terms

- relation
- set
- ordered pair
- function
- domain
- range
- discrete data
- continuous data
- rate of change
- trend line
- scatter plot

ESSENTIAL QUESTIONS

? Why is it important to consider domain, range, and intercepts in problem situations?

? Why is it important to be able represent functions as tables, graphs, algebraically, and verbally?

EMBEDDED ASSESSMENTS

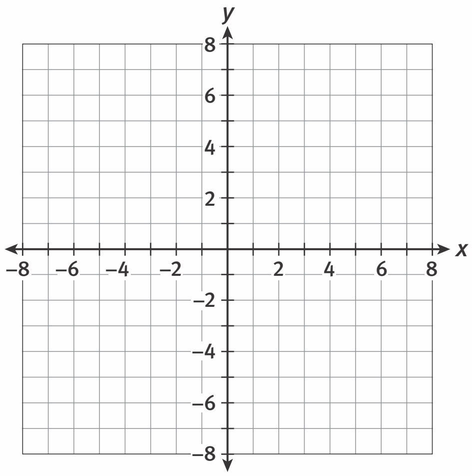
These assessments, following activities 29 and 31, will give you an opportunity to demonstrate your understanding of functions.

Embedded Assessment 1:	
Functions	p. 408
Embedded Assessment 2:	
Scatter Plots and Trend Lines	p. 440

UNIT 4

Getting Ready

1. On the grid below, draw a figure that illustrates the meaning of linear.



2. Name five ordered pairs that would be on a graph made from the following table.

Input	Output
5	1
10	2
15	3

3. If $x + 2y = 13$ find the value of y when x is

a. 3 **b.** 8 **c.** 2.1

4. Name 3 ordered pairs that satisfy each of the following equations.

a. $2y = 6$ **b.** $3x + 2y = 9$ **c.** $3x = 12$

5. Which of the following equations represents the data in the table?

x	1	3	5	7
y	1	8	14	19

A. $y = 2x - 3$ **B.** $y = 2x + 3$

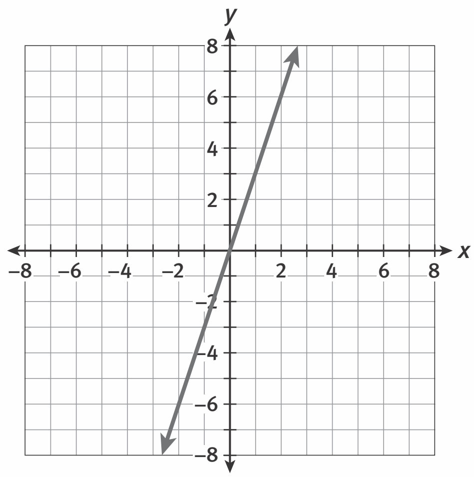
C. $y = 3x - 2$ **D.** $y = 3x + 2$

6. Harry earns \$300 a week plus \$25 for each insurance policy he sells.

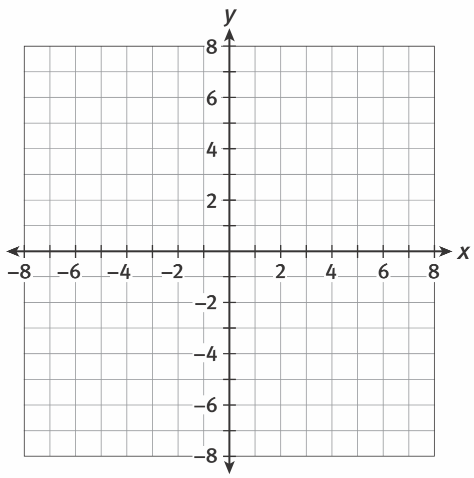
a. Write an equation to determine how much Harry can earn each week.

b. How many policies did Harry sell last week if he earned \$550?

7. What is the relationship between x and y values in the following?



8. On the grid provided draw a representation that is not linear.



Introduction to Functions

ACTIVITY 27

It's All Related

Lesson 27-1 What Is a Function?

My Notes

Learning Targets:

- Define relation and function.
- Evaluate functions.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Visualization, Discussion Groups, Create Representations, Note Taking

Mathematicians study relationships in the world around us in order to learn how things work. For example, the relationship between gasoline, air, and fire is what makes a motorcycle move. Understanding the relationship between carbonation and motion allows us to know that opening a bottle of carbonated drink that has been shaken is probably not a good idea.

1. In a relationship, one item depends on another. In the example below, a car taking us home depends on there being enough gas in the car to reach the destination. List other relationships that exist in the real world.

Desired Result:	Depends On:
1. Car taking us home	Having enough gas in the car

A ***relation*** is a ***set*** of one or more ***ordered pairs***. For example, the pairing of having enough gas in the car with the car taking us home is a relation. This relation can be written: {(having enough gas in the car, car takes us home)}.

2. Use the table you completed in Item 1 to write other relations.

MATH TERMS

A **set** is a collection of objects, like points, or a type of number, like the real numbers. The symbols {} indicate a set.

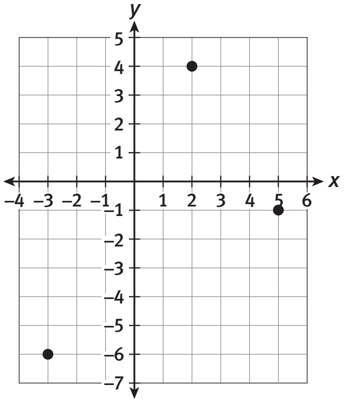
An **ordered pair** is two numbers written in a certain order. Most often, the term *ordered pair* will refer to the *x* and *y* coordinates of a point on the coordinate plane, which are always written (x, y). The term can also refer to any values paired together according to a specific order.

My Notes

Example A

Write a relation using the table or graph.

Input (x)	Output (y)
2	4
-3	-6
5	-1



Step 1: Look at the input (x) and output (y) values.

Step 2: Create each ordered pair (x, y) by writing the input value and corresponding output value in parentheses.

Step 3: Use braces to write the ordered pairs as a set.

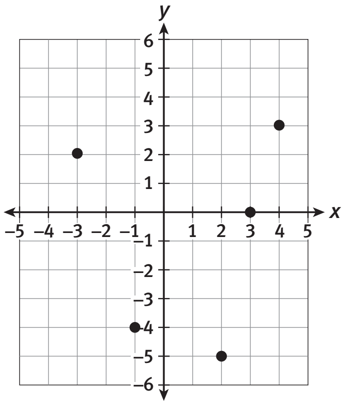
Solution: {(2, 4), (−3, −6), (5, −1)}

Try These A

a. Write a relation from the table of values.

x	y
-10	5
20	-10
-30	-15
40	20

b. Write a relation from the graph.



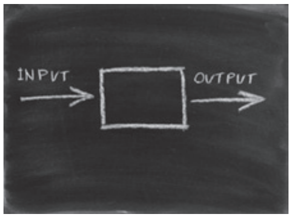
A ***function*** is a special kind of relation in which one input has only one output. One way to think of a function is as a machine. A machine has an input and an output and a relationship exists between the input and the output. The machine receives the input and transforms it into the output. For example, a toaster is a machine. When bread is the input, the machine toasts the bread and the output is toast.

3. With your group, list other machines along with their inputs and outputs.

Input	Machine	Output
1. Bread	Toaster	Toast

In mathematics, if x is the input and y is the output, the value of y depends on the value of x. The relationship between x and y is determined by the function, or rule (the machine). The function changes, or transforms, x into y, so y is a function of x.

4. A function represented by the equation $y = x + 5$ has inputs labeled x and outputs labeled y. The diagram below represents that function.



a. If $x = 5$ is used as an input in the diagram, what is the output?

b. If $x = -3$ is used as an input in the diagram, what is the output?

c. If $x = 0.03$ is used as an input in the diagram, what is the output?

d. If $x = -\frac{1}{2}$ is used as an input in the diagram, what is the output?

e. **Reason abstractly.** For this particular function, is it possible to have the same y-value when using two different x-values? Explain.

My Notes

CONNECT TO AP

Functions and relations, which describe how two varying quantities are related, form the basis of much of the work you will do in calculus.

DISCUSSION GROUP TIPS

Working with a partner or in small groups provides an opportunity for you to clarify your understanding of relations and functions.

My Notes

5. Complete the function table. Use the given input (x) values and determine the corresponding output (y) values based on the given function.

Input (x)	Function	Output (y)
	$y = 2x - 1$	

6. Create your own function. Choose any values for x and determine the corresponding values for y based on the function you write.

Input (x)	Function	Output (y)

7. Write the x- and y-values from the function that you created in Item 6 above as ordered pairs.

Check Your Understanding

8. Complete the function table below. Choose any values for x and determine the corresponding values for y.

Input (x)	Function	Output (y)
	$y = x - 9$	

9. Write the x- and y-values from the function in Item 8 as a set of ordered pairs.

My Notes

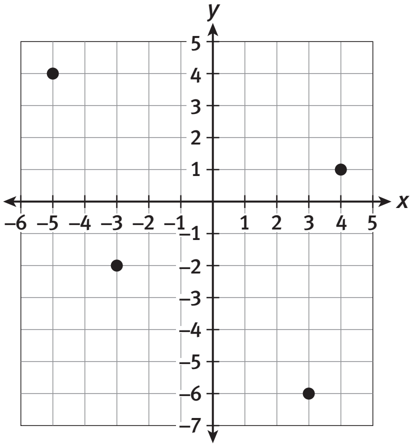
LESSON 27-1 PRACTICE

For Items 10 and 11, write each relation as a set of ordered pairs.

10.

x	*y*
−15	3
2	−11
−3	−7
12	4

11.



For Items 12 and 13, make a function table of values for each function with the given input values.

12. $y = 5x - 1$ for $x = -4$, $x = 12$, $x = 0$, and $x = 0.5$

13. $y = 1.7x$ for $x = 1.7$, $x = 5$, $x = -5$, and $x = 100$

14. **Critique the reasoning of others.** Josie used the function $y = 4x$ to complete the table below.

x	1	3	5	7
y	4	12	20	28

Then she wrote these ordered pairs: (4, 1), (12, 3), (20, 5), (28, 7). Do you agree with Josie's work? Justify your reasoning.

My Notes

Learning Targets:

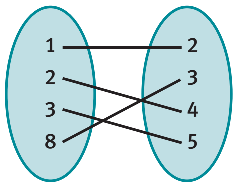
- Understand that a function is a rule that assigns exactly one output to each input.
- Identify functions using ordered pairs, tables, and mappings.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Visualization, Discussion Groups, Create Representations, Note Taking

One type of representation that helps to determine if a relation is a function is a *mapping*. The diagram in the My Notes column is a mapping. This particular mapping is a function because every input (x-value) is mapped to exactly one output (y-value).

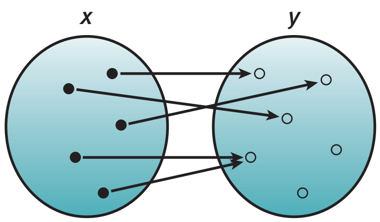
In the mapping below, all input values are written in an oval, and all output values are written in another oval. Each input value is connected with its output value.

{(1, 2), (2, 4), (3, 5), (8, 3)}



1. Is this relation a ***function***? Justify your response.

MATH TERMS

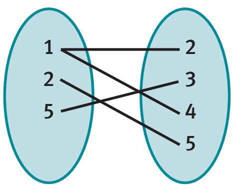


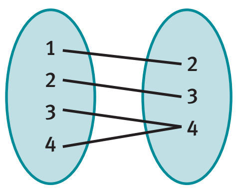
A **function** is a special kind of relation in which each input value is paired with exactly one output value.

In functions, y-values can be associated with more than one x-value. However, each input (x) has exactly one output (y).

2. Construct viable arguments. One of the relations mapped below is a function, and one is not. Determine which relation is a function and which is not. Justify your responses.

a. {(1, 2), (1, 4), (5, 3), (2, 5)} **b.** {(1, 2), (3, 4), (2, 3), (4, 4)}





My Notes

Check Your Understanding

Use mapping to determine if the following are functions. Explain.

3. $\{(1, 3), (1, 4), (1, 5)\}$

4. $y = x + 2$ for $x = \{0, 1, 2, 3, 4\}$

5.

x	1	3	5	7
y	2	5	3	2

6. Confirm that the relation you created in Item 6 in Lesson 27-1 is a function using mapping.

It is also possible to determine if a relation is a function by looking at ordered pairs. There should be only one output for each input. The ordered pairs can also be represented in a table.

7. Determine if the following relations are functions. Explain why they are or are not.

a.

x	y
−3	2
5	5
2	8
3	−3
−5	−5
6	2
−2	5
8	8

b.

x	y
−5	5
−4	4
−3	3
−2	2
−1	1
0	0
1	1
2	2

c.

x	y
2	3
4	5
6	7
8	9
3	11
4	13
5	15
6	17

My Notes

8. a. Create two tables of values to represent two relations, one that is a function and one that is *not* a function.

b. Justify your answers to part a using a mapping.

9. Make use of structure. Explain how you can identify from a table a relation that does not represent a function.

Check Your Understanding

10. Which relation represents a function? Justify your choice.

A.

x	y
1	5
3	7
3	8
4	10

B.

x	y
1	5
3	7
4	8
5	8

My Notes

LESSON 27-2 PRACTICE

Use mappings to determine if each relation represents a function.

11. {(7, 5), (8, 6), (9, 7), (10, 8)}

12. {(4, 24), (8, 18), (12, 2), (16, 6)}

13. {(−1, 1), (−1, 0), (−1, −1), (−1, −2)}

For Items 14–16, explain why each relation is or is not a function.

14.

x	2	4	6	8
y	14	28	42	56

15.

x	*y*
−1	−2
5	−6
−1	−3
5	−7

16.

x	0	10	20	30
y	5	5	5	5

17. **Model with mathematics.** Draw a mapping to prove the following relation is a function. Explain your reasoning.

{(1, 1), (2, 8), (3, 27), (4, 64)}

My Notes

Learning Targets:

- Define domain and range.
- Determine the domain and range of a relation.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Visualization, Discussion Groups, Create Representations, Note Taking

The set of all the input values is called the ***domain***. In a relation, all domain values must be matched with an output value. The set of all output values is called the ***range***.

DISCUSSION GROUP TIPS

As you read and define new terms, discuss their meanings with other group members and make connections to prior learning.

Example A

Find the domain and range of the relation:

$$\{(1, 2), (2, 4), (4, 5), (8, 3)\}$$

Step 1: Look at the x-coordinate in each ordered pair to identify the domain values. Write these numbers as a set.

Step 2: Look at the y-coordinate in each ordered pair to identify the range values. Write these numbers as a set.

Solution: The domain is $\{1, 2, 4, 8\}$ and the range is $\{2, 3, 4, 5\}$.

Try These A

Determine the domain and range of each relation.

a. $\{(2, 4), (2, 5), (2, 6), (2, 7), (2, 8)\}$

b.

x	y
3	12
4	12
12	12
1	8

1. Construct viable arguments. Use mappings to explain why each relation in Try These A is or is not a function.

My Notes

Check Your Understanding

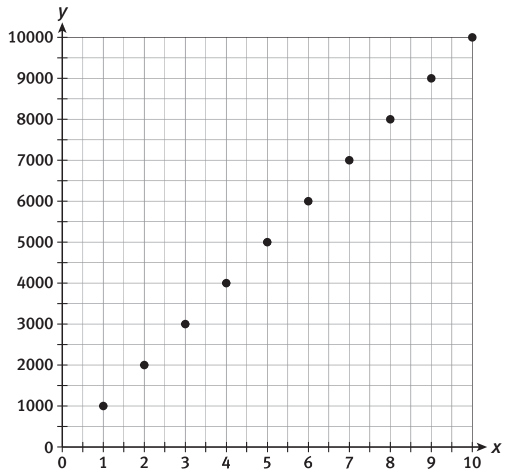
Determine the domain and range in each relation. Then state whether the relation is or is not a function.

2. $\{(3, 5), (3, 6), (3, 7), (3, 8), (3, 9)\}$

3.

x	*y*
1	10
2	11
12	12
3	9

4.



Relationships can also exist between different sets of information. For example, the pairing of the names of students in your class and their heights is one such relationship.

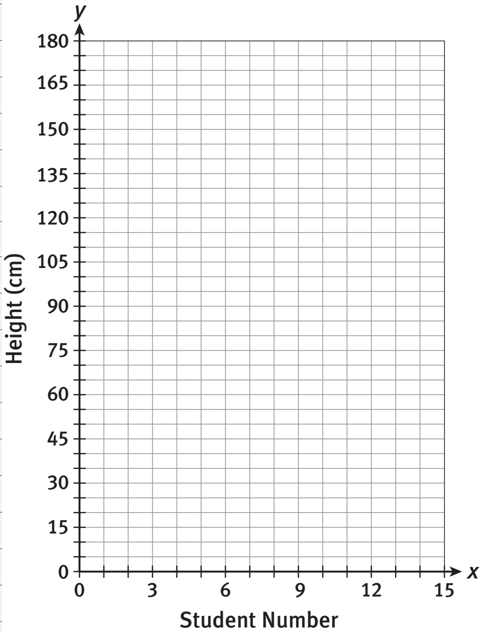
5. Collect the following information for 10 members of your class.

Student Number	First Name	Height (cm)	Length of Index Finger (cm)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			

CONNECT TO AP

When conducting observational studies in AP Statistics, the data collected are not always numerical. For example, a study might compare the fruit-juice flavor preferred by male students compared with the flavor preferred by female students.

My Notes



6. Write the student numbers and heights of 5 students in the class as ordered pairs (Student Number, Height).

7. Graph and label the coordinates of a point for each of the five ordered pairs you wrote in Item 6.

8. **Use appropriate tools strategically.** Using the information from the table, what relationships can you create other than student number to height?

9. Using one of the relationships you described in Item 8 that contains only numeric values, create five ordered pairs of students in your class.

10. Using the table in Item 5, what would be the domain of the relation that associates length of index finger to height?

11. What would be the range of the relation that associates length of index finger to height?

12. Is the relation that associates student numbers to their height a function? Use a mapping to justify your response.

My Notes

13. In your mapping in Item 12, label the two ovals with the terms *domain* and *range*.

Check Your Understanding

The table shows the results of a survey of 5 students about the number of siblings they have and the number of pets they have. Use the table to answer Items 14 and 15.

Number of Siblings	1	0	2	4	3
Number of Pets	2	0	1	1	3

14. Use a mapping to determine if the relation that associates number of siblings to number of pets is a function. Explain your reasoning.

15. Use a mapping to determine if the relation that associates the number of pets to the number of siblings is a function. Explain your reasoning.

LESSON 27-3 PRACTICE

16. What are the domain and range of the relation in Item 14?

17. What are the domain and range of the relation in Item 15?

For Items 18 and 19, determine the domain and range of each relation.

18. $\{(3, -5), (5, 6), (-3, 7), (4, 8), (2, -8)\}$

19.

x	y
−3	8
4	12
12	12
4	8

20. Use mappings to determine which of the relations in Items 18 and 19 is a function. Justify your response.

21. **Reason abstractly.** Ryan copied the table below.

x	5	7	8	4	?
y	2	0	1	6	1

He forgot to copy one of the x-values into the table. If he knows the table represents a function, what are the possible values for the x-value that is missing in the table? Explain your reasoning.

My Notes

MATH TIP

Summarize in your own words the ways to recognize a function when it is represented graphically.

Learning Targets:

- Identify functions using graphs.
- Understand the difference between discrete and continuous data.

SUGGESTED LEARNING STRATEGIES: Graphic Organizer, Visualization, Discussion Group, Create Representations, Note Taking

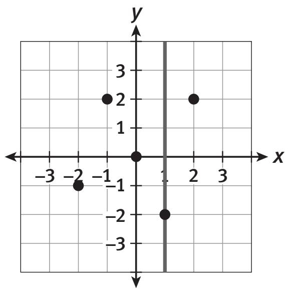
You can also determine if a relation is a function by looking at its graph. Knowing that in a function every x-value has only one y-value allows the vertical line test to be used to determine if a graph represents a function. If any vertical line intersects a graph at only one point, then the graph represents a function.

Example A

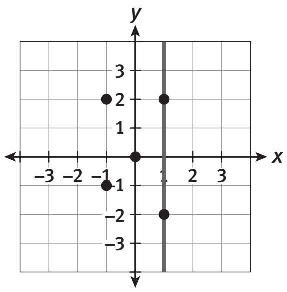
Determine which of the following graphs of relations represents a function.

Step 1: Apply the vertical line test.

Relation A:
$\{(-2, -1), (-1, 2), (0, 0), (1, -2), (2, 2)\}$



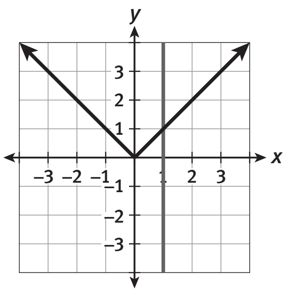
Relation B:
$\{(-1, -1), (-1, 2), (0, 0), (1, 2), (1, -2)\}$



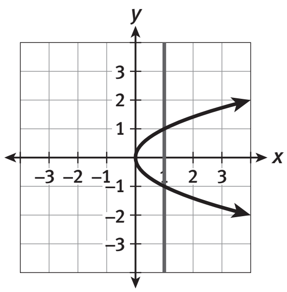
Step 2: Look at the graph to determine if any of the domain values have more than one range value.

Solution: Relation A is a function since each input has only one output. Relation B is not a function because at least one input has two different outputs.

Relation C: $y = |x|$



Relation D: $x = y^2$



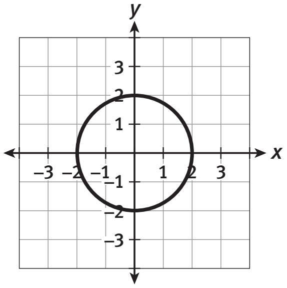
Solution: Relation C is a function since each input has only one output. Relation D is not a function because at least one input has two different outputs.

My Notes

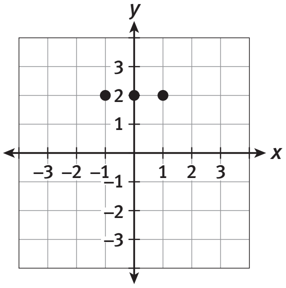
Try These A

State whether each graph represents a function. Explain your reasoning.

a.



b.



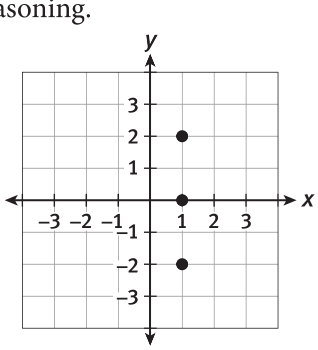
1. What does a vertical line in the vertical line test represent?

2. **Reason abstractly.** Explain how the vertical line test works. In other words, when looking at a graph of a relation, how can you determine if it is a function?

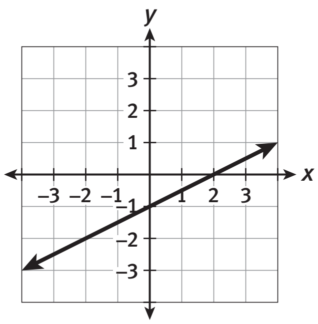
Check Your Understanding

3. Which of the following graphs represents a function? Explain your reasoning.

A.



B.



My Notes

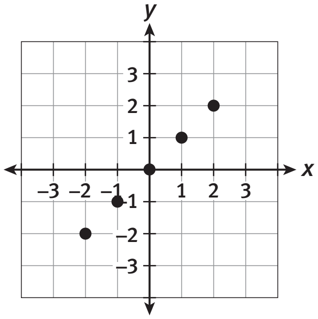
MATH TERMS

Data are **discrete** if there are only a finite number of values possible or if there is a space on the number line or on a graph between each 2 possible values.

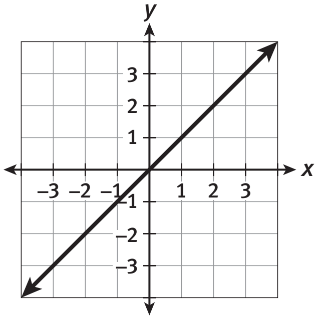
Data are **continuous** if there are no breaks in their domain or range or if the graph has no breaks, holes or gaps.

Discrete data are data that can only have certain values such as the number of people in your class. On a graph there will be a space between every two possible values. ***Continuous data*** can take on any value within a certain range, for example, height. On a graph, continuous data and continuous functions have no breaks, holes, or gaps. In the following example, Function A is discrete and Function B is continuous.

Function A



Function B



4. Functions can be represented by equations such as $y = 2x + 3$.

a. Is there any limit to the number of input values that can be used with this equation? Explain your reasoning.

b. Is the function discrete or continuous? Justify your response.

Mr. Walker collected the following data about shoe size and height from 5 students in his class.

Shoe Size	Approximate Height (in centimeters)
6	140
6.5	144
7	148
8	156
9.5	168

My Notes

5. a. Use a mapping of the relation above to determine if the relation is a function. Be sure to label the domain and range.

b. Create a graph and explain how it confirms your answer to part a. Use the My Notes column if needed.

c. Reason quantitatively. An equation that can be used to represent the relation is $y = 8x + 92$, where x represents a student's shoe size and y represents a student's height. Is there any reasonable limit to the domain values that can be used with this expression? Justify your response.

d. Is the relation discrete or continuous? Explain your reasoning.

My Notes

Mr. Walker also coaches the middle school track team and collected the following sample of data regarding the times that one of his athletes clocked while running certain distances.

Time	Distance (m)
8 minutes	1,500
9 minutes	1,700
10 minutes	1,900
11 minutes	2,100
12 minutes	2,300

6. a. Use a mapping of the relation above to determine if the relation is a function.

b. Create a graph and explain how it confirms your answer to part a. Use the My Notes column if needed.

c. An equation that can be used to represent the relation is $y = 200x - 100$, where x represents the athlete's time. Is there any reasonable limit to the input values that can be used with this expression? Explain your reasoning.

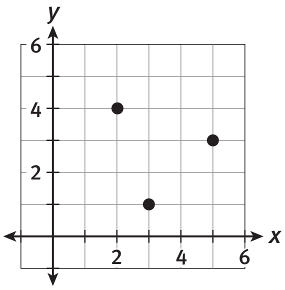
d. Is the relation discrete or continuous? Explain your reasoning.

My Notes

Check Your Understanding

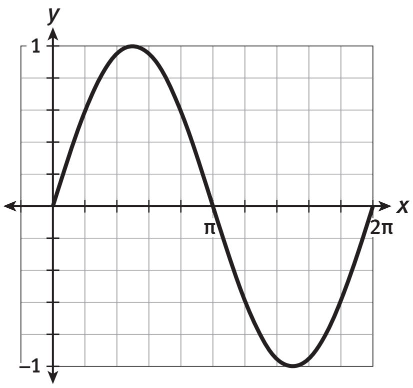
Determine if each relation shown below is discrete or continuous.

7.



8. $y = 2x - 7$

9.



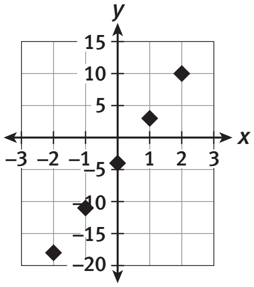
LESSON 27-4 PRACTICE

For each relation in Items 10–13:

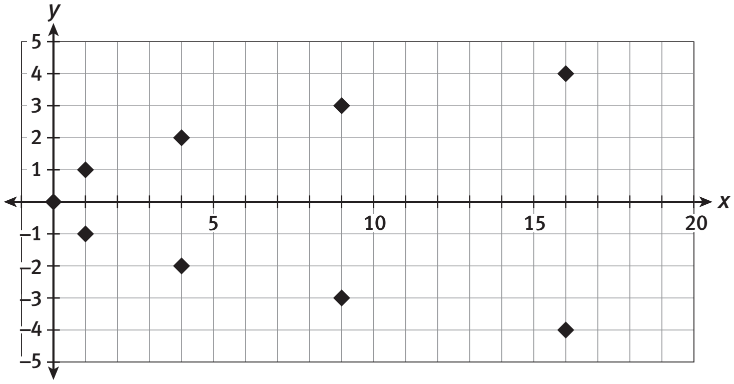
a. Determine if it is a function.

b. Determine if it is discrete or continuous.

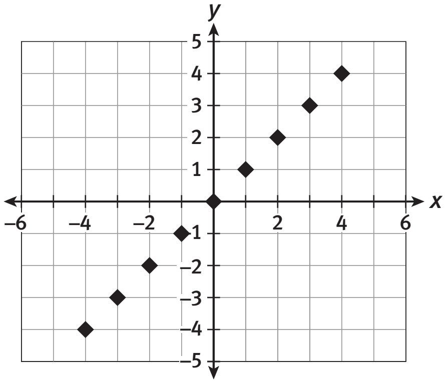
10.



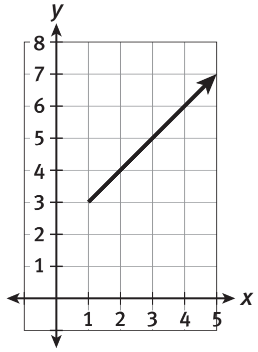
11.



12.



13.



14. Use appropriate tools strategically. Use a graphing calculator to plot the relation {(4, −1), (1, −4), (4, −7), (7, −4)}. Determine if it is a function. Explain your reasoning.

ACTIVITY 27 PRACTICE

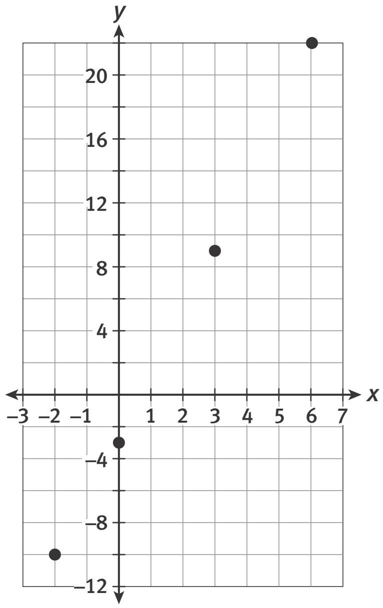
Write your answers on notebook paper.
Show your work.

Lesson 27-1

Evaluate each function for the given domain values.

1. $y = 2x + 4$; for $x = 3$, $x = 4$, and $x = 0.25$
2. $y = -6x + 2$; for $x = 5$, $x = \frac{1}{2}$, and $x = -7$
3. $y = 7 + (x - 9)$; for $x = 8$, $x = -1$, and $x = 10$

Write each relation as a set of ordered pairs.

4. 

5.

x	–6	2.5	11	15
y	6	14.5	23	27

Lesson 27-2

Use mappings to determine if each relation in Items 6–11 is a function.

6. $\{(-2, 2), (-3, 4), (-4, 5), (-2, 6), (-3, 7)\}$
7. $\{(-3, 4), (-6, 1), (6, 0), (-1, 5), (-6, 4)\}$
8.

x	5	6	7	8
y	3	3	4	4

9.

x	5	6	7	8
y	3	4	5	6

10. $y = x + 5$ for $x = 3, 5, 7, 9, 11$
11. $y = x - 9$ for $x = -1, -3, -5, -7, -9$

Lesson 27-3

State the domain and range for each relation in Items 12–16.

12. $\{(-1, 7), (-2, 4), (-2, -3), (6, -3)\}$
13. $\{(11, 2), (2, -14), (-5, 13), (58, 33)\}$
14.

x	y
5	2
9	3
1	0
–2	9

15.

x	y
1	–8
3	–6
5	–4
7	–3

16.

x	y
–1	9
–5	0
9	0
–1	4

For Items 17 and 18, determine if each relation is a function. Justify your response.

17.

x	y
–2	5
–5	7
8	8
22	17
–1	32
0	76
–12	0
17	22

18.

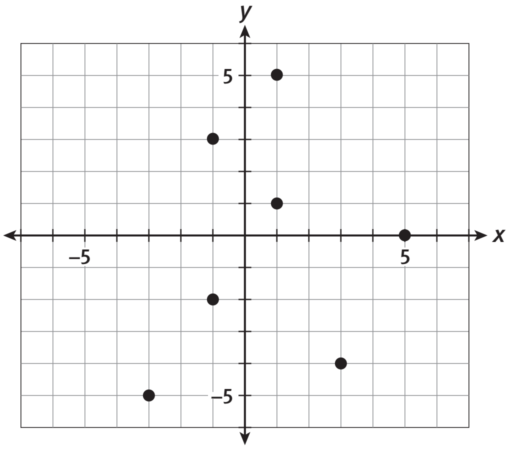
x	y
0	7
2	5
–7	0
6	–5
0	12
5	2
–1	4
1	8

19. How do the domain and range of a relation help to determine if it is a function?

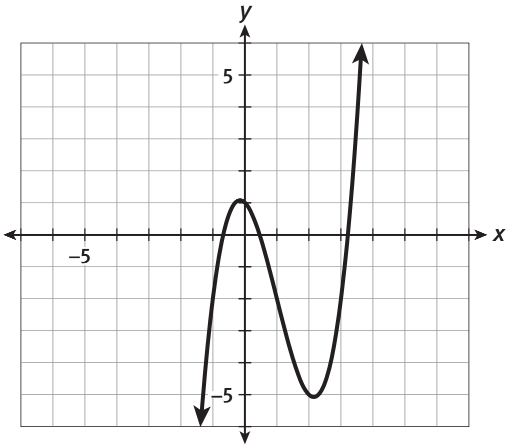
Lesson 27-4

For Items 20–23, determine if each relation is a function. Explain your reasoning.

20.



21.



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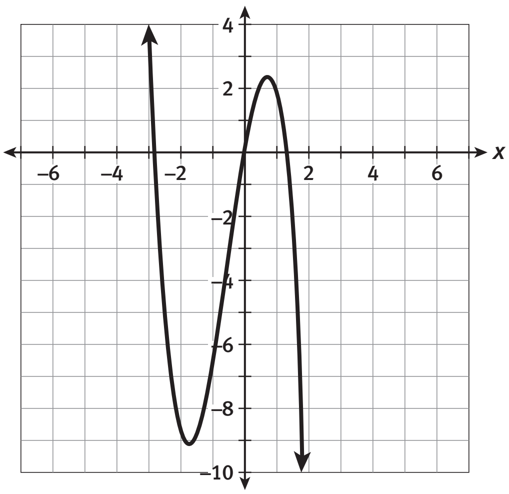
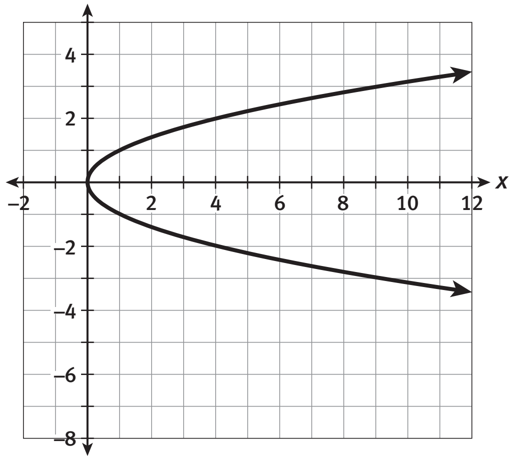
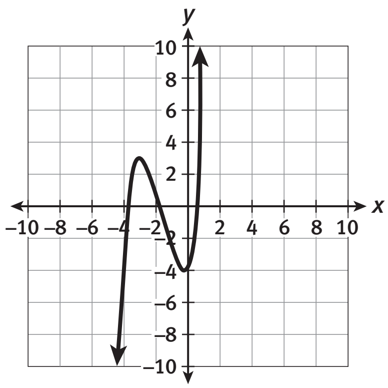


23.

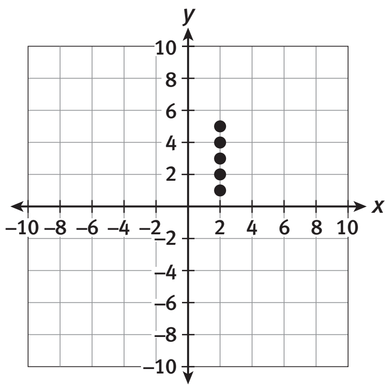


For Items 24 and 25, determine if the relations are discrete or continuous. Explain your reasoning.

24.



25.



MATHEMATICAL PRACTICES
Look for and Make Use of Structure

26. Describe the various methods to determine if a relation is a function.

Comparing Functions

ACTIVITY 28

Which Car Wins?

Lesson 28-1 Representing Functions

My Notes

Learning Targets:

- Represent functions algebraically, graphically, tabularly, or verbally.
- Compare properties of two or more functions.

SUGGESTED LEARNING STRATEGIES: Discussion Groups, Group Presentations, Graphic Organizer, Construct an Argument, Sharing and Responding, Create Representations

Functions can be represented in a variety of ways. Pages 385-388 contain graphs, equations, tables, and verbal descriptions representing a race among five model cars, each using a different type of propulsion.

CONNECT TO PHYSICS

Propulsion is the force that propels or pushes something forward or onward.

1. Cut the representations apart on the dotted lines and sort the graphs, equations, verbal descriptions, and tables to create sets. The representations in each set should show the same function describing the distance and speed of one of the model cars.

2. Attach the verbal description for car #1 to the grid labeled Function 1 on the next page. Attach the graph, table, and equation that goes with car #1 as well. Repeat for the remaining sets using the grids on the following pages. Write three or four justifications for your choice in creating each set.

3. How did you make decisions about which graphs, tables, equations, and verbal descriptions to group together?

4. **Critique the reasoning of others.** In pairs or small groups, share your work with your peers.
   a. Do you agree with the work of your peers? Explain why or why not.

   b. Ask any questions you have to any of your peers about their justifications or their sets.

   c. How was your reasoning the same and different from the reasoning of the other groups in your class?

Function 1:

Justifications:

Function 2:

Justifications:

Function 3:

Justifications:

Function 4:

Justifications:

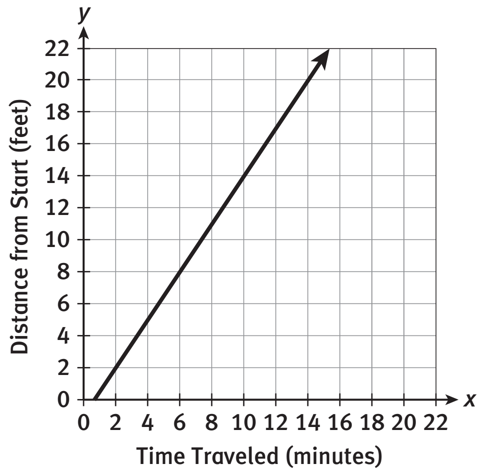
Function 5:

Justifications:

#1 A push car travels $\frac{1}{2}$ foot per minute. It has a head start of 10 feet at the beginning of the race.

$y = 1\frac{1}{4}x + 5$

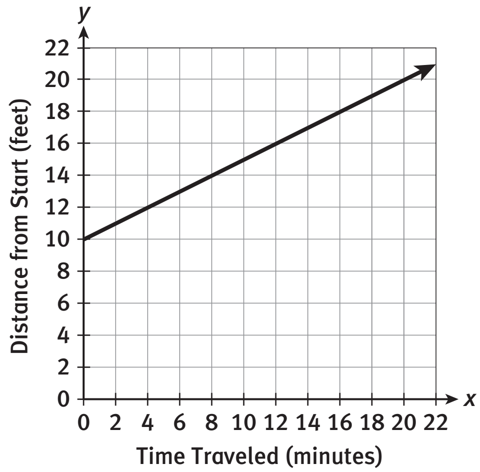
Time (in minutes)	Distance from starting line (in feet)
0	0
5	$6\frac{2}{3}$
7	$9\frac{1}{3}$
13	$17\frac{1}{3}$
27	36
50	$66\frac{2}{3}$
75	100
100	$133\frac{1}{3}$



#2 A mouse trap car travels at a rate of $1\frac{1}{3}$ feet per minute and starts on the starting line at the beginning of the race.

y = 17

Time (in minutes)	Distance from starting Line (in feet)
0	**5**
5	$11\frac{1}{4}$
7	$13\frac{3}{4}$
13	$21\frac{1}{4}$
27	$38\frac{3}{4}$
50	$67\frac{1}{2}$
75	$98\frac{3}{4}$
100	130



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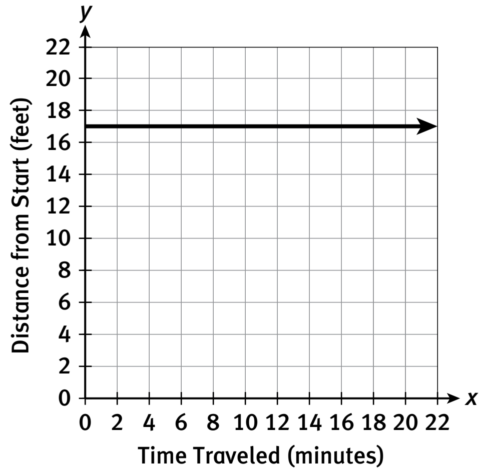
#3 A wind up car travels $1\frac{1}{2}$ feet per minute. It starts the race a foot behind the starting line.

$y = \frac{1}{2}x + 10$

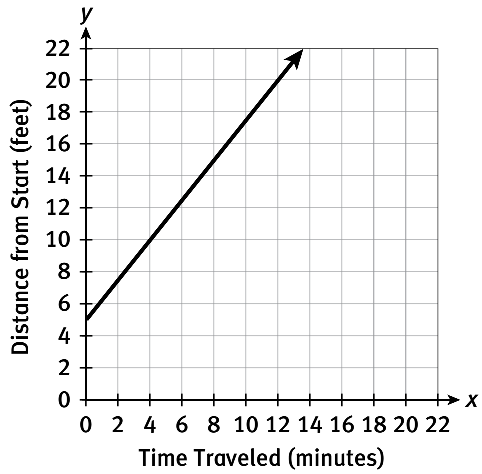
Time (in minutes)	Distance from starting line (in feet)
0	17
5	17
7	17
13	17
27	17
50	17
75	17
100	17



#4 A battery-powered car does not move but has a head start of 17 feet at the beginning of the race.

$y = 1\frac{1}{2}x - 1$

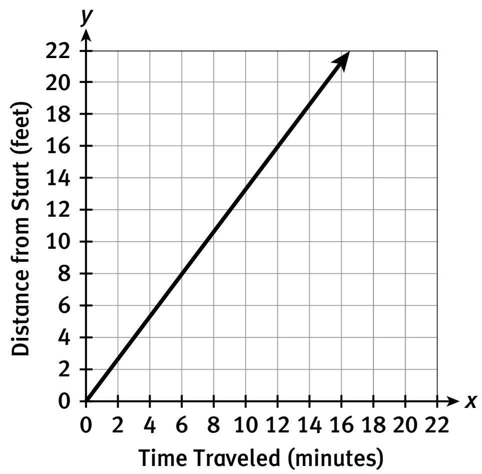
Time (in minutes)	Distance from starting Line (in feet)
0	10
4	12
8	14
13	$16\frac{1}{2}$
28	24
50	35
75	$47\frac{1}{2}$
100	60



#5 A rubber band car travels at a rate of $1\frac{1}{4}$ feet per minute and has a head start of 5 feet at the beginning of the race.

$y = \frac{4}{3}x$

Time (in minutes)	Distance from starting line (in feet)
4	5
8	11
10	14
13	$18\frac{1}{2}$
24	$35\frac{3}{4}$
50	74
75	$111\frac{1}{2}$
100	149



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My Notes

Check Your Understanding

Match each verbal description with its corresponding function representation.

5. Adam is allowed 15 text messages a day on his parents' cell phone plan.

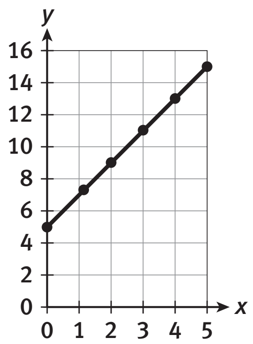
6. Ana Lucia rents a car for $20 a day.

7. Jabar starts a new job with 5 vacation days. He earns 2 additional days for every year he works.

A.

x	y
1	20
2	40
3	60
4	80
5	100

B.



C. $y = 15x$

LESSON 28-1 PRACTICE

For Items 8–10, match each verbal description with its corresponding equation and table of values.

8. Nina starts 1.5 miles from the park path entrance and inline skates at 10 miles per hour.

9. Sasha starts 0.5 mile from the park entrance and runs at 5 miles per hour.

10. Tavon starts at the park entrance and rides his bike at 15 miles per hour.

A. $y = 5x + 0.5$

B. $y = 15x$

C. $y = 10x + 1.5$

D.

x	y
0	0
1	15
2	30
3	45
4	60

E.

x	y
0	1.5
1	11.5
2	21.5
3	31.5
4	41.5

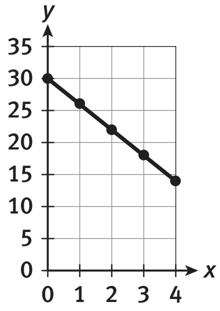
F.

x	y
0	0.5
1	5.5
2	10.5
3	15.5
4	20.5

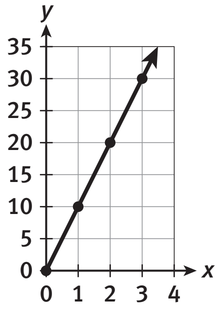
My Notes

Model with mathematics. For Items 11-13, match each graph of a function with its corresponding equation, verbal description, and table of values.

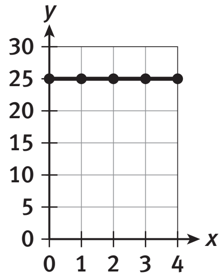
11.



12.



13.



A. $y = 10x$

B. $y = 30 - 4x$

C. $y = 25$

D. Javier has $30 in his savings account. He spends $4 a week.

E. Serena has $25 in her savings account. She doesn't save or spend any money.

F. Lila saves $10 a week.

G.

x	y
0	30
1	26
2	22
3	18
4	14

H.

x	y
0	0
1	10
2	20
3	30
4	40

I.

x	y
0	25
1	25
2	25
3	25
4	25

14. For Items 11–13, how did you decide which equation, verbal description, and table of values corresponds to each graph?

My Notes

Learning Targets:

- Compare properties of two or more functions, each represented in a different way.
- Identify examples of proportional and nonproportional functions.

SUGGESTED LEARNING STRATEGIES: Visualization, Think-Pair-Share, Create Representations, Predict and Confirm, Marking the Text

Answer the following items regarding the five cars in the race in Lesson 28-1 and their respective functions.

1. Notice that each equation is written in the form $y = mx + b$.
   a. Which part of each equation represents the speed of the car?

   b. How is this speed represented on each graph?

   c. Where is the speed found in each table?

   d. Which car is going the fastest? Explain your reasoning.

2. In all five situations, determine how the starting place of the car is represented.
   a. Which part of the equations represent where the car started?

   b. How is this represented on each graph?

   c. Where is this information found in each table?

   d. Which car has a starting place that is farthest from the starting line? Explain your reasoning.

3. What numbers make sense for the domain and range of each of these functions?

4. Are these functions discrete or continuous? Justify your response.

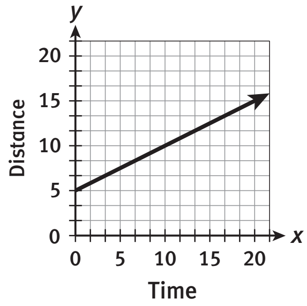
My Notes

5. **Reason quantitatively.** Determine the winner of the race if the track is 60 feet long. Use multiple representations to justify your choice.

Check Your Understanding

Each pair of representations in Items 6 and 7 describes the function of a traveling vehicle. Which vehicle in each pair is traveling faster? Justify your choices. In each case, consider the time and distance units to be the same.

6. A.

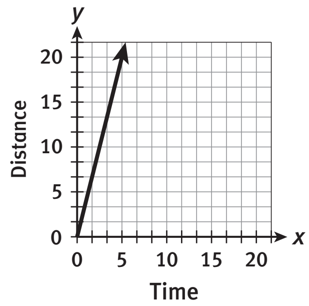


B.

Time	Distance
5	0
10	10
15	20
20	30

7. A. The push car traveled at 5 mph.

B.



In a directly proportional function, the rate, or constant of proportionality, is steady and unchanging. Proportional functions have a distinctive equation and graph. All directly proportional functions have the form $y = kx$ and form a straight line that begins at or passes through the origin.

8. Which of the functions representing the cars in the race in Lesson 28-1 are directly proportional functions? Justify your answer using all four representations.

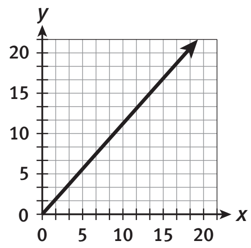
9. Which of the functions representing the cars in the race are not proportional functions? Justify your answer using all four representations.

Check Your Understanding

10. Determine if each function is or is not directly proportional. Justify your responses.

a. $y = -3x - 1$

b.



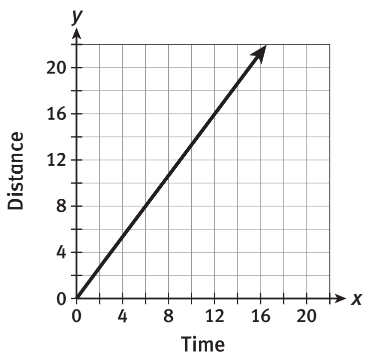
c.

Time	Distance
0	5
10	10
20	15
30	20

LESSON 28-2 PRACTICE

Each pair of representations describes the function of a traveling vehicle. Which vehicle in each pair is traveling faster? Justify your choices. In each case, consider the time and distance units to be the same.

11. A.



B. $y = 3x + 2$

12. A. $y = 2.5x + 10$

B.

Time	Distance
0	3
2	4.5
4	6
6	7.5

13. Determine which functions represented in Items 11 and 12 are directly proportional. Justify your response.

ACTIVITY 28 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 28-1

Determine which graph, table, and equation match with each of the stories. You may use square tiles or cubes to model each situation.

1. A walkway is being laid using square tiles with no edges touching. Determine the total perimeter of the tiles in the walkway based on the number of tiles.
2. A building is being painted on all four sides as well as the flat square roof. Each floor is shaped like a cube with four square walls. In order to know how much paint to purchase, determine how many square faces need to be painted depending on the number of floors.
3. Another walkway is being laid, but in this case the square tiles each touch the one laid previously. Determine the total perimeter of the tiles in the walkway based on the number of tiles and not counting sides that are touching.

A.

x	y
1	5
2	9
3	13
4	17
5	21

B.

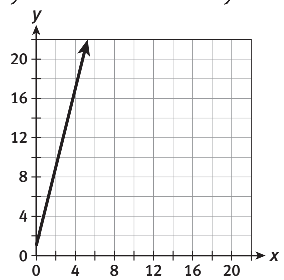
x	y
1	4
2	6
3	8
4	10
5	12

C.

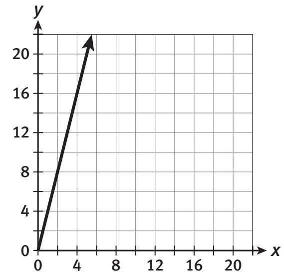
x	y
1	4
2	8
3	12
4	16
5	20

D. $y = 4x$ **E.** $y = 4x + 1$ **F.** $y = 2x + 2$

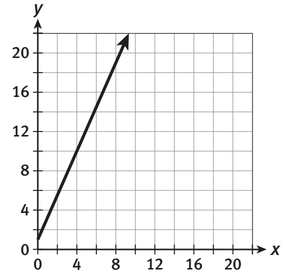
G.



H.

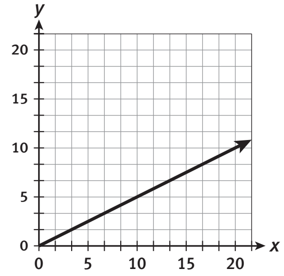


I.



Lesson 28-2

4. Give the speed of each vehicle represented below as well as the vehicle's position at the start of the race.
   a. $y = -5x - 1$
   b.



   c.

x	y
−1	2
0	5
1	8
2	11
3	14

5. Kaneesha's new job pays her a wage of \$75 each week plus an additional \$8 for each hour worked. Give the rate of change for this situation and Kaneesha's base pay.
6. Which equation represents Kaneesha's pay rate in Item 5?
   A. $y = \$8x - \$75$ **B.** $y = \$8x + \$75$
   C. $y = \$75x - \$8$ **D.** $y = \$75x + \$8$
7. For each of the functions in Item 4, determine if it is directly proportional or not. Justify your answer for each.
8. A vehicle at the starting line travels at a constant speed of 0.5 miles per minute. Which function could describe this vehicle?
   A. $y = 0.5$ **B.** $y = 0.5x$
   C. $y = x + 0.5$ **D.** $y = 0.5x + 1$

MATHEMATICAL PRACTICES
Look For and Make Use of Structure

9. When comparing functions, which representation of a function is the easiest for you to use? Justify your choice using an example.

Constructing Functions

Hold On to Your Hats

Lesson 29-1 Construct a Function

My Notes

Learning Targets:

- Construct a function to model a linear relationship between two quantities.
- Graph functions that model linear relationships.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Sharing and Responding, Identify a Subtask, Look for a Pattern, Note Taking

Wind turbines create energy from the power of the wind. Turbines can be small and power a home or small building, or they can be large and contribute power to an entire power grid. A large group of wind turbines are run by the Tennessee Valley Authority on Buffalo Mountain near Oak Ridge, Tennessee.

Buffalo Mountain holds three older turbines that are 213 feet tall and have blades that are 75 feet in length. Each of these older turbines generates 660 kilowatts of power per hour when they are producing their maximum amount of power.

There are newer, larger turbines as well that have a height of 260 feet with blades that are 135 feet long. Each of these newer turbines creates 1,800 kilowatts of power per hour.

CONNECT TO SCIENCE

Wind power is one of many types of power currently being used. There are wind turbines in sites across the globe.

1. Determine how much energy, in kilowatt-hours (kWH), each of the newer turbines generates in a 24-hour day.

My Notes

2. Write a function to represent the amount of energy generated by a newer turbine for x number of hours. Be sure to define the variables you use.

3. Reason abstractly. Explain why the function you wrote in Item 2 is directly proportional.

Unfortunately, wind turbines only produce at top capacity at a small range of wind speeds. It has been suggested that wind turbines only produce 40% of the power that they theoretically should.

4. a. How much energy would be generated in one 24-hour day with the turbines producing only 40% of the expected power?

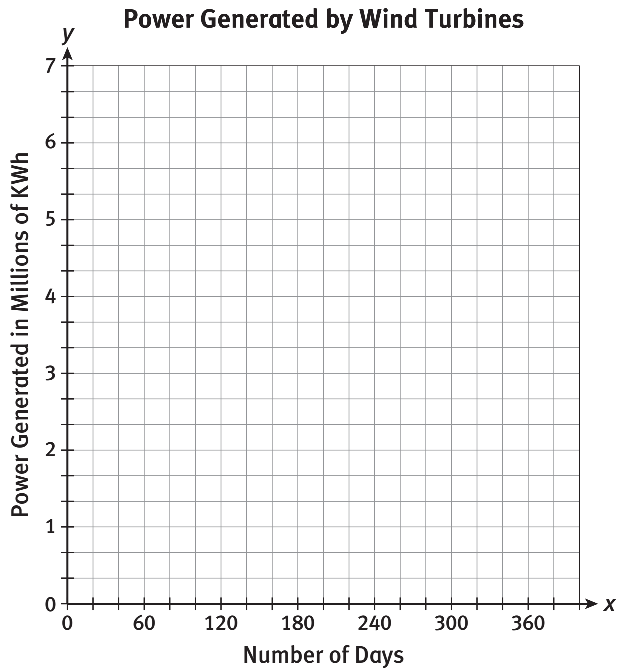
b. Rewrite your function from Item 2 to reflect that wind turbines produce only 40% of the power they are rated to produce.

5. Use the function you wrote in Item 4b to complete the table of values below.

Days	Power Generated
30	
60	
90	
120	
180	
365	

My Notes

6. Graph the power generated by each of these newer turbines below.



7. Consider the equation you wrote in Item 4b.

a. What are the domain and range of this function?

b. Is this function discrete or continuous?

8. a. How would your equation change to find the power generated by all 18 of these newer turbines on Buffalo Mountain?

b. Rewrite your function to reflect the total number of newer turbines on the mountain.

My Notes

Check Your Understanding

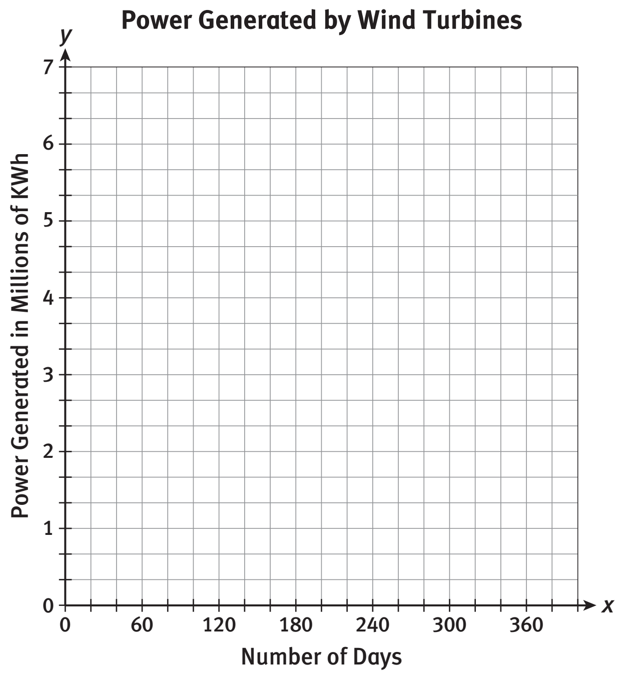
9. Determine how much energy is produced by the three older turbines.

a. Write a function to represent the amount of power generated by the three older turbines.

b. Complete the table using the function you wrote in part a.

Days	30	60	90	120	180	365
Power Generated						

c. Graph the function you wrote in part a.



d. What are the domain and range of this function?

10. How much energy is generated by all the turbines on Buffalo Mountain in one year? Show your work.

11. An average home in Oak Ridge, Tennessee, uses approximately 11,496 kilowatt-hours of electricity per year. For about how many homes could the Buffalo Mountain turbines provide energy per year?

My Notes

Check Your Understanding

12. How much more energy per year does one newer turbine generate than one older turbine? Justify your response.

13. If the cost of electricity is $.09 per kilowatt-hour, what is the value in dollars of the additional electricity generated using the newer turbine over one year?

LESSON 29-1 PRACTICE

Water flows from the faucet at a rate of 2.5 gallons per minute (gpm). Use this information to answer Items 14–17.

14. Write a function to represent the problem context. Be sure to define the variables you use.

15. Use the function you wrote in Item 14 to complete the table.

Minutes	0	5	10	15	18
Gallons					

16. Use the completed table in Item 15 to graph the function you wrote in Item 14. Be sure to title the graph and write the scale on the axes.

17. Suppose that low water pressure causes the flow rate to run at 80% of the normal flow. Rewrite your function from Item 14 to reflect that the water flow is only 80% of the normal flow rate.

Connie babysits afterschool and on weekends to earn extra money. She charges $5 per hour and works 6 hours every week. Use this information to answer Items 18 and 19.

18. **a.** Write a function to represent Connie's situation. Be sure to define the variables you use.
b. What are the domain and range of the function?
c. Is the function directly proportional? Explain why or why not.

19. **Reason quantitatively.** Connie is saving her babysitting money to buy a new tablet that costs $250. How many weeks does Connie need to work to earn enough money to buy the tablet?

My Notes

Learning Targets:

- Determine the rate of change and initial value of a function.
- Interpret the rate of change and initial value of a linear function in terms of the situation it models.
- Identify examples of proportional and nonproportional functions that arise from mathematical and real-world problems.

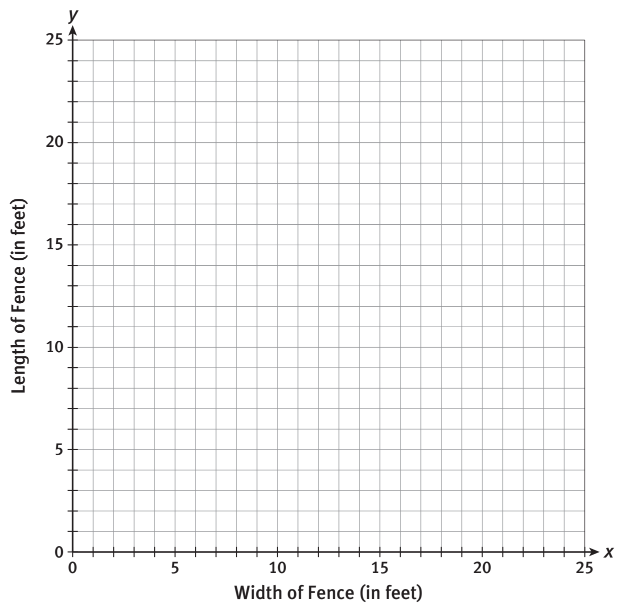
SUGGESTED LEARNING STRATEGIES: Marking the Text, Create Representations, Think-Pair-Share, Visualization

Nikita is adding a small wind turbine to her home. She wants to fence off the area around the wind turbine so that children do not get hurt. She has 60 feet of fencing and wants to use it to create a rectangular border around the wind turbine. Nikita is considering various combinations of length and width for her rectangular fence.

1. Complete the table below with the possible lengths and widths of the rectangular fence Nikita plans to put around the wind turbine.

Width of Fence	Length of Fence
5	
10	
15	
	10
	5

2. Plot the data from the table on the grid provided.



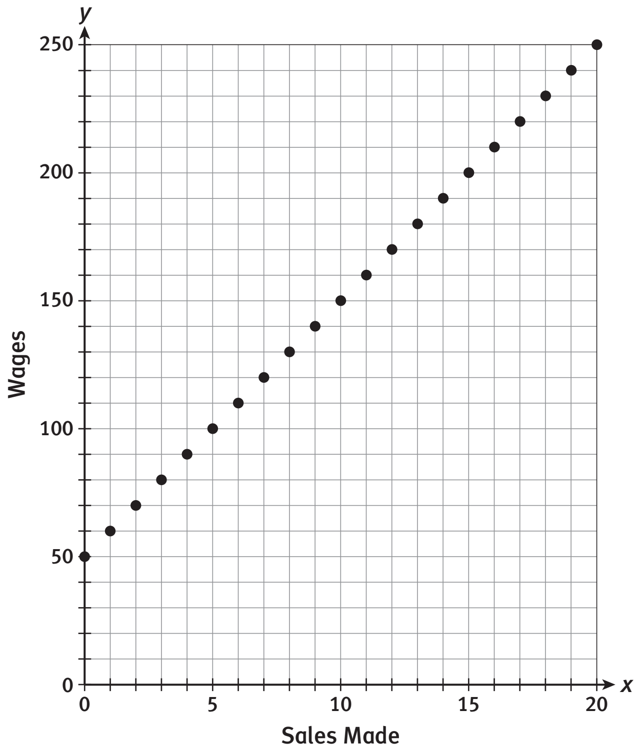
My Notes

3. Examine the graph you have plotted. How are the length and width of the fence related to each other?

Example A

When she is not at home working on her wind turbine, Nikita works part-time as a salesperson.

The graph below shows the relationship between the number of sales Nikita makes and the wages she earns. Determine and explain the meaning of the rate of change and the initial value in terms of Nikita's situation. Then determine whether the function is proportional or nonproportional.



Step 1: Find the slope to find the rate of change:

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{70 - 60}{2 - 1} = \frac{10}{1}$$

Nikita earns \$10 for each sale she makes.

Step 2: The initial value is found by finding the y-value when the x-value is 0. Looking at the graph, when $x = 0$, $y = \$50$. The initial value is (0, 50). This is her base salary, which she earns regardless of how many sales she makes.

My Notes

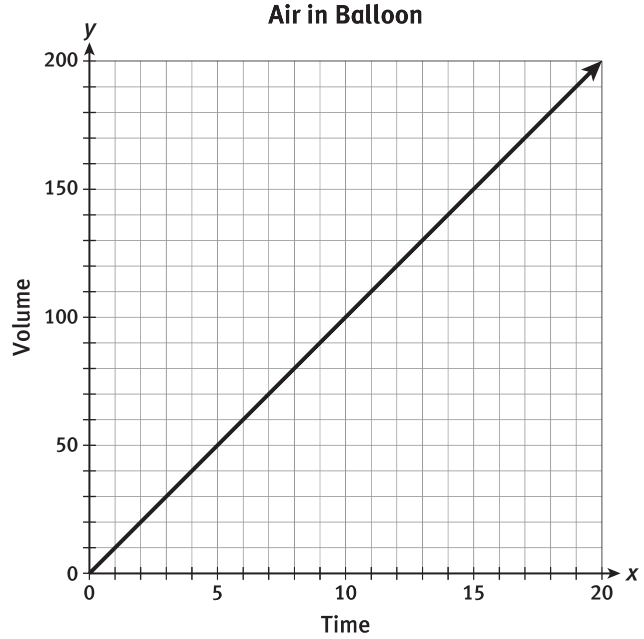
Step 3: Determine if the function is proportional or nonproportional. The graph shows a straight line that does not go through the origin, and the initial value is not (0, 0). So the function is nonproportional.

Solution: Nikita earns $10 per sale and starts with a base salary of $50 in wages, and her function of wages earned per sale is nonproportional.

Try These A

Determine and explain the meaning of the rate of change and the initial value for each of the functions represented below. State whether each function is proportional or nonproportional.

a.



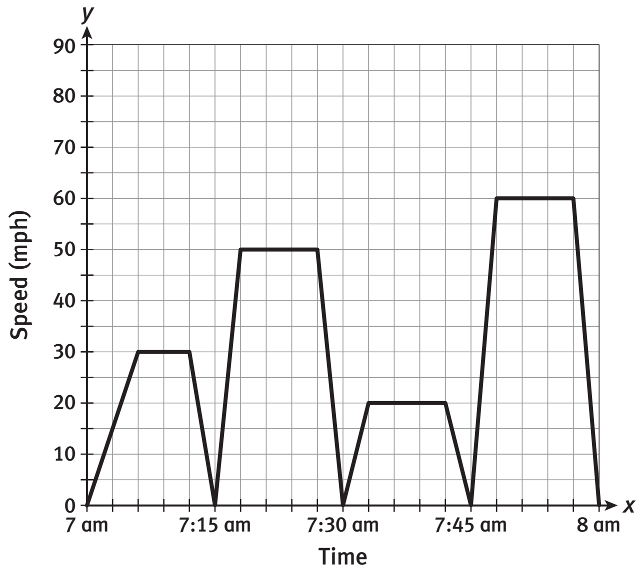
b.

Time in Seconds	0	1	2	3	4
Distance Mouse Runs in Meters	0	2.6	5.2	7.8	10.4

c. In 1 yard there are 3 feet, in 2 yards there are 6 feet, and so on.

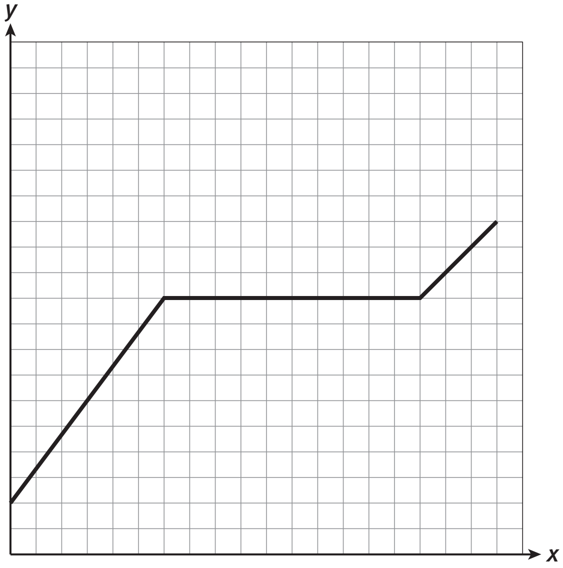
My Notes

4. Make sense of problems. The graph below shows the times and speeds for a commuter train with many morning stops. Determine between which two times the train displayed the greatest rate of change and explain how you know.



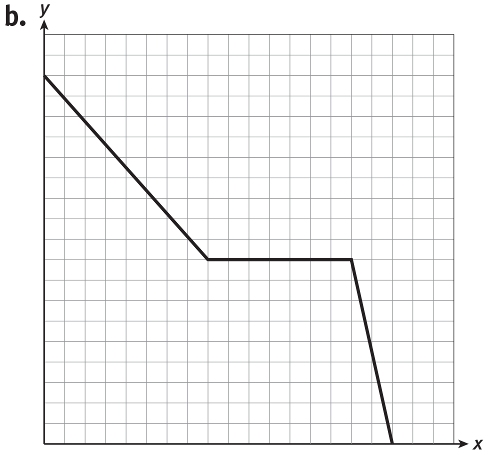
5. Describe a situation for each graph given. Share your descriptions with your group. Compare and contrast your situation choices with those of other group members.

a.



My Notes

b.



Check Your Understanding

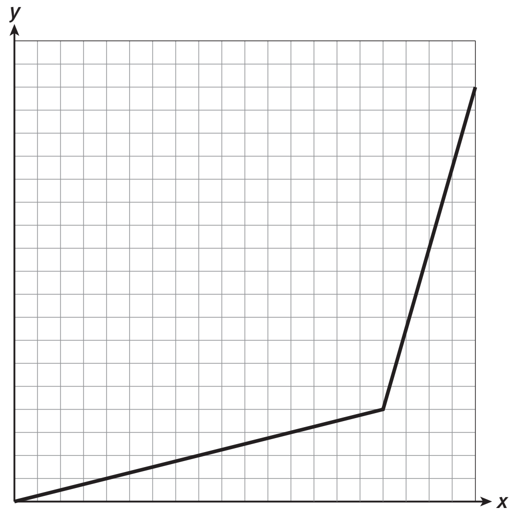
6. For each situation represented below, determine the rate of change and initial value of the function that models the situation. Give units for these values where appropriate. Then state whether the function in each situation is proportional or nonproportional:

a. Tommy Lee runs in marathons. When he practices, he tries to pace himself so that he runs 10 miles in 90 minutes.

b.

x	*y*
0	12.5
2	37.5
4	62.5
6	87.5

7. Describe a situation for the data modeled in this graph.



My Notes

LESSON 29-2 PRACTICE

The table below shows common text messaging charges that Mrs. Troy pays.

Number of Texts	0	100	500	1,000	1,500
Charge	$20	$22	$30	$40	$50

8. Determine the rate of change of the function represented in the table. Explain the meaning of the rate of change in terms of Mrs. Troy's situation.

9. Determine the initial value of the function represented in the table. Explain the meaning of the initial value in terms of Mrs. Troy's situation.

10. Is Mrs. Troy's function for text messaging charges proportional or nonproportional? Justify your response.

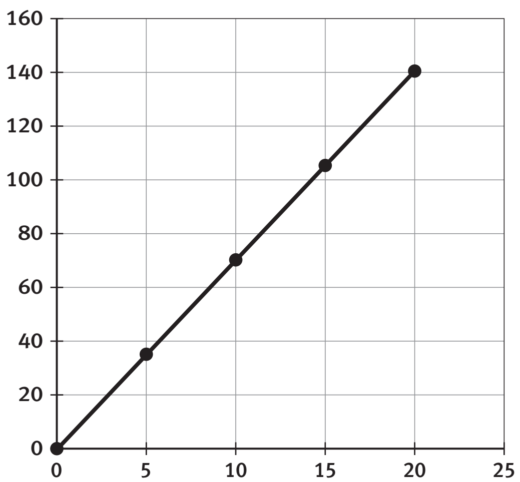
Caden is saving up his money to buy a set of classic comic books. He started with $50 he got for his birthday, and now he is putting away $20 a week that he receives for doing his chores.

11. What is the rate of change of the function that models Caden's savings?

12. What is the initial value of the function?

13. Is the function proportional or nonproportional?

14. **Critique the reasoning of others.** Darren looked at the graph below and determined that the function is proportional. Do you agree with Darren? Explain your reasoning.



ACTIVITY 29 PRACTICE

Write your answers on notebook paper. Show your work.

Lesson 29-1

A plant grows at a constant rate of 12 mm each day. Use this information to answer Items 1–4.

1. Make a table of the plant's height over an 8-day period.
2. Graph the plant's growth over time in days.
3. Write a function to model the plant's growth.
4. Determine the plant's height on the 80th day.

Another plant has a height of 20 millimeters when it is first measured and grows at a constant rate of 6 mm per day. Use this information to answer Items 5–8.

5. Make a table of the plant's height over an 8-day period.
6. Graph the plant's height over an 8-day period.
7. Write a function to model the plant's growth.
8. Determine the plant's height on the 80th day.
9. Compare and contrast the two functions you wrote in Items 3 and 7. Which function is directly proportional? Explain your reasoning.

For Items 10–12, use pattern blocks to create trains of polygons. Determine the perimeter of each train assuming that each side not touching another polygon is part of the perimeter and has a length of 1 unit.

10. Using squares:
    a. Sketch the pattern.
    b. Complete a table.
    c. Sketch a graph of the relation.
    d. Write an equation for the linear relation.
11. Using triangles:
    a. Sketch the pattern.
    b. Complete a table.
    c. Sketch a graph of the relation.
    d. Write an equation for the linear relation.
12. Using hexagons:
    a. Sketch the pattern.
    b. Complete a table.
    c. Sketch a graph of the relation.
    d. Write an equation for the linear relation.

Lesson 29-2

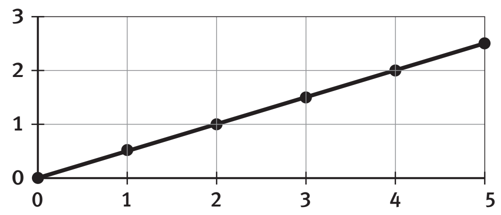
13. Consider the equations you wrote in Items 10d, 11d, and 12d. Compare and contrast these equations in terms of rate of change and initial value.

Use the equation below to answer Items 14–16.

$$y = 10 + 2x$$

14. What is the initial value?
    A. (0, 0) **B.** (0, 2)
    C. (0, 10) **D.** (0, 12)
15. What is the rate of change?
    A. 0 **B.** 2
    C. 10 **D.** 12
16. Is this function proportional or nonproportional? Explain your reasoning.

Use the graph below to answer Items 17–19.



17. What is the initial value?

18. What is the rate of change?

19. Is this function proportional or nonproportional? Explain your reasoning.

Use the table below to answer Items 20–22.

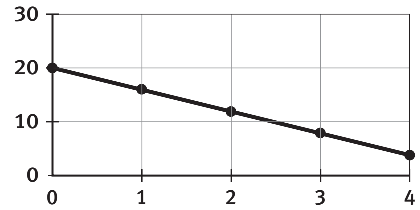
Distance (miles)	Time (min)
0	0
1	6
2	12
3	18
4	24

20. What is the initial value?

21. What is the rate of change?

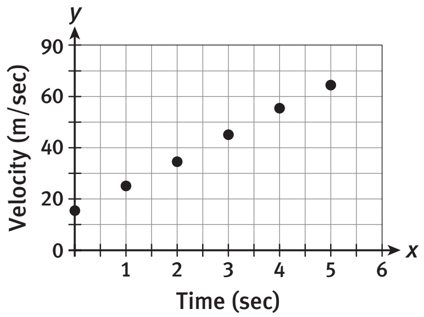
22. Is this function proportional or nonproportional? Explain your reasoning.

23. Describe a situation for the data modeled in the graph below:



MATHEMATICAL PRACTICES
Model with Mathematics

24.



The graph above shows a skydiver's falling speed in meters per second.

a. Is this relationship proportional? Approximately what is the initial value here, and what does it represent in this situation?

b. Approximately what is the rate of change here, and what does it represent in this situation?

c. Consider what you think will happen to this graph after $x = 5$ seconds. Will the graph continue to look the same, or will it change at 6 or 7 or 8 seconds or some point after that? What is happening at that point in this situation?

Memory cards are used in many devices. You probably own several devices that use these cards to hold photos, music, or documents. The table below shows the number of songs, the minutes of video, and the number of photos that different-size memory cards can hold.

CONNECT TO TECHNOLOGY

MB = megabytes
GB = gigabytes
1,024 MB = 1 GB

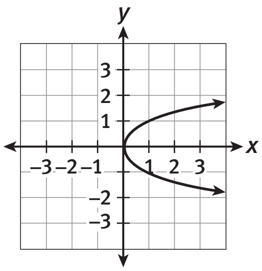
Size of Memory Card	Songs	Minutes of Video	Photos
32 MB	6	2	26
64 MB	12	4	52
128 MB	24	8	104
256 MB	48	16	208
512 MB	96	32	416
1 GB	192	64	832
2 GB	384	128	1,664

1. The data in the table relating memory card size to the number of songs the card can hold represent a function. Explain why this is true.
2. The data in the table relating memory card size to the number of photos the card holds represent a function. Is this function proportional? Explain your reasoning.
3. Graph the relationship between the size of the memory card and the minutes of video the card holds.
4. How would your graph change if you were graphing the number of photos rather than the minutes of video?

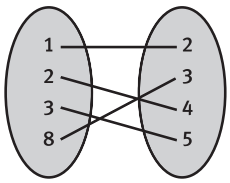
5. State whether each of the following represents a function. Explain your reasoning.

a. {(1, 2), (2, 4), (4, 2), (8, 4)}

b.



c.

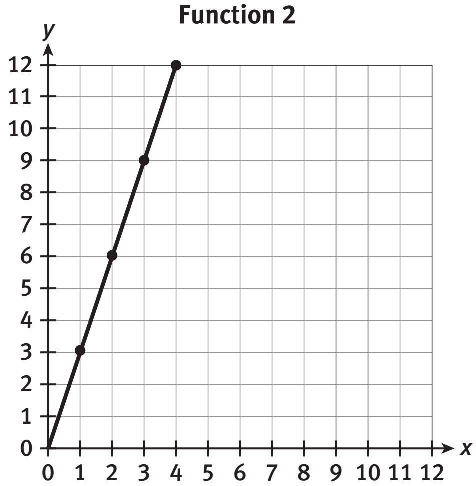


6. Two functions are shown below.

Function 1

$y = 2x + 4$

Function 2



a. Use a representation different from the one given to describe each of the functions.

b. Evaluate each function for $x = 5$.

c. Compare and contrast the two functions. Use appropriate math terminology from this unit in your answer.

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 1, 2, 3, 4, 5a-c, 6a-c)	• Clear and accurate understanding of functions, proportional functions, and evaluating functions.	• Correct understanding of functions, proportional functions, and evaluating functions.	• Partial understanding of functions, proportional functions, and evaluating functions.	• Inaccurate or incomplete understanding of functions, proportional functions, and evaluating functions.
Problem Solving (Item 4)	• An appropriate and efficient strategy that results in a correct answer.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers.	• No clear strategy when solving problems.
Mathematical Modeling / Representations (Items 1, 2, 3, 4, 5a-c, 6a)	• Clear and accurate understanding of representing a function using a table, a graph, a list of ordered pairs, a diagram, or an equation.	• An understanding of representing a function using a table, graph, list of ordered pairs, diagram, or equation.	• Partial understanding of representing a function using a table, graph, list of ordered pairs, diagram, or equation.	• Inaccurate or incomplete understanding of representing a function using a table, graph, list of ordered pairs, diagram, or equation.
Reasoning and Communication (Items 1, 2, 4, 5a-c, 6a, 6c)	• Precise use of appropriate math terms and language to explain why a relation is a function or a proportional function. • Precise use of appropriate math terms and language to describe a function or to compare and contrast two functions.	• An adequate explanation of why a relation is a function or a proportional function. • An adequate description of a function or comparison of two functions.	• A misleading or confusing explanation of why a relation is a function or a proportional function. • A misleading or confusing description of a function or comparison of two functions.	• An incomplete or inaccurate explanation of why a relation is a function or a proportional function. • An incomplete or inaccurate description of a function, or comparison of two functions.

Linear Functions

Get in Line

Lesson 30-1 Rate of Change

My Notes

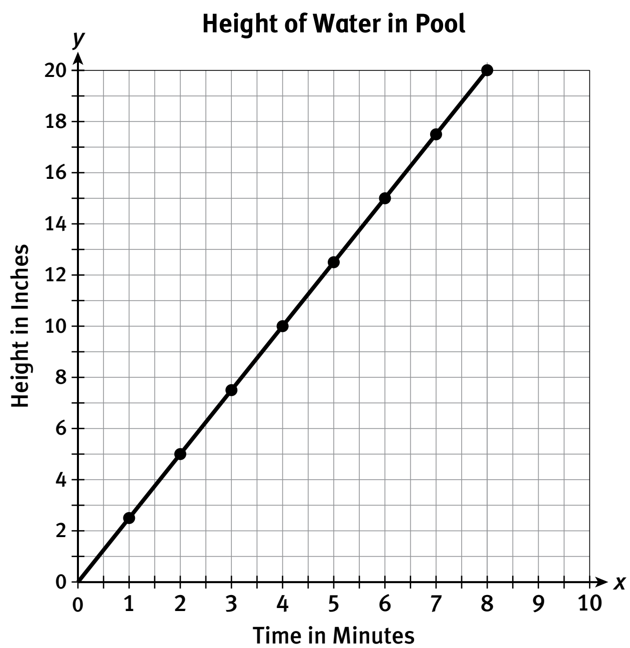
Learning Targets:

- Model linear relationships between quantities using functions.
- Identify and represent linear functions with tables, graphs, and equations.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Note Taking, Sharing and Responding, Interactive Word Wall, Create Representations, Look for Patterns

The ***rate of change*** in a relationship represents the ratio of the vertical change in y (output) to the horizontal change in x (input).

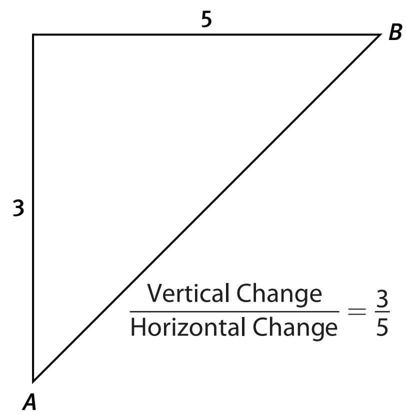
1. The height of water in a pool is shown in the graph below.



a. Determine the rate of change of the height of water in a pool.

b. Describe the rate of change in the graph.

MATH TERMS



The **rate of change** between points A and B is $\frac{3}{5}$.

My Notes

You can also see rate of change from a table of values.

Example A

The table shows the data from the graph above. How is the rate of change from the graph above represented in this table?

Time (minutes)	0	1	2	3	4	5	6	7	8
Height (inches)	0	2.5	5	7.5	10	12.5	15	17.5	20

Step 1: Look at the change in time between consecutive minutes.

Step 2: Look at the change in height between consecutive heights.

Step 3: Write the rate of change as a ratio of the vertical change to the horizontal change.

$$\frac{\text{vertical change}}{\text{horizontal change}} = \frac{2.5 \text{ inches}}{1 \text{ minute}} = 2.5 \text{ inches for each minute}$$

Solution: The rate of change is represented in the table as 2.5 inches in height for every minute. This is represented by a constant ratio of the difference between each pair of y-values to the difference between each pair of x-values.

Try These A

The table below represents the fare schedule for a taxi cab driver's fares.

Number of Miles	Cab Fee ($)
0	5
1	8
2	11
3	14
4	17
5	20

a. Determine the rate of change.
b. Interpret the meaning of the rate of change in this context.

You can also see rate of change by looking at equations.

2. The equation for the data represented in Item 1 and Example A is $y = 2.5x$. How is the rate of change you determined in Item 1 and Example A represented in this equation?

My Notes

Check Your Understanding

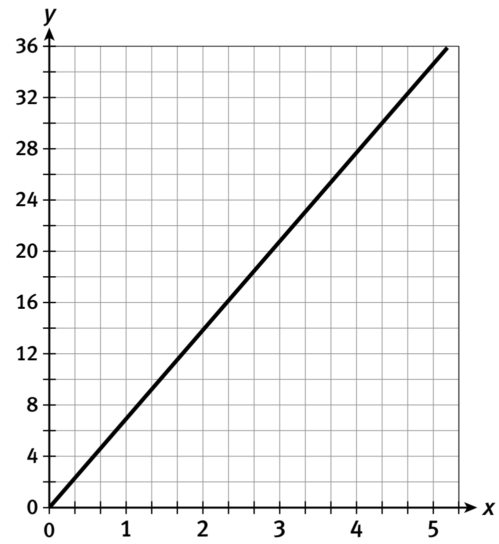
3. Each representation describes the cost for beach bike rental from a different shop. Find the rate of change for each representation.

a. $y = 20x + 2$

b.

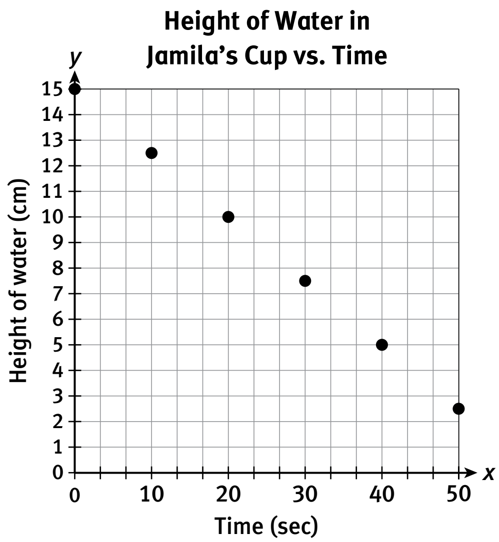
Number days	Cost ($)
0	15
1	18
2	21
3	24
4	27
5	30

c.



Data points are linear if they show a constant rate of change. When you plot the points of linear data on a coordinate plane, they lie on a straight line. The data points represented in Item 1 and in Example A are linear.

Jamila is using a straw to drink water from a cylindrical cup. The height of the water remaining in the cup is a function of the time the person has been drinking. The following graph gives the height of the water at different time intervals.



4. Using the data from the graph, complete the table.

Time (sec)	Height (cm)
0	
10	
20	
30	
40	
50	

My Notes

5. **Express regularity in repeated reasoning.** Describe any patterns you observe in the table about the height of the water over time.

6. Is the relationship between time and the height of the water linear?
   a. Explain using the graph.

   b. Explain using the table.

7. What is the change in the water level from 20 seconds to 30 seconds?

8. Determine the rate of change of the water level over time.

9. **Reason quantitatively.** Predict the height of the water at 52 seconds. How did you make your prediction? If you wanted to look at many different times, would your method still be effective?

10. Write a function that gives the height of the water, h, in terms of the time, t.

11. How long will it take for Jamila to drink all her water and completely empty out her cup? Use multiple representations to justify your response.

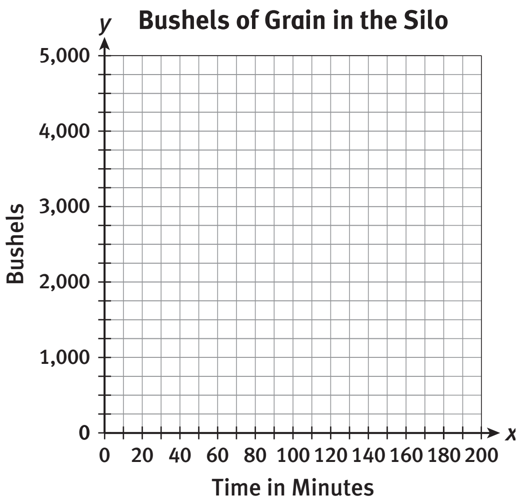
My Notes

Check Your Understanding

A farmer is filling a cylindrical silo with grain. The amount of grain in the silo is a function of the time the grain has been pouring into the silo.

Time (minutes)	0	20	40	60	80	100	120	140	160
Bushels	0	500	1,000	1,500	2,000	2,500	3,000	3,500	4,000

12. Complete the graph below to represent this data.



13. What is the rate of change shown in the table and graph above?

14. Is this function linear? Justify your response.

15. Write a function for this situation using t for the time and b for the number of bushels of grain.

16. If the silo holds 7,000 bushels, how many hours will it take to fill the silo?

CONNECT TO FARMING

A **silo** holds grain in storage until it is moved to a final destination to be used.

A **bushel** is a unit of dry volume, equivalent to 8 gallons. It is used most often in agriculture. It is abbreviated as bsh. or bu.

My Notes

LESSON 30-1 PRACTICE

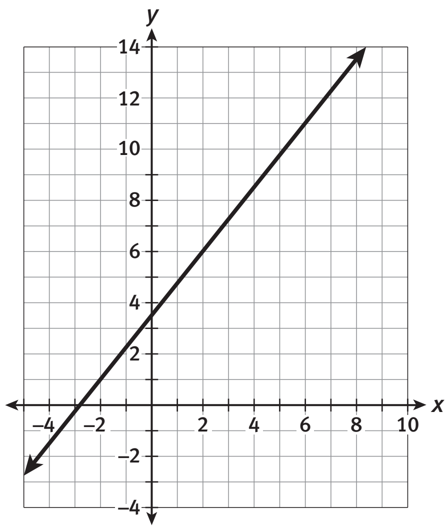
17. Sam reads the following number of pages per week for his science class.

Week	1	2	3	4
Number of Pages Read	75	150	225	300

a. What is Sam's rate of change in the number of pages he reads each week?

b. At this rate, how many pages will Sam have read at the end of the class, 8 weeks?

18. Why is the data represented in this graph linear?

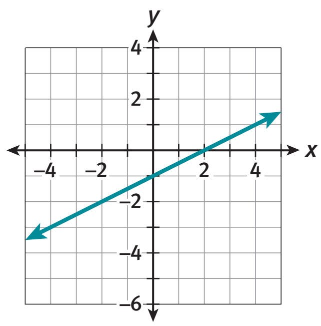


19. A writer's last four articles and fees paid have been recorded below. What is the rate of change in the writer's fee from writing a 700-word article to an 800-word article?

Total Words	500	600	700	800	900
Fees Paid	$45	$54	$63	$72	$81

20. Model with mathematics. A function representing the life expectancy at birth for those living in warm weather is $y = 0.32x - 0.03$, where x is the number of years since birth. How is the rate of change of the life expectancy represented in this function?

21. What rate of change is represented by the following graph?



My Notes

Learning Targets:

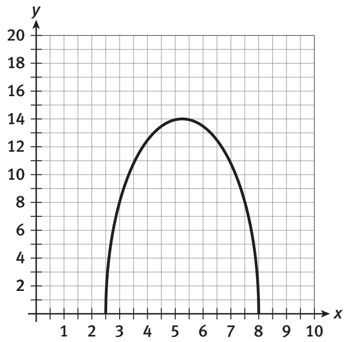
- Identify linear and non-linear functions from tables, graphs, and equations.
- Graph a linear function from a verbal description.
- Understand that $y = mx + b$ defines a linear equation.

SUGGESTED LEARNING STRATEGIES: Summarizing, Visualization, Create Representations, Think/Pair/Share

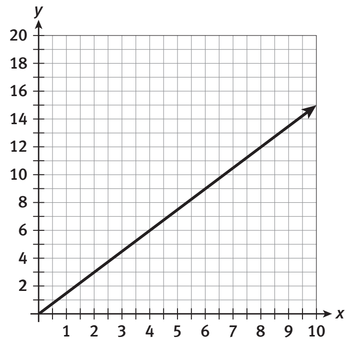
A linear relationship has a constant rate of change.

1. **Make use of structure.** In each pair below, determine which representation describes a *linear* relationship. Justify your choice.

   a. Graph 1



Graph 2



   b.

Table 1

x	0	1	2	3	4	5
y	0	3.25	6.5	9.75	13	16.25

Table 2

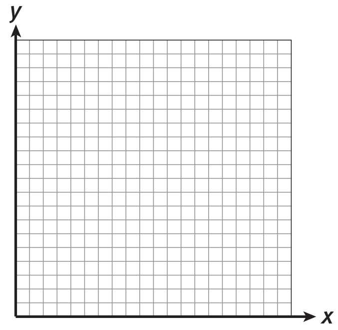
x	0	1	2	3	4	5
y	0	3.5	6.5	10	13	16.5

   c. Equation 1: $y = 3x + 2$ Equation 2: $y = x^2$

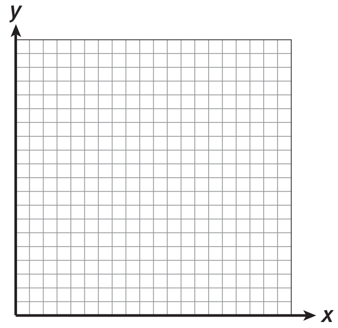
My Notes

2. Model with mathematics. Sketch a graph for the functions described below.

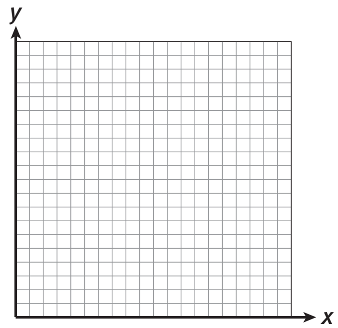
a. At Sundae King, the price of a sundae is a function of the weight of the sundae. The king charges the same amount for each ounce of weight.



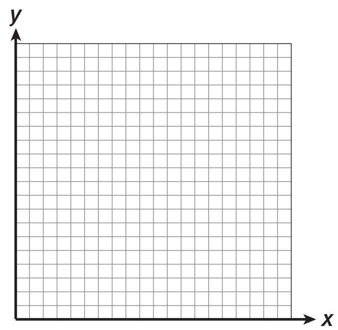
b. The cost to join an online gaming site is \$12. The player is then charged 1.75 for each hour played.



c. $y = 3x$



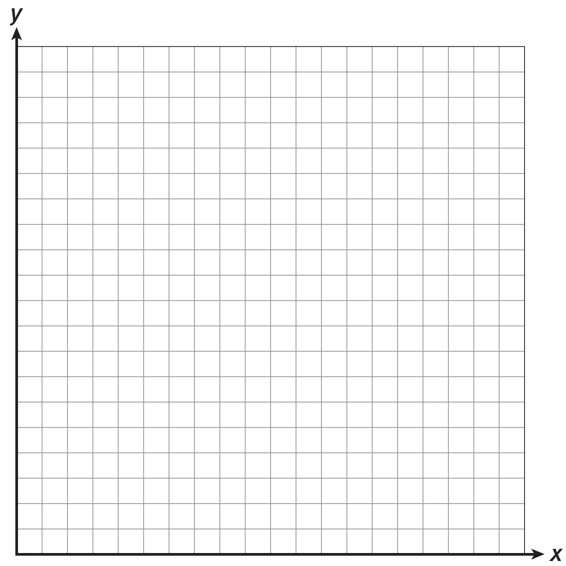
d. A cell phone bill is a flat monthly rate of \$50.



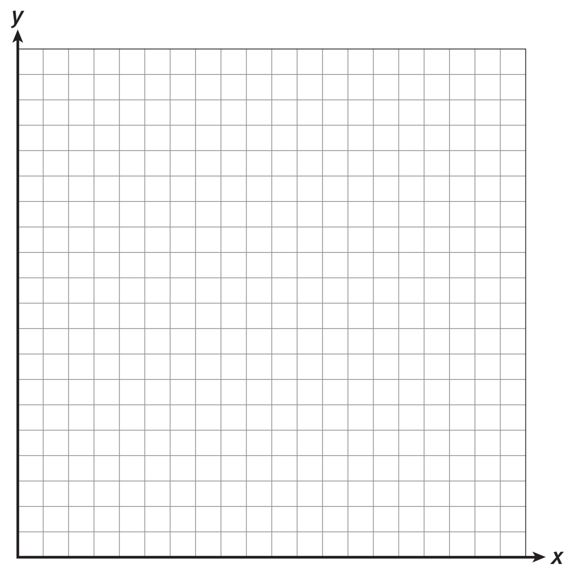
Check Your Understanding

Sketch a graph for the functions described in Items 3 and 4.

3. A fighter jet appears to be descending at a steady rate. The altitude of the jet is a function of time.



4. The value of y decreases as the value of x increases.



5. Sketch a pair of graphs, one representing a linear function and one representing a non-linear function.

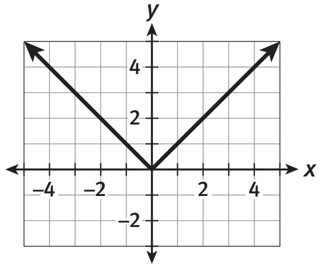
My Notes

My Notes

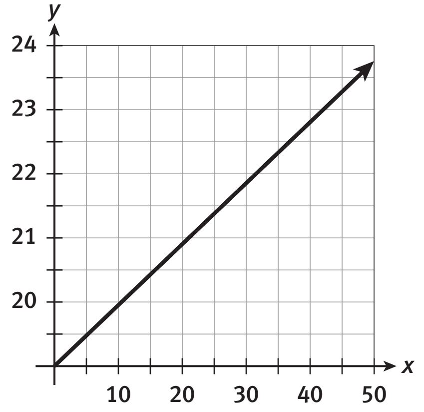
LESSON 30-2 PRACTICE

6. Which graph is linear? Explain your reasoning.

A.



B.



7. The average cost of a computer can be represented by a linear function with a rate of change of $700 per year. Which equation could represent this function? Explain your reasoning.

A. $y = 700x - 30$ **B.** $y = 700x^2$

8. Construct viable arguments. Which table of data represents a linear function? Explain your reasoning.

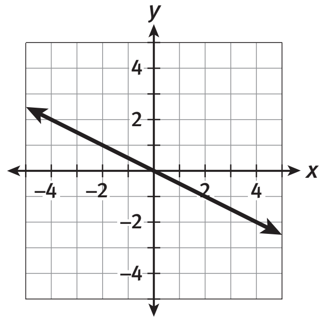
Months	Number of Books purchased
1	10
2	15
3	18
4	28
5	29
6	17
7	21

Table 1. The number of books purchased per month.

Hours	Number of Shirts Washed
0.5	2
1	6
1.5	10
2	14
2.5	18
3	22
3.5	26

Table 2. The number of shirts washed per hour at a local laundry.

9. What can be said about the x- and y-values in this line?



10. Sketch a graph to represent each scenario:

a. The distance traveled at a constant rate increases as time passes.

b. As the x-values increase the y-value stays the same.

c. $y = -3x + 2$

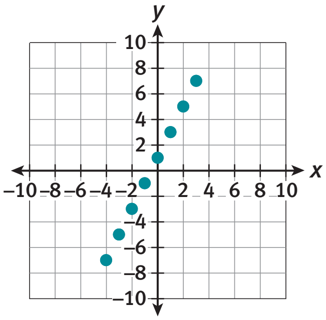
ACTIVITY 30 PRACTICE

Write your answers on notebook paper. Show your work.

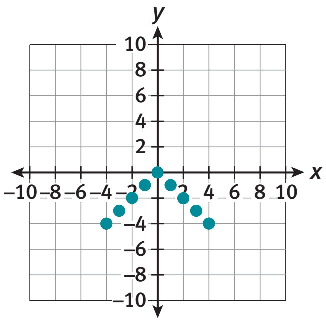
Lesson 30-1

Examine the graphs in Items 1 and 2. Describe the relationship between x and y.

1.



2.



Graph the following data sets and identify each as linear or non-linear.

3. {(2,−3), (4, −2), (−2, −5), (0, 4)}

4. {(3, 0), (2, 4), (−1, −4), (−2, 1)}

5. {(0, 5), (4,−3), (3, −1), (2, 1)}

6. {(5, −3), (7, −1), (9, 1), (11, 3)}

7. Which of the following tables display linear data? Explain your reasoning.

A.

x	y
0	45
10	40
20	35
30	30
40	35
50	40
60	45
70	50

B.

x	y
−5	−2.5
−3	−5.5
−1	−8.5
1	−11.5
3	14.5
5	17.5
7	20.5
9	23.5

8. Identify the rate of change in each equation:

a. $y = 8.3x - 1$

b. $y = 5 - 0.5x$

9. Find the rate of change in the table below:

x	y
0	0
3	90
6	180
9	270
12	360

10. The rate of change for a linear equation must be ______.

A. constant
B. negative
C. positive
D. varying

Lesson 30-2

11. Determine which of the following expressions displays a linear relationship. Use multiple representations to explain your reasoning.
A. $2x$
B. $-2x + 2$
C. $x(4x)$
D. $4 - 3x$

12. Explain how you can determine if an equation represents a linear relationship.

13. Find the rate of change for the data in each table.

a.

x	y
0	5
1	9
2	13
3	17
4	21
5	25
6	29
7	33

b.

x	y
−2	6
−1	4
0	2
1	0
2	−2
3	−4
4	−6
5	−8

14. Which equation shows a linear relationship with a rate of change of 8?
A. $y = x - 8$
B. $y = 8x - 12$
C. $y = 8 + 4x$
D. $y = \frac{1}{8}x + 16$

15. Sketch a graph for the following situations:
a. y increases as x increases
b. $y = 5x$
c. The temperature increases as the day progresses.
d. The amount of water in a tub decreases after the stopper is pulled out.

MATHEMATICAL PRACTICES
Construct Viable Arguments

16. In this activity, you explored three ways to represent linear data: in a table, graph, and with an equation. Compare and contrast the different representations of linear data. Provide examples to support your claims.

Linear and Non-Linear Functions

Measure Up

Lesson 31-1 Bean Experiment

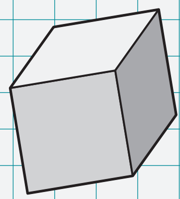
My Notes

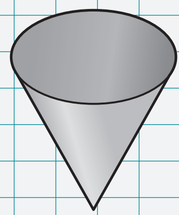
Learning Targets:

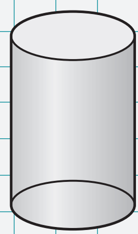
- Determine if a function is linear or non-linear.
- Represent functions with tables, graphs, and equations.
- Find a trend line to represent data.

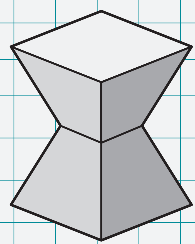
SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Use Manipulatives, Create Representations, Look for a Pattern, Visualization, Sharing and Responding, Interactive Word Wall

Using the template provided by your teacher, create the cube and cone. You will use them in an experiment to determine the volume of several solids. Before you begin the experiment, make the following predictions.









1. Predict how many groups of 20 beans you can add to the cube until it is full.

2. Predict how many groups of 20 beans you can add to the cone until it is full.

3. Explain how the shape of the figure affected your predictions.

Perform the following experiment using the cube. Each stage of the experiment consists of three steps. Each time you complete the three steps you complete a stage. Repeat the steps to complete another stage.

Step 1: Add 20 beans to the cube.

Step 2: Shake the cube gently to allow the beans to settle.

Step 3: Measure the height of the beans in centimeters.

4. Model with mathematics. Complete the experiment for the cube.

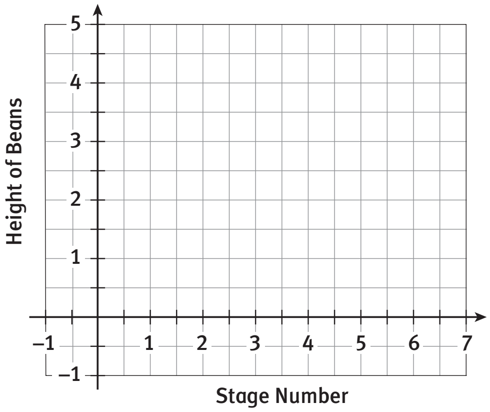
a. Enter the data in the table.

b. Describe patterns you notice in the data in the table.

Stage #	Height of Beans (cm)
1	
2	
3	
4	
5	
6	
7	

My Notes

c. Plot the ordered pairs from the table onto the graph below.



d. Looking at the graph, what do you notice about the relationship between the stage number and the height of the beans?

e. Is this data linear? Explain why or why not.

f. Does this data represent a function? Justify your answer.

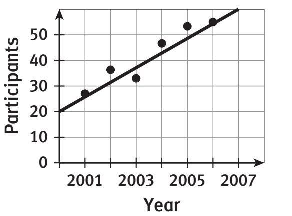
MATH TERMS

A **trend line** is a line that most closely approximates the pattern of points in a graph.

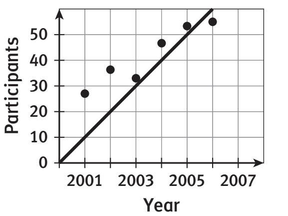
When data points are not completely linear, a ***trend line*** is used to allow us to make predictions from the data. A trend line indicates the general course, or tendency, of data.

5. Which trend line is placed correctly below? Justify your choice.

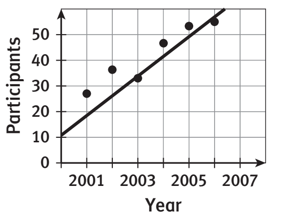
Snowboarding Competition



Snowboarding Competition



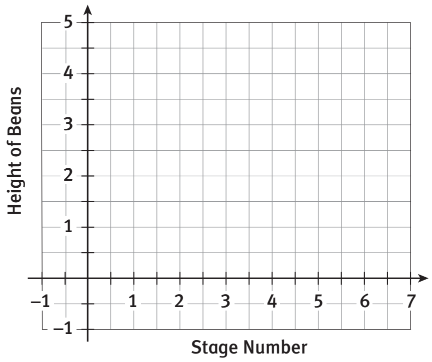
Snowboarding Competition



6. a. **Use appropriate tools strategically.** Use a straightedge and place it on the ***scatter plot*** in Item 4c in a position that has about the same number of points above and below the line. On the coordinate grid, mark two points that the line passes through. They do not have to be data points.
   b. Draw a line that passes through the two points. Does this line confirm the relationship that you noticed between the stage number and the height of the beans in Item 4d?

7. Complete the experiment for the cone.
   a. Complete the table for each stage and plot the data on the grid.

Stage #	Height of Beans (cm)
1	
2	
3	
4	
5	
6	
7	



   b. Look at the data in the table and the graph. What patterns do you notice about the relationship between the stage number and the height of the beans?
   c. Does the data for the cone represent a function? Explain why or why not.
   d. **Construct viable arguments.** Explain why a trend line cannot be drawn for this data.

My Notes

MATH TERMS

A **scatter plot** is a graph that displays bivariate data on a coordinate plane and may be used to show a relationship between two variables.

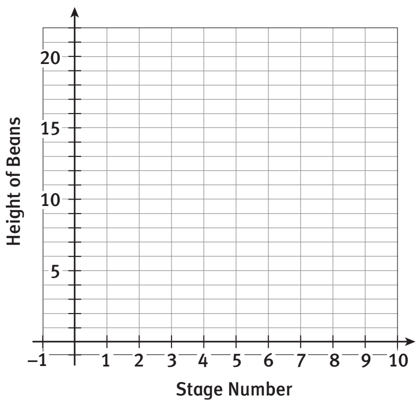
My Notes

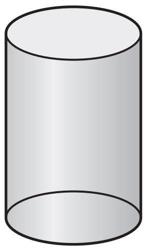
Check Your Understanding

The following data was collected as beans were added to a cylinder.

8. Plot the points on the grid for the cylinder.

Stage #	Height of Beans (cm)
1	3
2	6
3	9
4	12
5	15
6	18
7	21





9. Describe the patterns you observe in the data in the table.

10. Looking at the graph, what patterns do you notice about the relationship between the stage number and the height of the beans?

11. Is this data linear? Justify your response using ideas from your answer to Item 9.

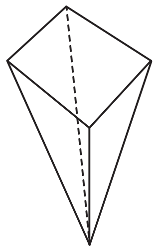
12. Can a trend line be drawn for this data? If so, draw one. If not, explain why not.

13. Make a conjecture about the rate of change of the height of the beans in the cylinder as the number of stages increases?

My Notes

Check Your Understanding

The following data was collected as beans were added to a pyramid.



Stage #	Height of Beans (cm)
1	1.5
2	2.4
3	3.1
4	3.6
5	3.9
6	4.1
7	4.2

14. What patterns do you notice about the relationship between the stage number and the height of beans?

15. Can a trend line be drawn for this data? Explain your reasoning.

16. What conjecture can you make about the rate of change of the height of beans in the pyramid as the stages increase?

LESSON 31-1 PRACTICE

Marisol fills bags of popcorn at the local movie theater. She counted the number of scoops it takes to fill each size of bag and recorded the data in the table below.

Size of Bag	Number of Scoops
Snack	0.75
Small	2
Medium	4
Large	8
Extra Large	12

17. Draw a scatter plot to represent the popcorn data.

18. Does this data represent a function? Explain your reasoning.

My Notes

19. Is this data linear? Explain your reasoning.

Marisol also fills drinks at the movie theater. The table shows the amount of time it takes to fill each size of drink.

Size of Drink (in oz.)	Time to Fill (in seconds)
8	2.5
16	5
24	7.5
32	10
40	12.5

20. **Model with mathematics.** Draw a scatter plot to represent the drink data.

21. Is this data linear? Explain your reasoning.

22. What patterns do you notice about the relationship between the time it takes to fill a drink and the drink size?

23. What conjecture can you make about the rate of change of the time to fill each drink as the drink size increases?

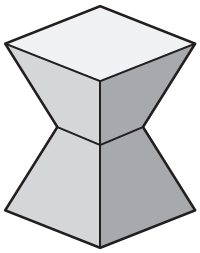
My Notes

Learning Targets:

- Define, evaluate, and compare functions.
- Recognize patterns in non-linear functions.
- Represent functions with tables, graphs, and equations.

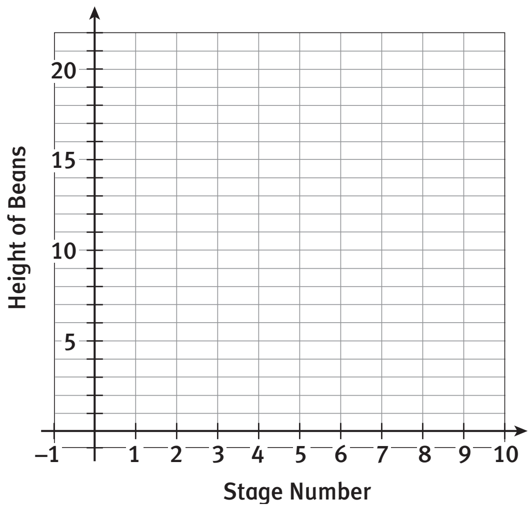
SUGGESTED LEARNING STRATEGIES: Summarizing, Use Manipulatives, Create Representations, Look for Patterns, Sharing and Responding, Note Taking, Think-Pair-Share

The following data was collected as beans were added to an irregular polyhedron.



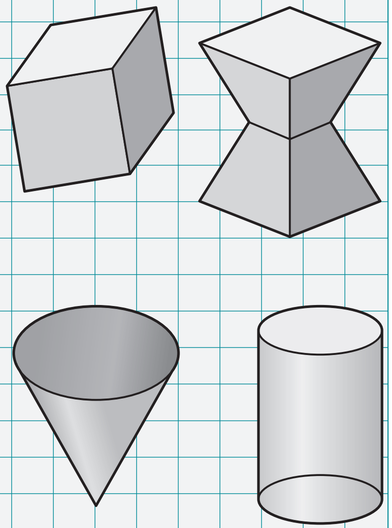
Stage #	Height of Beans (cm)
1	1
2	2
3	5
4	10
5	15
6	18
7	19

1. a. Plot the points on the grid for the irregular polyhedron.



b. Make use of structure. Describe the patterns you observe in the data in the table.

My Notes



c. Looking at the graph, what patterns do you notice about the relationship between the stage number and the height of the beans?

d. What conjecture can you make about the *rate of change* of the height of the beans as the stage numbers increase?

e. Can a trend line be drawn for this data? Why or why not?

2. How does the rate of change from the cylinder experiment differ from the rate of change for the irregular polyhedron experiment?

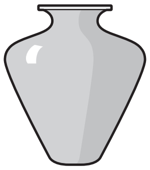
3. Construct viable arguments. Explain how the shape of the object affects the rate of change of the height of the beans for each stage of the experiment.

4. If the patterns of the data for the cylinder and the irregular polyhedron were extended indefinitely, explain how the height of the beans would change as the stage number increased.

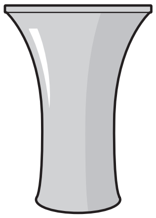
My Notes

5. Reason abstractly and quantitatively. The graphs and tables below show what happened when the bean experiment was performed with each of the vases shown. Match each vase to a graph and table. Explain the reasoning behind your choices.

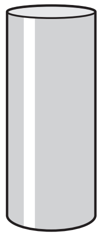
a.



b.



c.



I

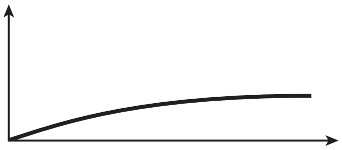
x	y
0	0
1	1
2	2
3	2.5
4	3
5	3.25
6	4
7	6

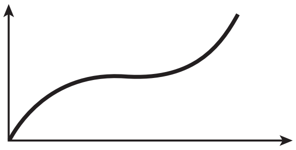
II

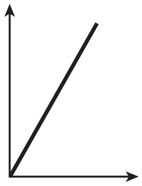
x	y
0	0
1	2
2	4
3	6
4	7
5	8
6	8.5
7	8.75

III

x	y
0	0
1	4
2	8
3	12
4	16
5	20
6	24
7	28



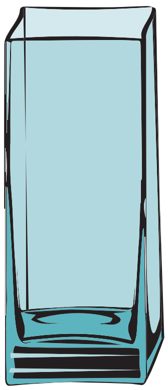




Check Your Understanding

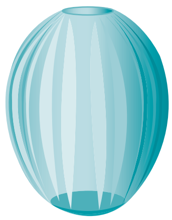
6. Sketch and describe a graph that might result from performing the bean experiment with each of the containers below.

a.



b.

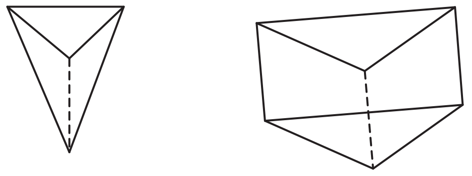
c.



My Notes

LESSON 31-2 PRACTICE

7. Which container would have a constant rate of change in the bean experiment: a triangular pyramid or a triangular prism? Explain your reasoning.



8. Sketch a graph that might result from doing the bean experiment with this container.

The table below shows the results of Hasan's bean experiment using a large can.

Stage #	Height of Beans (cm)
1	2
2	4
3	6
4	8
5	10
6	12
7	14

Use the table to answer Items 9–12.

9. Draw a scatter plot to represent Hasan's bean experiment data.

10. What is the rate of change of the height of beans per stage?

11. Write a function to represent the data in the table. Be sure to define the variables you use.

12. **Reason quantitatively.** Use your equation to predict the height of beans at the twentieth stage.

My Notes

Learning Targets:

- Recognize the relationship between verbal descriptions and graphs of linear and non-linear functions.
- Use a trend line to make predictions.

SUGGESTED LEARNING STRATEGIES: Predict and Confirm, Create Representations, Look for Patterns, Visualization, Note Taking, Think-Pair-Share

The astronauts who walked on the moon thirty years ago actually *weighed less* on the moon than they did on Earth. They did not diet or exercise to lose weight. So why did they weigh less on the moon?

Another experiment you can do involves weight. Begin by finding out how a scale works. Create a scale using the following supplies: a small paper cup, a ruler, string, tape, and part of a spring.

1. Set up the scale by doing the following:

a. Poke three holes around the top of the cup.

b. Tie the string through the three holes in the cup.

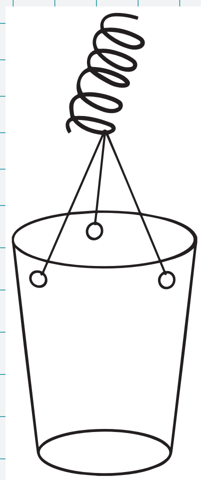
c. Tie the ends of the string to one end of the spring.

d. Tape the other end of the spring to the bottom of a desk or table so that the entire device hangs freely.

e. **Attend to precision.** Measure the distance from the floor to the bottom of the cup and put it in the table below. This is the starting weight for the empty cup.

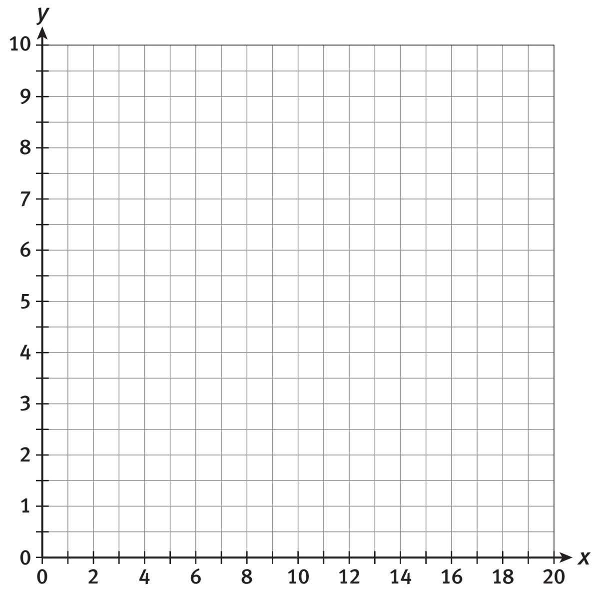
f. Place the weights provided from your teacher into the cup one at a time. After each cube is placed into the cup, measure the distance from the floor to the cup again and record these distances in the table below.

Number of Cubes	0	1	2	3	4	5
Distance from Floor (cm)						



My Notes

2. a. Graph the data as a scatter plot. Be sure to label the axes and title your graph.



b. Draw a trend line for the data in the graph.

3. a. Write the equation for the trend line describing the function between the number of cubes in the cup and the cup's distance from the floor. Define the variables you use.

b. What is the y-intercept in your equation, and what is its meaning in this situation?

c. What is the rate of change in your equation, and what is its meaning in this situation?

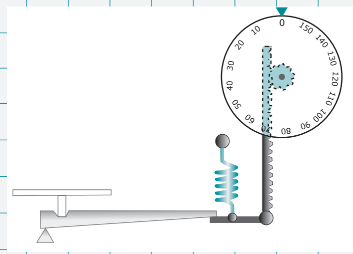
4. a. Use your equation to predict the cup's distance to the floor when 12 cubes are in the cup.

b. Use your equation to predict how many cubes it would take for your cup to reach the floor.

5. Use your trend line to confirm your answers to Item 4. Explain why the trend line could be used to confirm these values.

The scale you made works because it uses the same principle as a bathroom spring scale. This principle is known as Hooke's Law. Hooke's law says that the opposing force of a spring is **directly proportional** to the amount by which the spring is stretched.

In the illustration at the right, you can see the parts of a bathroom spring scale. Your weight pushes down on the lever, the lever pushes down on the spring contraption, and the spring contraption makes the gear turn which in turn makes the dial turn to show your correct weight.



6. Consider what you know about the moon and its gravity and how a scale works. Why might the astronauts have weighed less on the moon than they do on Earth?

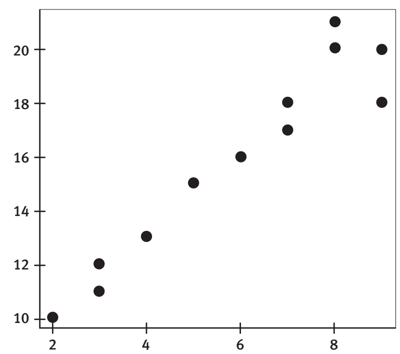
CONNECT TO PHYSICS

A ***contraption*** is a mechanical device or gadget.
Hooke's Law is named after the seventeenth century British physicist Robert Hooke. The law states that applied force, F, equals a constant, k, times the change in length, x : $F = kx$.

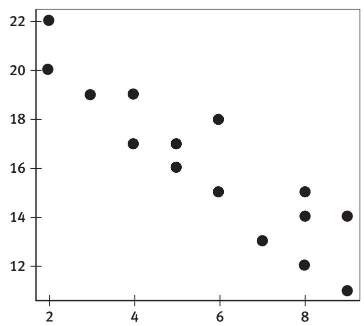
Here are the graphs for some other contraptions.

7. Reason abstractly. Add a trend line and describe in words the relationship between the two quantities in the scatter plots represented below.

a.



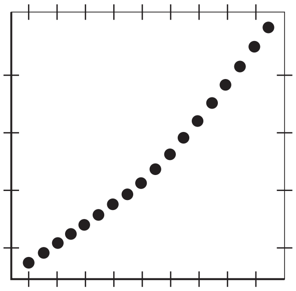
b.



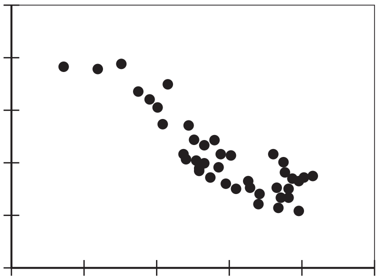
My Notes

Some graphs do not represent linear data; however, the relationship between the quantities can still be described.

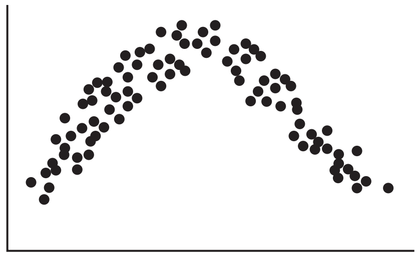
8. **Make sense of problems.** Match each scatter plot below with the situation that best describes it.



The weight of the cup increases as the amount of liquid poured into it increases.



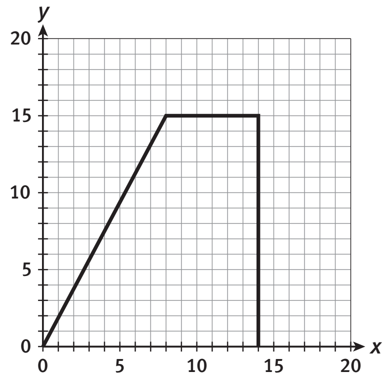
The weight of the liquid decreases at very high or very low temperatures.



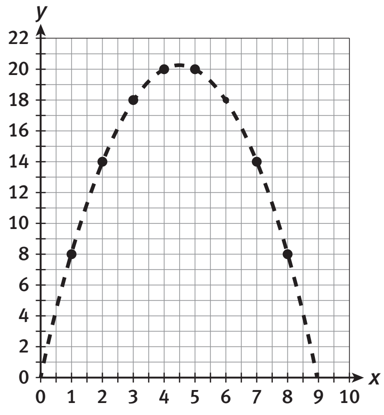
The weight of the cup decreases as the amount of liquid taken out of it increases.

9. Describe a situation that these graphs could represent. Share your descriptions with your group. Compare and contrast your situation choices with those of other group members.

a.



b.



Check Your Understanding

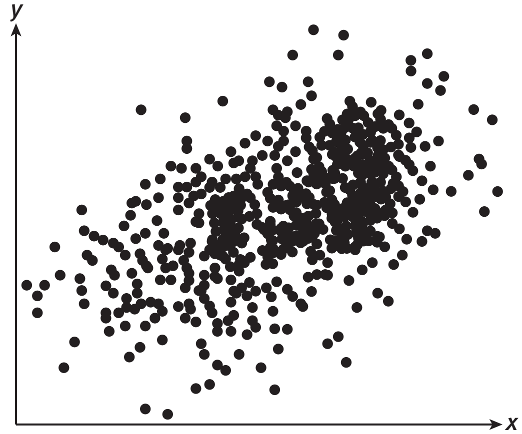
Sketch a graph that could represent each situation.

10. **a.** The amount of liquid a container will hold increases as the volume increases until it reaches the limit the container will hold.

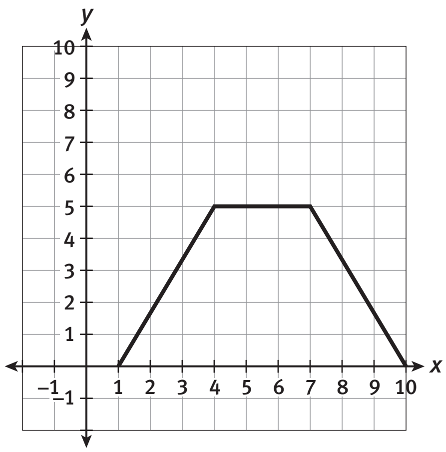
b. The number of times a frog croaks increases as the temperature rises until it reaches 96°F, and then the temperature stays constant.

11. Describe a situation that each graph could represent.

a.



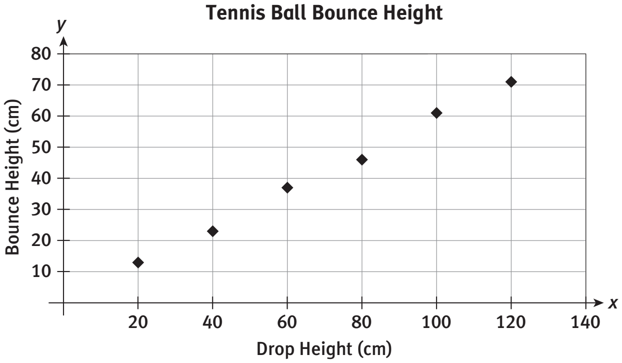
b.



LESSON 31-3 PRACTICE

Stella measured how high a tennis ball bounced after dropping it from different heights. The data she collected is shown in the scatter plot below.

Tennis Ball Bounce Height



Use the scatter plot to answer Items 12–15.

12. Draw a trend line for the data in the scatter plot.

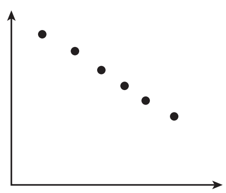
13. Write an equation for the trend line to describe the function between the drop height and the bounce height of the tennis ball. Be sure to define the variables.

14. **Critique the reasoning of others.** Stella says that the relationship between drop height and bounce height is directly proportional. Do you agree with her? Explain your reasoning.

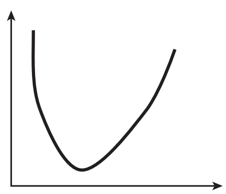
15. Use your equation to predict the bounce height at a drop height of 200 cm.

16. Describe a situation that each graph could represent:

a.



b.



ACTIVITY 31 PRACTICE

Write your answers on notebook paper.
Show your work.

Lesson 31-1

Shaina is filling a water tank for her horse, Buddy. She knows that the hose leading to the tank has an output of 2.25 gallons per minute.

1. Complete the table showing the amount of water in the tank over the first 6 minutes.

Time (minutes)	1	2	3	4	5	6
Water (gallons)						

2. a. Create a scatter plot to represent the data from Item 1.
b. Is this data linear? Explain why or why not.
c. Draw a trend line for the data.
d. Use your trend line to predict the amount of water in the tank after 8 minutes.

3. a. Write a function to represent this situation. Be sure to define the variables.
b. Use your function to determine the amount of water in the tank after 8 minutes.
c. Does your answer to part b confirm your prediction in Item 2d?

Shaina's friend Brittany has a puppy. Brittany buys dog food in large bags and empties them into smaller containers to store the food in her house. The table below shows the level of the dog food in a smaller container as she pours the dog food into it. Use this table to answer Items 4 and 5.

Time (seconds)	Food Level (inches)
0	0
1	$1\frac{3}{4}$
2	$3\frac{1}{4}$
3	$4\frac{1}{2}$
4	$6\frac{1}{2}$
5	8

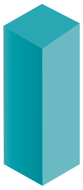
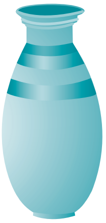
4. Is this data linear? Explain why or why not.

5. a. Create a scatter plot to represent the data in the table.
b. Draw a trend line for the data.
c. Use your trend line to predict the amount of food in the container after 8 seconds.

Lesson 31-2

6. a. Explain how you can determine if data is linear or non-linear.
b. Give an example of data that represents a linear pattern and an example of an equation that represents a non-linear pattern.

7. Sketch a graph representing the data gathered by filling each container with small marbles.

a.

b.

c.


A chef uses rice as an ingredient in many of his dishes. The graph below shows the amount of rice left in the container after he prepares each dish.

Dishes Prepared	1	2	3	4	5
Rice Remaining in Container (cups)	21.5	18	15	11.75	8.5

8. a. Create a scatter plot to represent the data in the table.

b. Draw a trend line for the data.

c. Use your trend line to predict the amount of rice in the container after seven dishes were prepared.

Lesson 31-3

9. Sketch a graph for the following situations.

a. A family left from their home to go on vacation. After several miles, they realized that they had forgotten their camera. They drove back to their home, and then drove away again.

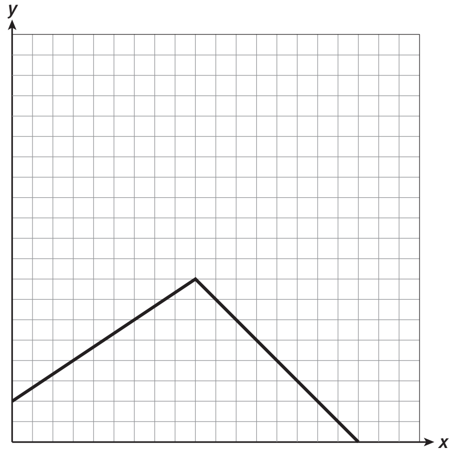
b. As *x* increases, *y* increases and then decreases.

c. As *x* increases, *y* decreases and then increases.

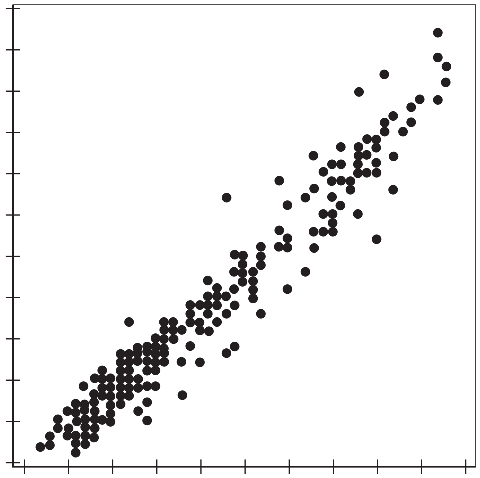
d. As the temperature of the water increases, the steam it produces increases.

10. Create a situation for each graph below.

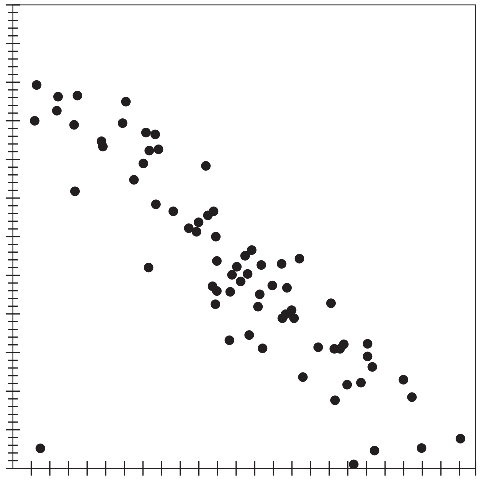
a.



b.

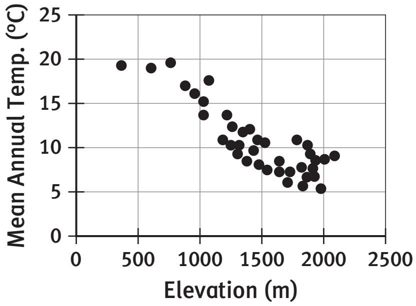


c.

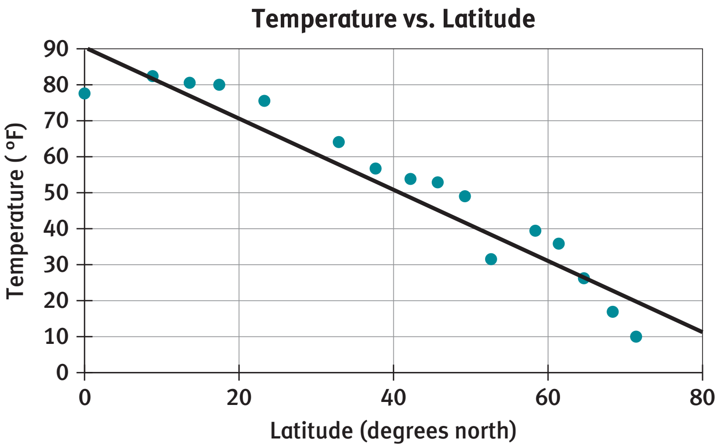


11. Reason abstractly. Explain what you can tell about a situation by looking at the graph that represents it. Use appropriate mathematical vocabulary to communicate your ideas precisely.

This graph shows the relationship between the elevation of a location and its mean annual temperature.

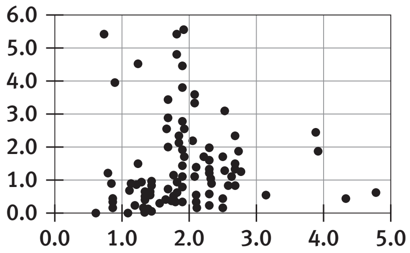


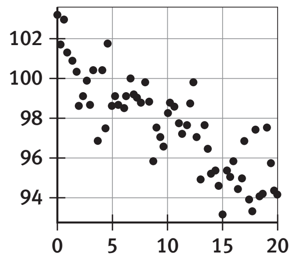
1. Draw a trend line for the graph above.
2. Use the trend line to determine the mean annual temperature for a location that has an elevation of 1,000 meters.
3. Use the trend line to predict the elevation of a location with a mean annual temperature of 10°C.
4. What relationship between latitude and temperature is suggested by the graph below?



5. Sketch a graph showing the relationship between the height of a mountain and annual number of days of snow on the ground if the number of days of snow on the ground increases by 10 days for every 1,000 feet of elevation.

6. Which scatter plot below suggests a linear relationship? Explain your choice.

A.


B.


7. Which equation below will produce a linear graph? Justify your choice.
A. $a = s^2$ **B.** $y = 2.3x + 10$

The data in the table below shows the latitudes of several locations and the number of daylight hours recorded for each location on December 21, 2004.

City	Latitude	Number of Daylight Hours
Portland, OR	45°N	7
San Francisco, CA	37°N	8
Anchorage, AK	61°N	0
Jacksonville, FL	30°N	9
Santo Domingo, Dominican Rep.	18°N	11
Charlotte, NC	35°N	10
Sioux Falls, SD	43°N	9
Calgary, Alberta, Canada	51°N	8
Mexico City, Mexico	19°N	11
Quito, Ecuador	0°	12

8. a. Create a scatter plot to represent the data.
b. Draw a trend line for the data.
c. Use the trend line to predict the number of hours of daylight at a location with latitude of 9°N.
d. Write an equation for this situation, and use the equation to confirm the prediction made in part c.

Scatter Plots and Trend Lines

GEOGRAPHICALLY SPEAKING

Scoring Guide	Exemplary	Proficient	Emerging	Incomplete
	The solution demonstrates these characteristics:			
Mathematics Knowledge and Thinking (Items 2, 3, 4, 5, 6, 7, 8c-d)	• Clear and accurate understanding of linear relationships, scatter plots, and trend lines.	• A functional understanding of linear relationships, scatter plots, and trend lines.	• Partial understanding of linear relationships, scatter plots, and trend lines..	• Inaccurate or incomplete understanding of linear relationships, scatter plots, and trend lines.
Problem Solving (Items 8c-d)	• An appropriate and efficient strategy that results in a correct answer. • Correct checking of a prediction.	• A strategy that may include unnecessary steps but results in a correct answer.	• A strategy that results in some incorrect answers. • Partial checking of a prediction.	• No clear strategy when solving problems. • No understanding of checking a prediction.
Mathematical Modeling / Representations (Items 1, 2, 3, 4, 5, 6, 7, 8a-d)	• Clear and accurate understanding of plotting data, drawing a trend line, and writing an equation from a trend line.	• Largely correct plotting of data, drawing a trend line, and writing an equation from a trend line.	• Partial understanding of plotting data, drawing a trend line, and writing an equation from a trend line.	• Inaccurate or incomplete understanding of plotting data, drawing a trend line, or writing an equation.
Reasoning and Communication (Items 4, 5a-c, 6a, 6c)	• Precise use of appropriate math terms and language to characterize a relationship from a scatter plot or trend line. • Making clear and accurate predictions from a graph.	• Correct characterization of a relationship from a scatter plot or trend line. • Making reasonable predictions from a graph.	• Misleading or confusing characterization of a relationship from a scatter plot or trend line. • Making partially correct predictions from a graph.	• Incomplete or inaccurate characterization of a relationship from a scatter plot or trend line. • Making incomplete or inaccurate predictions from a graph.